System Level Performance Evaluation of Dynamic Relays in Cellular Networks over Nakagami-\(m\) Fading Channels

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Abstract—The performance of dynamic relays in different types of cellular networks is investigated under the presence of inter-cell interference (ICI). In particular, the gains of dynamic relaying are assessed in different cellular environments which are accurately modeled with the aid of the Nakagami-\(m\) distribution. For the system under consideration, mobile stations (MSs) can relay signals intended for other MSs. Assuming the triangular relaying model, the best relay partner for each target MS is identified and utilized only if it provides gains over the non-relay assisted transmission. The considered channel model includes path-loss and small-scale fading with different fading statistics. It is shown that the gain in terms of average system capacity and probability of outage when dynamic relays are employed increases as the number of MSs in the cell grows. Furthermore, it turns out that the gains from utilizing dynamic relays become larger as the experienced fading becomes more severe. Therefore, dynamic relays can boost performance of cellular systems plagued by severe fading.

I. INTRODUCTION

It is well known that cooperative communications can exploit the spatial diversity inherent in multiuser systems offering increased capacity, fairness and coverage, under several resource constraints [1], [2]. However, the utilization of either static or dynamic relay stations (RSs) in a cellular environment still remains a challenging task due to power limitations and high implementation complexity [3], [4]. Utilization of dynamic RSs is acknowledged to be more cost efficient as relay nodes are not elements of the network infrastructure but user terminals which can relay signals intended for other users [5]. Their topology changes in time as users move and on the one hand this hardens the process of relay selection. On the other hand, users’ mobility provides a significant advantage as multi-user diversity can be exploited for increasing relaying efficiency and system performance [6].

A versatile statistical distribution which accurately models a variety of cellular environments, e.g. microcellular or macrocellular, is the Nakagami-\(m\) distribution [7]. It describes the small-scale fading process under different line-of-sight (LOS) conditions and directions of arrival (DOA) of the incoming signals [8], [9]. Recently, the performance of cooperative networks over Nakagami-\(m\) fading channels has been investigated (e.g. see [10], [11] and references therein). In [11], the bit error rate probability of a system utilizing Amplify-and-Forward (AF) relaying has been investigated over Nakagami-\(m\) fading channels. The outage probability performance has been studied in [10] for Decode-and-Forward (DF) relaying and Nakagami-\(m\) fading. Neither of the aforementioned contributions examines the impact of the cellular environment type on relaying and both works assume that multiple relays transmit to a single destination employing repetition diversity. However, transmission from multiple relays in a repetitive fashion incurs a significant spectral efficiency loss [12]. Moreover, selection of more than one relay node in a real system becomes a very complex problem [4].

The motivation of the present contribution is to investigate the performance of dynamic relaying in different cellular environments so that to apprehend under which conditions relaying is more beneficial. Cellular systems employing the triangular relaying model are considered, and for a specific target user at most one relay partner is selected. The considered performance metrics are the average achievable rate of the system or the outage probability of the system depending on whether the employed relay selection scheme is proactive or reactive. In the former, relay selection is performed before the transmission of the source whereas in the latter it takes place after the source transmission. To the best of our knowledge, such evaluation of dynamic relaying in different types of cellular systems prone to inter-cell interference (ICI) has not been pursued. The different cellular environments are modeled by adjusting the fading parameter of the Nakagami-\(m\) distribution. This parameter determines the severity of small-scale fading of the base station (BS) to mobile station (MS) (BS-MS), MS-MS, and ICI channels. Interestingly, the more severe the fading is, the greater the gains of proactive relaying are. Thus, dynamic relaying is a promising solution for boosting the achievable rate in systems plagued by fading.

The paper is structured as follows. In Sections II and III,
the system and the considered channel model are presented, respectively. In Section IV, the employed relay selection schemes are detailed and in Section V, numerical results are presented and discussed. Section VI concludes the paper.

II. SYSTEM MODEL

We consider a cellular network consisting of two tiers of cells, where $L$ is the total number of cells. BSs are located in the cell center and each cell contains $K$ single antenna MSs which are uniformly distributed in the cell area. It is assumed that all BSs have one omni-directional antenna and they communicate on the same frequency (full frequency reuse). Downlink communication is taken into account although our consideration is equally valid for the uplink.

A. Non-relay-assisted communication

In the downlink, when the $s$-th BS transmits directly to the $d$-th user the mutual information between the source and the destination user and the considered channel model are presented, respectively. In Section IV, the employed relay selection

where $h_{s,d}$ is the channel coefficient between the source and the destination user and $S$ is the transmit power of the source. In the above equation, $N_d = \mathbb{E} \left[ \sum_{j=1}^{L} |h_{j,d}|^2 p_j \right] + \sigma^2$ is the noise plus average ICI power received by the destination where $\mathbb{E}[.]$ denotes the expectation operator, $p_j$ is the transmit power of the $j$-th BS, and $\sigma^2$ is the variance of the zero mean circularly symmetric additive Gaussian noise. The probability of outage (OP) for a given transmit rate $R$ is

\[
P_{\text{out}}^{\text{d}} = \Pr \left[ I_D^d < R \right] = \Pr \left[ |h_{s,d}|^2 < \frac{2R - 1}{S/N_d} \right].
\]

B. Relay-assisted communication

Transmission in cellular systems can be enhanced by permitting user terminals to relay signals intended for other users [2]. Let $\mathcal{G}$ be the set comprising all users in our cell of interest. We assume that the transmission to a specific target user $d \in \mathcal{G}$ can be assisted at most by another user $r \in \mathcal{G}$, $r \neq d$, of the cell which acts as a RS. We take into account half-duplex relaying where the RSs cannot receive and transmit concurrently on the same resource (time or frequency). This is a valid assumption when cell users become RSs as user terminals are subject to hardware complexity constraints; therefore, it is hard to incorporate full-duplex capabilities permitting RSs to transmit and receive on the same resource simultaneously. When relaying is enabled, transmission takes place in two time slots (dual-hop transmission). In the first slot, the source transmits to RS $r$ and to destination $d$. In the next slot, the RS $r$ transmits to destination $d$, while the source remains silent (orthogonal transmission). We also assume that the receivers at the destination node and at the RSs possess perfect channel state information (CSI) so that maximum-likelihood combining is employed. The relaying protocol considered is the DF one as it has been shown to attain greater gains than AF in the cellular context [2],[5]. More specifically, the orthogonal version of the DF protocol (ODF) is taken into account, where the source and the relay node do not transmit simultaneously. It has been observed in [5] that non-orthogonal DF does not provide any significant capacity gains compared to ODF when the number of RS candidates is sufficiently large.

We assume equal power allocation and that the power stemming out of a cell in each time slot is constrained to $P$. Therefore both the source and the RS transmit with power $S \leq P$. Hence, the mutual information between the source, i.e. the $s$-th BS, and relay $r \in \mathcal{G}$ (first hop of the transmission), is given by

\[
I_{s,r} = \frac{1}{2} \log_2 \left[ 1 + S \left( \frac{|h_{s,r}|^2}{N_r} \right) \right].
\]

\[
I_{s,r,d} = \frac{1}{2} \log_2 \left[ 1 + S \left( \frac{|h_{s,d}|^2 + |h_{r,d}|^2}{N_d} \right) \right].
\]

Using (3) and (4), the end-to-end mutual information of the ODF scheme with relay $r$ is given by

\[
I_{s,r,d}^{\text{ODF}} = \min \{ I_{s,r}, I_{s,r,d} \}
\]

as relay $r$ has to decode the source message. Therefore, if the source has the relevant CSI, it can adapt its rate to $I_{s,r,d}^{\text{ODF}}$. If the source does not possess neither the source-RS nor the RS-destination CSI and transmits at a constant rate $R$, the probability $A_r$ that a relay $r$ does not decode the source signal is given by

\[
A_r = \Pr \left[ I_{s,r} < R \right] = \Pr \left[ |h_{s,r}|^2 < \frac{2R - 1}{S/N_r} \right].
\]

Clearly, the probability that relay $r$ decodes its received signal is $1 - A_r$. In this case, the mutual information between the relay $r$ and destination $d$ is

\[
I_{r,d} = \frac{1}{2} \log_2 \left[ 1 + S \left( \frac{|h_{r,d}|^2}{N_d} \right) \right].
\]

\[\text{[Unless indicated otherwise, the indices } s, d \text{ take values } 1, 2, \cdots, L \text{ and } 1, 2, \cdots, K, \text{ respectively.}\]
where $h_{r,d}$ is the channel coefficient between the relay $r$ and the destination $d$. It must be noted that the $N_d$ at the destination is the same whether or not relay-assisted transmission takes place. Moreover, it is assumed that $N_d$ remains the same during the two times slots of transmission. The end-to-end OP between the source $s$ and the destination user $d$ through the decoding relay $r$, when source transmits with constant rate $R$, is

$$P_{out}^s = \Pr \left[ |h_{r,d}|^2 < \frac{2^{2R} - 1}{S/N_d} \right].$$

### III. CHANNEL MODEL

A generic flat fading channel model that includes antenna power gain, path-loss (PL), local scattering, and fast fading with different fading statistics is considered. In particular, the channel coefficient between the $k$-th and the $\ell$-th node, $k, \ell = 1, 2, \ldots, L + K$ (a node can be either a BS or a MS), of the network is given by

$$h_{k,\ell} = |h_{k,\ell}| \exp(i \phi_{k,\ell}) \sqrt{G \beta d_{k,\ell}^\mu}$$

where $|h_{k,\ell}|$ is the fading envelope and $\phi_{k,\ell}$ is the random phase of the channel between the aforementioned nodes that is assumed to be uniformly distributed over the range $[0, 2\pi]$. Moreover, $G$ is the antenna power gain of the radiating node, $d_{k,\ell}$ is the distance between the $k$-th and the $\ell$-th node, $\mu$ is the PL exponent, and $\beta$ is the PL constant. For the BS-MS channels, $G$ is assumed to be 9 dB (gain on the elevation), while for the MS-MS channels, $G$ is assumed to be 0 dB (Long Term evolution (LTE) evaluation parameters). Furthermore, for the path-loss, the 3GPP LTE PL model has been used (dB scale)

$$PL_{k,\ell}^{(dB)} = 148.1 + 37.6 \log_{10} \left[ d_{k,\ell} \right]$$

where $d_{k,\ell}$ is in kilometers.

The fading envelopes $|h_{k,\ell}|$’s, are assumed independent, not necessarily identically distributed (INID) Nakagami-$m$ random variables (RVs) with marginal probability density functions (PDFs) [7, eq. (22)]

$$f_{|h_{k,\ell}|}(x) = \frac{2 x^{m_{k,\ell} - 1}}{\Gamma(m_{k,\ell})} \Omega_{k,\ell}^{m_{k,\ell}} \exp \left( -\frac{x^2}{\Omega_{k,\ell}} \right), \quad x \geq 0$$

where $m_{k,\ell} \geq 1/2$ is the fading parameter, $\Gamma(\cdot)$ is the Gamma function [13, eq. (8.310)], and $\Omega_{k,\ell} = \mathbb{E} \left[ |h_{k,\ell}|^2 \right]/m_{k,\ell}$ is a parameter related to the average fading power.

The Nakagami-$m$ distribution is a versatile statistical distribution that describes multipath scattering with relatively large delay-time spreads and with different clusters of reflected waves [14]. The PDF of (11) is very general as it can describe other well-known distributions, e.g. for $m_{k,\ell} = 1$ the Rayleigh and for $m_{k,\ell} = 0.5$ the one-sided exponential distribution. Moreover, it can be used to model the Rician distribution with sufficient accuracy by setting [15]

$$m_{k,\ell} = \left[ 1 - \left( \frac{K_{k,\ell}}{K_{k,\ell} + 1} \right)^2 \right]^{-1}$$

where $K_{k,\ell}$ denotes the Rice factor, i.e. the ratio of the average direct power over the average scattered power. Its fading parameter $m_{k,\ell}$, can describe the absence or presence of line of sight (LOS) between any $k$-th and $\ell$-th node for $m_{k,\ell} \leq 1$ and $m_{k,\ell} > 1$ respectively. Moreover, extensive measurement campaigns have shown that the relationship between a signal and its direction of arrival (DOA) can be embodied by $m_{k,\ell}$. Hence, varying degrees of fast fading and local scattering can be approximated for any BS-MS and MS-MS channel with the correct choice of $m_{k,\ell}$’s, leading to accurate modeling of different cellular channel conditions. For example, in macrocells where the cell radius is usually $2-20$ km and the antenna radiating power is in the order of $0.6-10$ W from high towers, LOS is usually blocked $(0.5 \leq m_{k,\ell} \leq 1$ for every BS-MS channel) [9]. The same rule applies with high probability to MS-MS channels. In microcells, the antenna height is a few meters, the radiating power is less than $20$ mW and the cell radius is $0.4-2$ km. In such systems, there usually exist some BS-MS and/or MS-MS channels with $m_{k,\ell} \geq 1$. For both aforementioned environments, ICI channels are usually non-LOS (NLOS) ones, i.e. for any node $i$ the $m_{\text{ICl},i}$ parameter of its ICI channel is $0.5 \leq m_{\text{ICl},i} \leq 1$.

### IV. RELAY SELECTION SCHEMES

Two relay selection schemes are considered, a proactive and a reactive one. In the proactive scheme, it is assumed that the source, i.e., the BS, possesses all the relevant system CSI and the relay node is selected before the transmission of the source. The source selects the relay that maximizes mutual information and transmits at a rate equal to this mutual information without any outage probability. Thus, the employed evaluation metric for the proactive scheme with CSI is the maximum attained mutual information (achievable capacity). In the reactive scheme, relay selection is performed after the transmission of the source. In the latter case, source transmits at a constant rate and the best relay node is selected amongst the ones that have decoded the source message. As the source lacks CSI before transmission, the OP is our considered metric for this scheme.

#### A. Proactive relay selection with CSI

The BS gathers all the BS-MS and MS-destination channel coefficients (perfect CSI) as well as the average ICI received by each MS of the system. The best relay partner $r_d$, for destination $d$, is

$$r_d = \arg \max_{r \in \mathcal{P}, r \neq d} T_{d,r,d}^{\text{DF}}.$$  

Transmission towards a destination user $d$ through its best relay partner $r_d$ might not be the best strategy; the direct transmission from the BS without the intervention of any relay partner might be preferable. One reason for this is that half-duplex relaying incurs a pre-log penalty of $\frac{1}{2}$ at the capacity expression. Therefore, the BS not only selects the $r_d$ for a specific transmission, but also decides whether to employ it or transmit directly to the destination $d$. Hence, the final
mutual information between $s$ and $d$ is given by the following expression

$$T_{final}^d = \max \left\{ I_{s,r,d}^O, I_{D_D}^D \right\}. \tag{14}$$

In order to guarantee system fairness, users are considered to be served in a round-robin fashion. The considered performance metric for the proactive scheme is the average system capacity (ASC)

$$\bar{C} = E \left[ T_{final}^d \right]. \tag{15}$$

B. Reactive relay selection

The BS transmits at a constant rate $R$ towards a destination $d$. The relays that decode the source message form the set $\mathcal{C} \subseteq \mathcal{G}$, $d \notin \mathcal{G}$. These relays together with the destination feed back to the BS the CSI between them and destination $d$, so that the relay selection takes place. The best relay partner $r_d$, for destination $d$, is the one minimizing the end-to-end OP

$$r_d = \arg \max_{r \in \mathcal{C}, r \neq d} T_{r,d}^D = \arg \min_{r \in \mathcal{C}, r \neq d} P_{out}^d. \tag{16}$$

Hence, the final OP between $s$ and $d$ is

$$P_{out}^d = \min \left\{ P_{out}^{D_D}, P_{out}^{r_d,d} \right\}. \tag{17}$$

Again, users are considered to be served in a round-robin fashion. The considered evaluation metric for the reactive scheme is the average OP

$$P_{out} = E \left[ P_{out}^d \right]. \tag{18}$$

It must be noted that this scheme requires that only the relay to destination channel coefficients of the nodes that have decoded the source message (nodes of $\mathcal{C} \subseteq \mathcal{G}$) are fed back to the BS. This feedback overhead is much smaller than the overhead entailed by the proactive scheme. The latter requires that both the BS-relay and the relay-destination coefficients of all the cell users (all cell users are relay candidates) are fed back to the BS.

V. Numerical Results and Discussion

The performance of the relaying schemes presented in IVIs studied over INID Nakagami-$m$ fading channels. In particular, the ASC and the OP have been evaluated for the proactive and the reactive relaying schemes. A two-tier cell network (19 BSs overall with radius of 2 km) has been considered with the central cell being our cell of interest as it captures well the effect of ICI. Various fading conditions have been assumed modeling different macrocellular and microcellular environments. An important parameter defining the transmit power of the BSs is the System SNR which is the average SNR experienced at the edge of the cell without counting ICI. All plots have been drawn for a System SNR of 20 dB (ICI limited regime).

In Fig. 1, $\overline{C}$ is plotted versus the number of cell users $K$ for the proactive scheme and for two different types of fading environments, fading with NLOS and fading with LOS. For the former case, three different NLOS fading environments are considered: i) a bad urban environment, where BS-MS and MS-MS channels as well as ICI are subject to Rayleigh fading, i.e. $m_{k,\ell}, m_{\text{ICI}}, = 1, \forall k, \ell$, ii) a macrocellular one, where BS-MS ($D$ channels) and MS-MS channels as well as ICI are subject to Nakagami-$m$ fading with $0.5 \leq m_{k,\ell}, m_{\text{ICI}}, \leq 1$, and iii) an environment with severe fading, where BS-MS and MS-MS channels as well as ICI channels experience exponential fading, i.e. $m_{k,\ell}, m_{\text{ICI}}, = 0.5, \forall k, \ell$. As depicted in this figure, for all NLOS fading conditions under consideration, $\overline{C}$ increases as $K$ increases and for large $K$ all $\overline{C}$ curves
The utilization of dynamic relays in cellular systems (user terminals act as relays) is of particular interest as it is cost effective and it can leverage multi-user diversity. In this contribution, the capacity and OP performance of different types of cellular networks employing dynamic relays has been assessed under the presence of ICI. The widely applicable Nakagami-$m$ distribution has been employed in order to capture the fading behavior of different cellular environments such as macrocells or microcells. A channel model that includes path-loss and Nakagami-$m$ small-scale fading with different fading statistics has been considered. It has been shown that while employing a proactive scheme, the capacity gain becomes larger as fading conditions become more severe. The opposite trend has been observed for the OP and reactive relaying. Thus, the utilization of dynamic relays is an effective solution to tackle severe fading and boost system performance. In the future, we aim at performing theoretical analysis in order to complement our numerical results.

VI. CONCLUSIONS AND FUTURE WORK

The utilization of dynamic relays in cellular systems (user terminals act as relays) is of particular interest as it is cost effective and it can leverage multi-user diversity. In this contribution, the capacity and OP performance of different types of cellular networks employing dynamic relays has been assessed under the presence of ICI. The widely applicable Nakagami-$m$ distribution has been employed in order to capture the fading behavior of different cellular environments such as macrocells or microcells. A channel model that includes path-loss and Nakagami-$m$ small-scale fading with different fading statistics has been considered. It has been shown that while employing a proactive scheme, the capacity gain becomes larger as fading conditions become more severe. The opposite trend has been observed for the OP and reactive relaying. Thus, the utilization of dynamic relays is an effective solution to tackle severe fading and boost system performance. In the future, we aim at performing theoretical analysis in order to complement our numerical results.

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