Decentralized Control System Design for MIMO Processes with Integrators/Differentiators

Wuhua Hu, Wen-Jian Cai *, Gaoxi Xiao

School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798, Singapore

Abstract:
This paper extends the concept of relative normalized gain array (RNGA) and proposes a systematic approach to designing decentralized PID control for multi-input multi-output (MIMO) processes containing integrators and/or differentiators. By determining the input-output-pairing using relative gain array (RGA)-Nederlinski index (NI)-RNGA criterion, equivalent transfer functions (ETFs) for the selected input-output pairs are derived by utilizing the RGA and RNGA information, which have incorporated the information of loop interactions. Based on the ETFs, decentralized PID controllers are tuned to stabilize the MIMO system independently. The proposed approach is simple, easy to be understood and implemented by field engineers. Three industrial MIMO processes with different dimensions and interaction modes are employed to demonstrate the efficiency of the proposed approach.

Keywords: RNGA, MIMO, integrator, differentiator, input-output pairing, ETF, decentralized control, PID control.

1. Introduction

Integrating processes are encountered in chemical plants not only in the control of level loops but also in the control of processes with very large time constants.\textsuperscript{1-6} Control of such processes for single-input single-output (SISO) systems has been extensively studied and some excellent results have been obtained.\textsuperscript{1-3} The development of effective methods, especially for decentralized control of multi-input multi-output (MIMO) systems containing integrators, is, however, still in the primary stage.\textsuperscript{2}

Among the existing work for decentralized control of MIMO processes containing integrators, Woolverton first proposed a method to use relative gain array (RGA)\textsuperscript{7} in analyzing systems with integrating variables,\textsuperscript{6} which can be used to determine the input-output pairing for decentralized control design. Later, McAvoy proved that Woolverton’s method gives the exact RGA for a specific 3×3 example.\textsuperscript{4-5} Arkun and Downs also extended Woolverton’s method to general systems containing integrators described by both transfer function matrices and state-space
models.\textsuperscript{4} Though such work provided means to compute RGA for analyzing the loop interactions and determining the input-output pairing, no control strategies were investigated. Recently, Huang, et al. proposed a new method to compute the RGA value and consequently design decentralized controllers for MIMO processes containing integrators.\textsuperscript{2} While this approach may be valid for general MIMO processes containing integrators, the controller design procedures are not straightforward.

In this paper, we propose a systematic approach to designing the decentralized controllers for MIMO processes containing integrators and/or differentiators by extending the concept of relative normalized gain array (RNGA)\textsuperscript{8} through proper factorization of the transfer function matrix. RGA-Nederlinski index (NI)-RNGA criterion\textsuperscript{8} is proposed to determine the input-output pairing which minimizes the cross loop interactions. By using the information conveyed in RNGA and RGA, an equivalent transfer function (ETF) is derived for each selected input-output pair when other loops are closed. These ETFs have properly taken the loop interactions into account such that a given MIMO process can be perceived to be decomposed into a set of SISO processes with their transfer functions represented by ETFs.\textsuperscript{9-11} Furthermore, the ETFs are modified so that the control system integrity can be maintained (i.e., the stability of the system is maintained when any of the input-output loops is taken in or out of service).\textsuperscript{9-10} Finally, PID control is tuned for each loop based on the modified ETFs. The proposed decentralized control design thus follows a systematic approach and is easy to be understood and implemented by field engineers. The usefulness of the novel approach is illustrated by several industry processes.

2. The input-output pairing

Consider an $n \times n$ system with a decentralized feedback control structure as shown in Fig. 1, where $\mathbf{r} = [r_1, r_2, \ldots, r_n]^T$, $\mathbf{u} = [u_1, u_2, \ldots, u_n]^T$ and $\mathbf{y} = [y_1, y_2, \ldots, y_n]^T$ are vectors of
references, inputs and outputs respectively; $G(s) = [g_{ij}]_{n \times n}$ is system’s transfer function matrix and $C(s) = \text{diag}\{c_1(s), c_2(s), \ldots, c_n(s)\}$ is the decentralized controller; $i, j = 1, 2, \ldots, n$ are integer indices.

2.1. Loop pairing for normal processes. Rewrite $G(s)$ as $G(s) = K \otimes \bar{G}(s)$, where $\otimes$ denotes the element-by-element multiplication, $K := [k_{ij}]_{n \times n} := G(0)$, and $\bar{G}(s) = [\bar{g}_{ij}(s)]_{n \times n}$ with $\bar{g}_{ij}(0) = 1$. Assume that the $\bar{g}_{ij}(s)$, $\forall i, j$, is open-loop stable and its output $\bar{y}_i = \bar{g}_{ij}(s)u_j$ initially rests at zero. With $u_j$ being a unit step input, the average residence time (ART) is defined as

$$\tau_{\text{ar}} := \left| \int_0^\infty (\bar{y}_i(\infty) - \bar{y}_i(t))dt \right|. \quad (1)$$

ART is closely related to the time constant of a process as referred to Table 1. The derivation of ART for a general process is given in Appendix A.

Let $T_{\text{ar}} := [\tau_{\text{ar}ij}]_{n \times n}$. The normalized gain matrix is defined as

$$K_N = K \odot T_{\text{ar}}, \quad (2)$$

where $\odot$ indicates the element-by-element division. Then the RNGA is defined as

$$A_N = K_N \otimes K_N^{-T}, \quad (3)$$

where the superscript $^{-T}$ means the transpose of the inverse of a matrix.

A special case is when $k_{ij} \bar{g}_{ij}(s) \equiv k_{ij}$, then $\tau_{\text{ar}ij} = 0$. In calculating RNGA, $\tau_{\text{ar}ij} := \varepsilon$, with $\varepsilon \to 0$ (usually set as a small constant say $\varepsilon = 10^{-6}$) is used. The ARTs and normalized gains for computing RNGAs of typical processes are explicitly obtained and summarized in Table 1. The analytical results avoid numerical computation of $\tau_{\text{ar}ij}$ in Eq. (1).

The RNGA provides reasonable information to indicate the interactions between the inputs and the outputs and is used in conjunction with RGA and NI to determine the input-output pairing.
The RGA and NI are used to eliminate any structurally unstable pairings. Given the transfer matrix \( G(s) \), the RGA (denoted by \( \Lambda \)) and NI are defined as follows\(^7\)\(^,\)\(^{12-13}\):

\[
\begin{align*}
\text{RGA:} \quad & \Lambda = K \otimes K^{-T} \\
\text{NI:} \quad & \text{NI} = \frac{\det K}{\prod_{i=1}^{n} k_{ii}} 
\end{align*}
\]

In the calculation of NI, the selected input-output pairs are reordered such that their transfer functions lie on the diagonal. The \textit{RGA-NI-RNGA criterion} requires that the inputs and outputs are paired in such a way that: i) all paired RGA elements are positive; ii) the NI is positive; iii) the paired RNGA elements are closest to 1.0; and iv) large RNGA elements are avoided.\(^8\)

\subsection*{2.2. Loop pairing for processes containing integrators/differentiators.} In Section 2.1, it is assumed that \( K \) and \( T_{ar} \) are finite and nonzero to validate the definition of RNGA in Eq. (3). If a MIMO process contains integrators, \( T_{ar} \) goes to infinity; on the other hand, if a MIMO process contains differentiators, \( K \) equals zero. For such a process, RNGA cannot be computed directly using the definition.

To deal with such processes, let a MIMO process be given by \( G(s) = [g_{ij}(s)]_{n \times n} \), where \( g_{ij}(s) = k_{ij} s^{m_{ij}} \bar{g}_{ij}(s) \), \( m_{ij} \) is an integer and \( \bar{g}_{ij}(s) \) does not contain any integrator or differentiator and satisfies \( \bar{g}_{ij}(0) = 1 \). If \( m_{ij} < 0 \), it indicates that the transfer function contains integrator(s); otherwise, the transfer function contains differentiator(s) if \( m_{ij} > 0 \); and it is a normal transfer function if \( m_{ij} = 0 \). Suppose that there are diagonal output scaling matrix \( S_1(s) \in \mathbb{R}^{n \times n} \) and diagonal input scaling matrix \( S_2(s) \in \mathbb{R}^{n \times n} \) for \( K \otimes \bar{G}(s) \), such that \( G(s) \) can be factorized as

\[
G(s) = S_1(s) \left( K \otimes \bar{G}(s) \right) S_2(s), \tag{5}
\]
where $\bar{G}(s) = [\bar{g}_i(s)]_{n \times n}$. Since RNGA is invariant to input/output scaling, the RNGA (denoted by $\Lambda_N$) for $G(s)$ is the same as that for $K \otimes \bar{G}(s)$, as expressed in Eq. (3). Once the RNGA is obtained, the RGA-NI-RNGA rules for normal processes can directly be applied to determine the input-output pairing for the MIMO process with integrators/differentiators.

The above loop pairing is limited to the class of MIMO processes that can be factorized in (5). (Similar constraint occurs in the loop pairing using the RGA analysis. 4) Indeed, many practical MIMO processes fall into this class. Typical examples will be shown in Section 5.

3. Equivalent transfer functions

Let $\Lambda = [\lambda_{ij}]_{n \times n}$ and $\Lambda_N = [\lambda_{Nij}]_{n \times n}$. In the RNGA, $\lambda_{Nij}$ is interpreted as the ratio between the normalized gain when other loops open and the normalized gain when other loops close except the $ij$th loop. 8 That is,

$$\lambda_{Nij} = k_{Nij}/\hat{k}_{Nij},$$

where $\hat{k}_{Nij}$ is explicitly expressed as

$$\hat{k}_{Nij} = \hat{k}_{ij} \tau_{arq}.$$  

Here $\hat{k}_{ij}$ and $\tau_{arq}$ are the gain and the ART of ETF respectively. Eqs. (2), (6) and (7) lead to

$$\hat{\tau}_{arq} = \gamma_{ij} \tau_{arq},$$

where $\gamma_{ij} := \lambda_{Nij}/\lambda_{ij}$, and $\lambda_{ij} = k_{ij}/\hat{k}_{ij}$ is the relative gain of loop $i$-$j$ defined as the ratio between the steady state gain when other loops open and the steady state gain when other loops close except the $ij$th loop. 7, 14

Let $\Gamma := [\gamma_{ij}]_{n \times n}$ be the relative ART array defined as
\[ \Gamma = \Lambda_N \odot \Lambda, \quad (9) \]

Then Eq. (8) can be expressed in a matrix form

\[ \hat{T}_{ar} = \Gamma \otimes T_{ar}, \quad (10) \]

Eq. (10) indicates that the ART array \( \hat{T}_{ar} \) can be calculated from the relative ART array \( \Gamma \) and the ART array \( T_{ar} \) of the open-loop transfer function.

An ETF is defined as the open-loop transfer function between an input-output pair when all other loops are closed.\(^{10-11}\) The derivation of an ETF is based on two assumptions: 1) the control is perfect that the output attains the steady state with no transient once an input is injected; and 2) the ETF has the same structure as the corresponding open-loop transfer function. Assumption 1) aims to derive an approximate ETF while avoiding the complexity in deriving a true ETF which involves specific controllers in all loops. Although there are always transients before reaching the steady states, this assumption has been widely adopted and is found to be effective for simplified control design in many MIMO processes.\(^{8-9, 14-15}\) Assumption 2) aims to make the derivation of an approximate ETF analytically tractable as a true ETF can usually be approximated by a typical process model, which does not introduce large deviations in most cases. The ETFs for typical processes are derived and summarized in Table 1.

Given \( K \otimes \bar{G}(s) \) in Eq. (5), there are two properties on its ETF of a selected input-output pair:

1. If \( \lambda_y \) is finite and nonzero, then the ETF of the \( j-i \) input-output pair is free of integrators;

2. If the sub-matrix of \( K_N, K_{N_{i,j}} \), is invertible, which is obtained from \( K_N \) by removing the \( i \)-th row and \( j \)-th column, and the ART \( \tau_{arg} \) equals zero and \( \lambda_{N_y} \) is finite and nonzero,

then the ART \( \hat{\tau}_{arg} \) of the ETF of the \( j-i \) input-output pair is also zero.
Property 1 is proposed by Huang et al.\textsuperscript{2} it provides a sufficient condition for the ETF not to contain integrators which may support the approximation of the ETF by a certain low-order model. Property 2 gives a deterministic relation between ART of an open-loop transfer function and ART of its ETF; the proof is given in Appendix B.

The derivation of an ETF for a selected input-output pair is illustrated as follows. Consider an integrating process with its normal transfer function described by a first-order plus time delay model

\begin{equation}
    g_{ij}(s) = \frac{k_{ij} e^{-\theta_{ij} s}}{s(\tau_{ij} s + 1)}.
\end{equation}

The ETF takes a similar form as

\begin{equation}
    \hat{g}_{ij}(s) = \frac{\hat{k}_{ij} e^{-\hat{\theta}_{ij} s}}{s(\hat{\tau}_{ij} s + 1)}.
\end{equation}

Since \( \tau_{ar} = \tau_{ij} + \theta_{ij} \), \( \hat{\tau}_{ar} = \gamma_{ij} \tau_{ar} = \hat{\tau}_{ij} + \hat{\theta}_{ij} \) and \( \lambda_{ij} = k_{ij} \hat{k}_{ij} \). If \( \lambda_{ij} \) is finite and nonzero, Eq. (12) becomes

\begin{equation}
    \hat{g}_{ij}(s) = \frac{k_{ij} e^{-\gamma_{ij} \theta_{ij} s}}{\lambda_{ij} s(\gamma_{ij} \tau_{ij} s + 1)}.
\end{equation}

4. Decentralized PID controller design

Since ETFs have incorporated the information of loop interactions, the MIMO process can be decomposed into a set of SISO processes and then decentralized PID controllers can be designed to stabilize these SISO loops independently. In application, however, it is desirable that the MIMO system remains stable if any of the loops is taken in or out of service. This requires that the controllers be designed conservatively. Such motivates the use of modified ETF which keeps
the same form of the ETF but has parameters taking the larger values of ETF and its corresponding open-loop transfer function, i.e.,

\[
\tilde{g}_y(s) = \frac{\tilde{k}_y e^{-\tilde{\theta}_y s}}{s(\tilde{\tau}_y s + 1)},
\]

(14)

where \( \tilde{g}_y(s) \) is the modified ETF in which

\[
\tilde{k}_y = \max \{ k_y, \hat{k}_y \}, \quad \tilde{\tau}_y = \max \{ \tau_y, \hat{\tau}_y \}, \quad \tilde{\theta}_y = \max \{ \theta_y, \hat{\theta}_y \}.
\]

(15)

Note that the larger parameters usually imply the more challenging situations for control, which sequentially implies that the controller design will be more conservative as compared to using the smaller parameters. In a similar manner, the modified ETFs can be determined for the processes summarized in Table 1.

As each controller design becomes a SISO case, any good PID tuning methods may apply. In this paper, the simple internal model control (SIMC) tuning method\(^{16}\) is adopted for simplicity and robustness. The SIMC method assumes the PID controller in the form of

\[
C_i(s) = k_c \left(1 + \frac{1}{\tau_i s}\right)(1 + \tau_D s),
\]

(16)

where \( k_c \), \( \tau_i \) and \( \tau_D \) are the proportional (P), integral (I) and derivative (D) parameters respectively. The tuning formulas of these three parameters for typical processes are given in Table 1 of the reference.\(^{16}\) The readers are referred there for details.

Since the proportional and derivative kicks can deteriorate the system performance a lot especially when the loop interactions are relatively strong,\(^{17}\) setpoint weighting\(^3\) is used to alleviate such effects in the implementation:

\[
u_i(t) = k_p (\mu_{r_i} r_i - y_i(t)) + k_i \int_0^t (r_i - y_i(v))dv + k_{D_i} \frac{d(\mu_{r_i} r_i - y_i(t))}{dt},
\]

(17)
where $r_i$ denotes the reference input, $u_i(t)$ the controller output, $y_i(t)$ the process output, $\mu_{p_i}$ and $\mu_{d_i}$ the setpoint weighting scalars for the proportional action and the derivative action respectively; and $k_{p_i} = k_{e_i}(1 + \tau_{d_i}/\tau_{i})$, $k_{I_i} = k_{e_i}/\tau_{I_i}$ and $k_{D_i} = k_{e_i}\tau_{D_i}$ are the P, I, D gains respectively. Both $\mu_{p_i}$ and $\mu_{d_i}$ take values in $[0, 1]$. The smaller the values are, the more sluggish the response of the $i$-th loop will be and the lesser serious the interactions will be inserted to the other loops. Thus, the choice of these two parameters is subject to a tradeoff between the loop performance and the interactions to other loops, extensive simulations have shown that it is sufficient to let $\mu_{p_i} = \mu_{d_i}$ and start the choice as $\mu_{p_i} = \mu_{d_i} = 0.5$. Then tune up the values, say at a step of 0.1, if the response of the $i$-th loop is sluggish while its interactions to other loops are modest; otherwise, tune down the values. It is observed that the system performance normally changes smoothly as these two scalars change.

5. Case studies

Three examples are used to illustrate the design procedures and demonstrate the effectiveness of the proposed decentralized PID control strategy. In the design, the tuning parameter $\tau_{e_i}$, $\forall i$, in the SIMC tuning rule is taken as recommended to be the time delay of $\theta_i$ for good performance and robustness, and $\varepsilon = 10^{-6}$ unless otherwise specified.

**Example 1.** Consider a $2 \times 2$ distillation column process described by

$$
G(s) = \begin{bmatrix}
\frac{3.04}{s} & \frac{-278.28}{s(s^2 + 36s + 180)} \\
\frac{0.052}{s} & \frac{319.47}{s(s^2 + 36s + 180)}
\end{bmatrix}.
$$

By factoring out the integrators, $G(s)$ is rewritten as

$$
G(s) = \begin{bmatrix}
\frac{3.04}{s} & \frac{-278.28}{s(s^2 + 36s + 180)} \\
\frac{0.052}{s} & \frac{319.47}{s(s^2 + 36s + 180)}
\end{bmatrix}.
$$
\[
G(s) = \begin{bmatrix}
\frac{1}{s} & 0 \\
0 & \frac{1}{s}
\end{bmatrix}
\begin{bmatrix}
3.04 & -278.28 \\
\frac{s^2 + 36s + 180}{s^2 + 36s + 180} & \frac{319.47}{s^2 + 36s + 180}
\end{bmatrix}.
\]

Thus the indices of interest about \( G(s) \) are computed as follows:

\[
K = \begin{bmatrix}
3.0400 & -1.5460 \\
0.0520 & 1.7748
\end{bmatrix}, \quad T_{ar} = \begin{bmatrix}
\varepsilon & 0.2 \\
\varepsilon & 0.2
\end{bmatrix},
\]

\[
\Lambda = K \otimes K^T = \begin{bmatrix}
0.9853 & 0.0147 \\
0.0147 & 0.9853
\end{bmatrix},
\]

\[
K_N = K \otimes T_{ar} = \begin{bmatrix}
3.0400/\varepsilon & -7.7300 \\
0.0520/\varepsilon & 8.8742
\end{bmatrix},
\]

\[
\Lambda_N = K_N \otimes K_N^T = \begin{bmatrix}
0.9853 & 0.0147 \\
0.0147 & 0.9853
\end{bmatrix},
\]

\[
\Gamma = \Lambda_N \otimes \Lambda = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}.
\]

Since \( \lambda_{N_{11}} \) is close to 1, \( \lambda_{11} \) is positive and NI=1.0149>0, the input-output pairs are selected as 1-1/2-2 according to the RGA-NI-RNGA pairing criterion. Consequently, the ETFs (after the scaling matrix being multiplied) corresponding to the 1-1 and 2-2 input-output pairs are obtained as

\[
\hat{g}_{11}(s) = \frac{3.09}{s}, \quad \hat{g}_{22}(s) = \frac{324.23}{s(s^2 + 36s + 180)}.
\]

By comparison with \( g_{11}(s) \) and \( g_{22}(s) \) in Eq. (18), the modified ETFs are obtained as \( \tilde{g}_{11}(s) = \hat{g}_{11}(s) \) and \( \tilde{g}_{22}(s) = \hat{g}_{22}(s) \). For \( \tilde{g}_{11}(s) \), using the SIMC method the PI controller is designed as

\[
c_{11}(s) = 16.181 + 202.265/s.
\]
(Here $\tilde{g}_{11}(s)e^{-0.01s}$ is used instead of $\tilde{g}_{11}(s)$ in order to apply the SIMC method.) For $\tilde{g}_{22}(s)$, before applying the SIMC method, it is approximated by an integrator with lag plus time delay transfer function using the least-square method:\textsuperscript{19}

$$
\tilde{g}_{22}(s) \approx \frac{1.804e^{-0.0231s}}{s(0.1789s + 1)}.
$$

Consequently, the PID controller is obtained as

$$
c_{22}(s) = 23.614 + 64.926/s + 2.147s.
$$

Consequently, the PID controller is obtained as

Set $\mu_{p_i} = \mu_{d_i} = 0.6$, $i = 1, 2$. Apply the PI and PID controllers with and without setpoint weighting respectively. The system step responses are shown in Fig. 2. It can be seen that the performance is better with $\mu_{p_i} = \mu_{d_i} = 0.6$ than that without setpoint weighting. In both cases, the change of the first setpoint has little effect on the output $y_2$; however, the effect is more obvious for the change of the second setpoint on the output $y_1$ although it is still acceptably small.

In addition, the obtained ETFs in Eq. (20) are compared with the true ETFs derived from the dynamical RGA\textsuperscript{14, 20} to verify its accuracy. The ETFs for the first and second loops are derived from dynamical RGA as follows:

$$
\hat{g}_{11}'(s) = g_{11}(s) - \frac{g_{12}(s)g_{21}(s)}{g_{22}(s)} \approx \frac{3.09}{s},
$$

$$
\hat{g}_{22}'(s) = g_{22}(s) - \frac{g_{12}(s)g_{21}(s)}{g_{11}(s)} \approx \frac{324.23}{s(s^2 + 36s + 180)}.
$$

Coincidently, these are the same as the ETFs derived by the proposed approach. This validates the proposed approach for deriving approximate ETFs.

**Example 2.** Consider a $3 \times 3$ reactor-splitter process given by\textsuperscript{4}
which can be factorized as

\[
G(s) = \begin{bmatrix}
\frac{-1-45s-4s^2}{4s(5+105s+100s^2)} & \frac{4}{25(1+20s)} & \frac{1+36s}{20s(1+20s)} \\
\frac{-5+25s+20s^2}{s(1+21s+20s^2)} & -16 & \frac{5+20s}{s(1+20s)} \\
\frac{6-4s}{1+21s+20s^2} & \frac{16s}{1+20s} & -4 \\
\end{bmatrix}
\] .

Hence, matrices of interest are computed as:

\[
G(s) = \begin{bmatrix}
\frac{-1-45s-4s^2}{4(5+105s+100s^2)} & \frac{4}{25(1+20s)} & \frac{1+36s}{20(1+20s)} \\
\frac{-5+25s+20s^2}{1+21s+20s^2} & -16 & \frac{5+20s}{1+20s} \\
\frac{6-4s}{1+21s+20s^2} & \frac{16s}{1+20s} & -4 \\
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
0 & s & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\]

Hence, matrices of interest are computed as:

\[
K = \begin{bmatrix}
-0.05 & 0.16 & 0.05 \\
-5 & -16 & 5 \\
6 & 16 & -4 \\
\end{bmatrix},
\quad \quad
T_{ar} = \begin{bmatrix}
24 & 20 & 16 \\
26 & 20 & 16 \\
21.67 & 20 & 20 \\
\end{bmatrix},
\quad \quad
\Lambda = K \otimes K^T = \begin{bmatrix}
0.25 & 0.50 & 0.25 \\
-2.25 & 0.50 & 2.75 \\
3.00 & 0 & -2.00 \\
\end{bmatrix},
\quad \quad
K_N = K \otimes T_{ar} = \begin{bmatrix}
-0.0021 & 0.0080 & 0.0031 \\
-0.1923 & -0.8000 & 0.3125 \\
0.2769 & 0.8000 & -0.2000 \\
\end{bmatrix},
\quad \quad
\Lambda_N = K_N \otimes K_N^T = \begin{bmatrix}
0.2393 & 0.4908 & 0.2699 \\
-1.0064 & 0.4581 & 1.5483 \\
1.7671 & 0.0511 & -0.8182 \\
\end{bmatrix},
\quad \quad
\Gamma = \Lambda_N \otimes \Lambda = \begin{bmatrix}
0.9573 & 0.9816 & 1.0795 \\
0.4473 & 0.9161 & 0.5630 \\
0.5890 & +\infty & 0.4091 \\
\end{bmatrix}.
According to the RGA-NI-RNGA based pairing criterion, the input-output pairs are selected as 1-3/2-1/3-2 (NI=0.666>0). Consequently, the ETFs (with the scaling matrix being multiplied) corresponding to these three input-output pairs are obtained as

\[
\hat{g}_{12}(s) = \frac{8}{25(1+19.632s)}, \quad \hat{g}_{23}(s) = \frac{1.82 + 4.09s}{s(1+11.26s)}, \quad \hat{g}_{31}(s) = \frac{2 - 0.79s}{1+12.37s + 20s^2}.
\] (27)

By comparing parameters of the ETFs with those of the corresponding open-loop transfer functions in Eq. (25), the modified ETFs are determined as

\[
\tilde{g}_{12}(s) = \frac{8}{25(1+20s)}, \quad \tilde{g}_{23}(s) = \frac{5 + 20s}{s(1+20s)}, \quad \tilde{g}_{31}(s) = \frac{6 - 4s}{1+21s + 20s^2}.
\] (28)

To apply the SIMC method for tuning the PID controllers, \( \hat{g}_{12}(s)e^{-0.1s} \) is used instead of \( \hat{g}_{12}(s) \) and \( \tilde{g}_{31}(s) \) is approximated as (using the least-square method\(^{19}\))

\[
\tilde{g}_{31}(s) \approx \frac{6.007e^{-1.47s}}{20.3s+1}.
\] (29)

In the design, \( \tilde{g}_{23}(s) \approx 1/s \) is used for the PID controller tuning, which is obtained using the model reduction method proposed by Skogestad.\(^{16}\) To apply the SIMC method, \( e^{-0.1s}/s \) is used. Consequently, the PID controllers are obtained as

\[
c_{12}(s) = 312.5 + 390.625/s, \\
c_{23}(s) = 5 + 6.25/s, \\
c_{31}(s) = 1.15 + 0.0977/s
\] (30)

Simulations indicate that the integral gain of \( c_{31}(s) \) given in (30) is too conservative to give fast response and it can safely be modified to be much larger while causing little impact on other loops. Hence for fast response \( c_{31}(s) = 1.15 + 10/s \) is used instead. Set \( \mu_{py} = 0, \ ij = 12, 23 \), and \( \mu_{pi} = 1 \). Apply these PI controllers with setpoint weighting. The system step responses are shown in Fig. 3. The results indicate that each output reaches the steady state quickly. For the
step change of the second setpoint, both \( y_2 \) and \( y_3 \) exhibit obvious changes which imply that strong interactions exist between the system variables. The interactions are however trivial when either the first or third input is subject to similar change. When load disturbances are injected at the outputs of the controllers (before entering the processes), it is observed that \( y_1 \) keeps smooth and \( y_2 \) and \( y_3 \) return to the setpoints after short deviations. Overall the proposed decentralized control works well.

**Example 3.** Consider a \( 4 \times 4 \) industrial reactor/recycle system given by\(^2\)

\[
G(s) = \begin{bmatrix}
0 & \frac{3.96(197s+1)}{49.4s^2+14.1s+1} & \frac{0.536(258s+1)}{83.3s^2+18.3s+1} & 9.7 \\
0.00111 & 0.044 & -0.0152e^{-32s} & 0.039e^{-20s} \\
\frac{s}{46.9s^2+s} & \frac{s}{-232.2} & \frac{s}{-15.96(529s+1)} & \frac{s}{139.2} \\
\frac{s}{32.2s+1} & \frac{s}{10417s^2+204s+1} & \frac{s}{7.27s+1} & \\
\frac{s}{-0.582} & \frac{-2.54(8.11s+1)}{6.25s^3+5s^2+s} & \frac{0.0426(45.6s+1)e^{-35s}}{306s^3+35s^2+s} & \frac{-0.0358e^{-30s}}{s}
\end{bmatrix}
\] (31)

\( G(s) \) can be factorized as

\[
G(s) = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1/s & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1/s
\end{bmatrix}
\begin{bmatrix}
0 & \frac{3.96(197s+1)}{49.4s^2+14.1s+1} & \frac{0.536(258s+1)}{83.3s^2+18.3s+1} & 9.7 \\
0.00111 & 0.044 & -0.0152e^{-32s} & 0.039e^{-20s} \\
0 & \frac{-232.2}{32.2s+1} & \frac{-15.96(529s+1)}{10417s^2+204s+1} & \frac{139.2}{7.27s+1} \\
\frac{-0.582}{6.25s^3+5s^2+s} & \frac{-2.54(8.11s+1)}{306s^3+35s^2+s} & \frac{0.0426(45.6s+1)e^{-35s}}{7.27s+1} & \frac{-0.0358e^{-30s}}{s}
\end{bmatrix}
\] (32)

Thus, the matrices of interest are computed as:
According to the RGA-NI-RNGA criterion, the input-output pairs are selected as 1-4/2-3/3-2/4-1, which has NI=1.5696>0. The ETFs for these input-output pairs are obtained as follows

\[
\hat{g}_{14}(s) = \frac{13.0}{28.4s+1}, \quad \hat{g}_{23}(s) = \frac{-0.0192e^{-41s}}{s}, \quad \hat{g}_{32}(s) = \frac{-320.9}{40s+1}, \quad \hat{g}_{41}(s) = \frac{-0.568}{s}. \quad (33)
\]

By comparing parameters of the above ETFs with those of the corresponding open-loop transfer functions in Eq. (31), the modified ETFs are determined as

\[
\tilde{g}_{14}(s) = \frac{13.0}{28.4s+1}, \quad \tilde{g}_{23}(s) = \frac{-0.0192e^{-41s}}{s}, \quad \tilde{g}_{32}(s) = \frac{-320.9}{40s+1}, \quad \tilde{g}_{41}(s) = \frac{-0.582}{s}. \quad (34)
\]

To apply the SIMC method, time delays of 0.1 are introduced to any delay-free processes above. Consequently, the PI controllers are obtained as
\[ c_{14}(s) = 10.923 + 13.654/s, \]
\[ c_{23}(s) = -0.635 - 0.002/s, \]
\[ c_{32}(s) = -0.623 - 0.779/s, \]
\[ c_{41}(s) = -8.591 - 10.739/s. \]  \hspace{1cm} (35)

Set \( \mu_{p_i} = 0.5, \ ij = 14, \ 32, \ 41, \) and \( \mu_{p_{23}} = 0.7. \) The system step responses are shown in Fig. 4. The results show that \( y_1, \ y_3 \) and \( y_4 \) reach the steady states very fast, while \( y_2 \) needs longer time to settle down due to the challenging processes in its loop. When load disturbances are injected at the outputs of the controllers, \( y_1, \ y_3 \) and \( y_4 \) all return to the setpoints after short deviations; \( y_2 \) keeps smooth when disturbances occur in other loops, but takes a long time to settle down when the disturbance occurs in its own loop. The results indicate that the interactions between the loops are well suppressed and do not degrade the performance much.

6. Conclusions

An approach was proposed for decentralized PID control design of MIMO processes with integrators and/or differentiators. Based on the RGA-NI-RNGA criterion, the input-output pairing was determined. Then ETFs were derived for the selected input-output pairs using the RGA and RNGA information. To maintain integrity, the ETFs were modified for controller tuning. Since the modified ETFs had properly taken account of the loop interactions, the MIMO process was perceived to be decomposed into a set of independent SISO processes so that the PID controllers were designed independently. The unique advantage of the proposed approach is its simplicity in carrying out a systematic decentralized control design, which can easily be understood and implemented by field engineers. Examples illustrated the design procedures and demonstrated the effectiveness of the approach.
Appendix A. Derivation of ART for a general process model.

Consider the process described by a general model:

\[
g_{ij}(s) = \frac{k_{ij} e^{-\theta_{ij}}}{\prod_{k=1}^{m} (z_{ij} s + 1) \prod_{k=1}^{n_1} (p_{ij} s + 1) \prod_{k=1}^{n_2} \left( \frac{1}{\omega_{ij}} s^2 + 2 \frac{\zeta_{ij}}{\omega_{ij}} s + 1 \right)},
\]

where \(z_{ij}, p_{ij}, \zeta_{ij} < 1\) and \(\omega_{ij}\) are all real numbers and \(m \leq n_1 + n_2\). Rewrite Eq. (36) as

\[
g_{ij}(s) = k_{ij} e^{-\theta_{ij}} \overline{g}_{ij}(s),
\]

where \(e^{-\theta_{ij}} \overline{g}_{ij}(s)\) has a steady-state gain of one and its ART equals the ART of \(g_{ij}(s)\). For simplicity, we ignore the time delay in deriving ART as can be added later.

Let \(r_j(s)\) and \(\overline{y}_i(s)\) be the input and output of the process, the unit step response of \(\overline{g}_{ij}(s)\) is given by \(\overline{y}_i(s) = \overline{g}_{ij}(s) r_j(s) = \overline{g}_{ij}(s) / s\). \(\overline{y}_i(s)\) can be expanded as the sum of fractions as follows:

\[
\overline{y}_i(s) = \frac{1}{s} + \sum_{k=1}^{n_1} \frac{a_k}{p_{ij} s + 1} + \prod_{k=1}^{n_1} \frac{b_k (s + \sigma_{ij}) + c_k \tilde{\omega}_{ij}}{s^2 + 2 \zeta_{ij} \omega_{ij} s + \omega_{ij}^2},
\]

where \(a_k, b_k\) and \(c_k\) are proper coefficients; \(\sigma_{ij} := \zeta_{ij} \omega_{ij}\) and \(\tilde{\omega}_{ij} := \omega_{ij} \sqrt{1 - \zeta_{ij}^2}\). By inverse Laplace transform, the time response is obtained as

\[
\overline{y}_i(t) = 1 + \sum_{k=1}^{n_1} \frac{a_k}{p_{ij}} e^{-\tau_{ij} t} + \sum_{k=1}^{n_1} b_k e^{-\sigma_{ij} t} \cos \left( \tilde{\omega}_{ij} t \right) + \sum_{k=1}^{n_1} c_k e^{-\sigma_{ij} t} \sin \left( \tilde{\omega}_{ij} t \right).
\]

Since \(\overline{y}_i(\infty) = 1\),
Thus, taking the time delay into account, the ART for the process described in Eq. (36) is obtained as
\[ \tau_{ar} := |\overline{\tau}_{ar} + \theta_j|. \]

In particular, if
\[ \eta = 0, \quad \tau = \tau_{ar} = 0, \]
ART is obtained as
\[ \tau_{ar} = \left| \frac{2 \zeta_{ij}}{\omega_{ij}} - \zeta_{ij} - z_{ij} + \theta_j \right|. \]

Appendix B. Proof of Property 2.

The proof proceeds similarly to the proof of Lemma 2 in Huang et al.\(^2\) Since the ART \( \tau_{ar} = 0 \), the normalized gain is expressed as \( k_{Ny} = k_{y} / \varepsilon, \varepsilon \to 0 \). With invertible \( K_{N_{i,p}} \), it has\(^2\)

\[ \lambda_{Ny} = \left( k_{Ny} / k_{Ny} - k_{Ny} \left[ K_{N_{i,p}} \right]^{-1} k_{Ny, i} \right) = \lim_{\varepsilon \to 0} \left( k_{y} / \varepsilon - k_{Ny, i} \left[ K_{N_{i,p}} \right]^{-1} k_{Ny, j} \right), \]

where \( k_{Ny, i} \) and \( k_{Ny, j} \) denote the \( i \)-th row and the \( j \)-th column of \( K_N \) with \( k_{Ny} \) being excluded respectively, and \( K_{N_{i,p}} \) denotes the sub-matrix of \( K_N \) with \( i \)-th row and \( j \)-th column being removed. Given that \( \lambda_{Ny} \) is finite and nonzero, it means
\[ k_{N_{ij}} \left[ K_{N_{i \rightarrow j}} \right]^{-1} k_{N_{ji}} = \beta / \varepsilon, \quad \beta \neq k_{ij}, \]  
\[(43)\]

where \( \beta \) is a proper constant. Since \( \lambda_{N_{ij}} = k_{N_{ij}} / k_{N_{ij}} \), we obtain

\[ \hat{k}_{N_{ij}} = k_{N_{ij}} - k_{N_{ij}} \left[ K_{N_{i \rightarrow j}} \right]^{-1} k_{N_{ji}} = \lim_{\varepsilon \to 0} \left\{ \frac{k_{ij} - \beta}{\varepsilon} \right\} \rightarrow \infty. \]  
\[(44)\]

Hence \( \hat{k}_{N_{ij}} = \hat{k}_{ij} / \hat{r}_{ij} \) together with Eq. (44) implies that \( \hat{r}_{ar} = 0 \). That is, the ART in ETF of the \( j \)-\( i \) input-output pair is zero.

**Literature Cited**


(13) Niederlinski, A. A heuristic approach to the design of linear multivariable interacting control systems. *Automatica* 1971, 7, 691.


Table 1. ARTs, normalized gains and ETFs for typical process models

<table>
<thead>
<tr>
<th>g_y(s)</th>
<th>τ_{ar,y}</th>
<th>k_{N,y}</th>
<th>\hat{g}_y(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>k_{ij}</td>
<td>\varepsilon (\varepsilon &gt; 0, \varepsilon \rightarrow 0)</td>
<td>\frac{k_{ij}}{\tau_{ar,y}}</td>
<td>\frac{k_{ij}}{\lambda_{ij}}</td>
</tr>
<tr>
<td>k_y e^{-\theta_y \tau_y s}</td>
<td>\theta_y</td>
<td>\frac{k_{ij}}{\tau_{ar,y}}</td>
<td>\frac{k_{ij}}{\lambda_{ij}} e^{-\gamma_{ij} \theta_y s}</td>
</tr>
<tr>
<td>\frac{k_y e^{-\theta_y \tau_y s}}{\tau_y s + 1}, \tau_y &gt; 0</td>
<td>\tau_y + \theta_y</td>
<td>\frac{k_{ij}}{\tau_{ar,y}}</td>
<td>\frac{k_{ij}}{\lambda_{ij} (\gamma_{ij} \tau_y s + 1)} e^{-\gamma_{ij} \theta_y s}</td>
</tr>
<tr>
<td>\frac{k_y \omega_y^2 e^{-\theta_y \tau_y s}}{s^2 + 2 \zeta_y \omega_y \tau_y s + \omega_y^2}, \zeta_y &gt; 0</td>
<td>\left</td>
<td>\frac{2 \tau_y}{\omega_y} + \theta_y \right</td>
<td></td>
</tr>
<tr>
<td>\frac{k_y e^{-\theta_y \tau_y s}}{s^2 + 2 \zeta_y \omega_y s + \omega_y^2}, \zeta_y &gt; 0</td>
<td>\left</td>
<td>\frac{2 \tau_y}{\omega_y} \right</td>
<td></td>
</tr>
<tr>
<td>\frac{k_y e^{-\theta_y \tau_y s}}{\omega_y^2 s^2 + 2 \frac{\tau_y}{\omega_y} s + 1}, \zeta_y &gt; 0</td>
<td>\left</td>
<td>\frac{2 \tau_y}{\omega_y} - z_{ij} \gamma_{ij} s + \theta_y \right</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 1. Closed loop multivariable control system

\[ \mu_{p_{ii}} = \mu_{d_{ii}} = 0.6 \]

\[ \mu_{p_{ii}} = \mu_{d_{ii}} = 1.0 \]

Fig. 2. Output responses: the unit step inputs 1 and 2 are injected at times of 0 and 0.7 respectively.
Fig. 3. Output responses: the unit step inputs 2, 3 and 1 are injected at times of 0, 10 and 20 respectively; the unit step load disturbances are injected at the outputs of the controllers $c_{31}(s)$, $c_{12}(s)$ and $c_{23}(s)$ at times of 30, 40 and 50 respectively; the small window in the figure shows the zoomed-in response of $y_1$. 
Fig. 4. Output responses: the unit step inputs 4, 3, 2, and 1 are injected at times of 0, 500, 1000 and 1250 respectively; load disturbances with magnitudes of 1 are injected at the outputs of the controllers $c_{41}(s)$ and $c_{44}(s)$ at times of 1500 and 2250 respectively, and load disturbances with magnitudes of 0.2 are injected at the outputs of the controllers $c_{32}(s)$ and $c_{33}(s)$ at times of 1750 and 2000 respectively; the small windows in the figure show zoomed-in responses of the related outputs respectively.