# Energy and Spectral Efficient Transmissions of Coded ARQ Systems

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Abstract-Energy efficiency (EE) and spectral efficiency (SE) feature one of the most fundamental tradeoffs in communication systems. In this paper, we propose an optimum transmission scheme for coded Type-I automatic repeat request (ARQ) systems to balance the EE-SE tradeoff. The optimization is performed with respect to a new design metric, the normalized EE (NEE), which is defined as the energy per bit normalized by the SE. Minimizing NEE means either reducing the energy per bit or increasing the SE, it thus yields a balanced tradeoff between the two. The system design incorporates a wide range of practical system parameters, such as circuit power, modulation, coding, and detection errors in the physical layer, and frame length and protocol overhead in the media access control layer. Under the constraints of fixed modulation level and data rate, the optimum transmission energy and frame length are identified as closed-form expressions of the system parameters. Simulation results show that minimizing the NEE instead of energy per bit almost doubles the SE with less than 1 dB loss in EE.

## I. INTRODUCTION

Energy efficiency (EE) and spectral efficiency (SE) are two essential design metrics for wireless communication systems. Energy efficient communications reduce energy consumption and extend the battery life of wireless terminals. Spectral efficient communications can support more simultaneous users or a higher data rate in a unit bandwidth, thus achieve better utilization of the scarce spectrum resource. With the increased demand of high rate broadband communication and the growing need of green communications, it is imperative to develop communication technologies that are efficient in term of both the energy and spectrum resources.

There are a large number of works in the literature devoted to the development of energy efficient communication systems, by considering circuit power, modulation, coding, resource allocation, scheduling, etc. [1] - [8]. The EE is usually achieved at the cost of SE. The tradeoff relationship between the two has attracted growing interests in the design of energy efficient communication systems [9] - [11]. The information theoretic study in [9] identifies the fundamental tradeoff between EE and SE in the wideband and low energy regime, and it is shown that SE is a decreasing function in EE for various channels. Incorporating more practical parameters, such as circuit power and orthogonal frequency division multiple access (OFDMA), Xiong et. al. discovered that the maximum EE is strictly quasiconcave in SE if there are sufficiently many subcarriers in a downlink OFDMA system [10]. A similar result is presented in [11], where it was shown that EE and SE can increase simultaneously for a band unlimited system under a rate constraint, and EE is a decreasing function in SE when the bandwidth is limited. All works use the Shannon channel capacity to measure rate, without considering practical operations such as coding, detection error, retransmission, protocol overhead, etc.

In this paper, we propose to balance the tradeoff between EE and SE by minimizing a new metric: normalized EE (NEE), which is defined as the energy per bit normalized by the SE. Minimizing the NEE will either reduce the energy per bit or increase the SE, thus achieves a balanced tradeoff between them. The optimum system design is performed for a practical coded Type-I automatic repeat request (ARQ) system operating in a Rayleigh fading channel. The energy efficient design of ARQ or hybrid ARQ (HARQ) systems are studied in [6] - [8], but none of them considered the EE-SE tradeoff. We propose to balance the EE-SE tradeoff in coded Type-I ARQ systems by jointly optimizing the transmission energy in the physical layer and the frame length in the media access control layer. The optimum transmission energy and frame length are expressed as closed-form expressions of a large number of practical system parameters, such as the efficiency of the power amplifier, the power consumption of digital hardware, data rate, modulation and coding schemes, frame length, frame error rate (FER), and the protocol overhead, etc.

#### **II. SYSTEM MODEL**

Consider a transmitter and a receiver separated by a distance d. The information bits at the transmitter are divided into frames. Each frame has L uncoded information bits and  $L_0$  overhead bits. The information and overhead bits are encoded with a channel encoder with a code rate r. For a system employing M-ary modulation, the number of symbols in each frame is  $L_s = \frac{L+L_0}{r \log_2 M}$ , where L is chosen in a way such that  $L_s$  is an integer.

The signal is transmitted through a flat Rayleigh fading channel, and the discrete-time samples at the receiver can be represented as

$$y_i = \sqrt{E_{\rm r}} h x_i + z_i, \qquad i = 1, 2, \cdots, L_{\rm s}, \tag{1}$$

where  $E_r$  is the average energy of a symbol observed at the receiver,  $x_i \in S$  is the *i*-th modulated symbol in a frame, with

S being the modulation constellation set with the cardinality  $M = |S|, y_i, h$ , and  $z_i$  are the received sample, the quasi-static fading coefficient between the transmitter and the receiver, and additive white Gaussian noise (AWGN) with single-sided power spectral density  $N_0$ , respectively.

# A. Energy Efficiency

The energy efficiency is measured as the average energy required to successfully deliver one uncoded information bit from its source to destination, including the energy consumption of retransmissions if the previous transmission attempts are not successful.

Let  $E_{\rm b}$  denote the average energy per uncoded information bit observed at the receiver during one transmission attempt, then the average  $E_{\rm b}/N_0$  at the receiver is

$$\gamma_{\rm b} \triangleq \frac{E_{\rm b}}{N_0} = \frac{E_{\rm r}}{rN_0 \log_2 M}.$$
(2)

The average transmission energy for each symbol at the transmitter can be represented as [1]

$$E_{\rm s} = E_{\rm r} G_1 d^{\kappa} M_{\rm l},\tag{3}$$

where  $\kappa$  is the path-loss exponent,  $G_1$  is the gain factor (including path-loss and antenna gain) at a unit distance, and  $M_1$ is the link margin compensating the hardware process variations and other additive background noise or interference.

In addition to the actual transmission energy, we also need to consider the hardware energy per symbol in the transmitter and receiver, which can be modelled as [1],

$$E_{\rm c} = \left(\frac{\xi_M}{\eta} - 1\right) E_{\rm s} + \frac{\beta}{R_{\rm s}},\tag{4}$$

where  $R_{\rm s} = \frac{1}{T_{\rm s}}$  is the gross symbol rate,  $\eta$  is the drain efficiency of the power amplifier,  $\xi_M$  is the peak-to-average power ratio (PAPR) of an *M*-ary modulation signal,  $\beta$  incorporates the effects of baseband processing, such as signal processing, encoding and decoding, and it can be treated as a constant determined by the particular transceiver structure. For *M*-ary quadrature amplitude modulation (MQAM) systems,  $\xi_M \simeq 3(\sqrt{M} - \frac{1}{\sqrt{M}} + 1)$  for  $M \ge 4$  [13].

From (2), (3), and (4), the energy required to transmit an information bit during one transmission attempt is

$$E_{0} = \frac{L_{\rm s}}{L} (E_{\rm s} + E_{\rm c}) = \frac{L + L_{0}}{L} \frac{\gamma_{\rm b} \xi_{\scriptscriptstyle M} N_{0} G_{d}}{\eta} + \frac{\beta}{R_{\rm b}}, \quad (5)$$

where  $G_d = G_1 d^{\kappa} M_l$ , and  $R_b = \frac{L}{L_s} R_s$  is the net bit rate of the uncoded information bit.

Due to the effects of channel fading and noise, the receiver might not be able to successfully recover the transmitted signal. The probability that a transmitted frame cannot be recovered equals to FER, which is a function of  $\gamma_b$  at the receiver, the frame length  $L_s$ , the modulation level M, and the channel code. The frame will be retransmitted if the transmitter receive a negative acknowledgement (NACK). Since the retransmissions are independent, the number of retransmissions is a geometric random variable with the parameter FER. The average number of retransmissions is thus

$$\Lambda = \frac{1}{1 - \text{FER}}.$$
(6)

The total average energy required to successfully deliver an information bit from the transmitter to the receiver can then be calculated by  $E_{\rm t} = \Lambda E_0$ , which can be expanded by combining (5) and (6) as

$$E_{t} = \frac{1}{1 - \text{FER}} \left[ \frac{L + L_0}{L} \frac{\gamma_{b} \xi_M N_0 G_d}{\eta} + \frac{\beta}{R_{b}} \right]. \tag{7}$$

# B. Spectral Efficiency

The spectral efficiency is defined as  $\eta_{\rm s} = \frac{R_{\rm bs}}{(1+\alpha)R_{\rm s}}$ , where  $R_{\rm bs}$  is the net data rate of the successfully transmitted information bit including the effects retransmissions,  $(1+\alpha)R_{\rm s}$  is the signal bandwidth with  $\alpha$  being the roll-off factor of the pulse shaping filter. The average time required to successfully transmit L information bits is  $T = \frac{\Lambda(L+L_0)}{rR_{\rm s}\log_2 M}$ , thus  $R_{\rm bs}$  can be calculated by

$$R_{\rm bs} = \frac{L}{\frac{\Lambda(L+L_0)}{rR_{\rm s}\log_2 M}} = \frac{L}{L+L_0} \frac{rR_{\rm s}\log_2 M}{\Lambda}.$$
 (8)

The spectral efficiency of the system can then be written as

$$\eta_{\rm s} = \frac{L}{L+L_0} \frac{r \log_2 M}{\Lambda(1+\alpha)}.\tag{9}$$

## C. Normalized Energy Efficiency

Instead of directly minimizing  $E_t$  or maximizing  $\eta_s$  without considering the other one, we propose a new design metric, the normalized energy efficiency, as  $E_m = \frac{E_t}{\eta_s}$ . Minimizing  $E_m$  will reduce  $E_t$  or increase  $\eta_s$ , thus achieve a balanced tradeoff between the EE and SE. Combining (7) and (9), we can express the NEE as

$$E_{\rm m} = \frac{1}{(1 - {\rm FER})^2} \frac{L + L_0}{L} \left(\frac{L + L_0}{L} \gamma_{\rm b} A + B\right) \eta_{\scriptscriptstyle M}, \quad (10)$$

where  $A = \frac{\xi_M N_0 G_d}{\eta}$ ,  $B = \frac{\beta}{R_b}$ , and  $\eta_M = \frac{1+\alpha}{r \log_2 M}$  is the largest SE of a system with *M*-ary modulation. The NEE relies on a number of system parameters, including  $E_b/N_0$  at the receiver (denoted as  $\gamma_b$ ), the number of information bits per frame *L*, the number of overhead bits per frame  $L_0$ , the modulation level *M*, the net data rate  $R_b$ , and the FER that inherently depends on all the above parameters and the code rate *r*, etc.

In (10), we can see that the value of  $\gamma_b$  has two opposite effects on  $E_m$ . On one hand, FER is a decreasing function in  $\gamma_b$ . Therefore, increasing  $\gamma_b$  will decrease the average number of retransmissions  $\Lambda$ , thus reduce  $E_m$ . On the other hand,  $E_0$  is a strictly increasing function in  $\gamma_b$ , thus it translates a positive relationship between  $\gamma_b$  and  $E_m$ . A similar observation can also be obtained for the relationship between  $E_m$  and L. Therefore, it is critical to identify the optimum values of  $\gamma_b$  and L that can minimize  $E_m$ .

## III. OPTIMUM SYSTEM DESIGN WITH RESPECT TO NEE

The optimum system design with respect to  $E_{\rm m}$  under the constraints of fixed M,  $R_{\rm b}$  and  $L_0$  are discussed in this section.

The optimization problem requires the information about the FER. In a quasi-static Rayleigh fading channel, the FER of a coded system can be accurately approximated as [12]

$$\operatorname{FER}(\gamma_{b}) \simeq 1 - \exp\left(-\frac{\gamma_{\omega}}{\gamma_{b}}\right),$$
 (11)

where  $\gamma_{\rm b}$  is the average  $E_{\rm b}/N_0$  at the receiver, and  $\gamma_{\omega}$  is a threshold value that can be calculated with a linear approximation as [8]

$$\gamma_{\omega} \simeq k_{_M} \log(L + L_0) + b_{_M}, \tag{12}$$

where  $k_{M}$  and  $b_{M}$  are determined by the modulation scheme and the actual channel code. It is shown in [8] that (11) and (12) yield a very accurate approximation of the FER over a large range of modulation and coding schemes. When the channel code is a rate  $r = \frac{1}{2}$  convolutional code with the generator polynomial  $[5, 7]_8$  and a constraint length 3, the parameters are  $k_{\scriptscriptstyle M} = 0.3744$  and  $b_{\scriptscriptstyle M} = -0.31.$ 

To simplify notations, denote  $\Delta_1 = \frac{1}{(1-\text{FER})^2}$ ,  $\Delta_2 =$  $\frac{L+L_0}{L}\left(\frac{L+L_0}{L}\gamma_bA+B\right)\eta_M$ , then  $E_{\rm m}=\Delta_1\Delta_2$ . The values of  $\gamma_{\rm b}$  and L that minimize  $E_{\rm m}$  are discussed in the next two subsections, respectively.

#### A. Optimum $\gamma_b$

The optimum value of  $\gamma_{\rm b}$  at the receiver that minimizes  $E_{\rm m}$ is studied in this subsection.

Before proceeding to the actual optimization, we present the following theorem about convexity [8], which will be used in identifying the optimum system parameters.

Theorem 1: Consider a decreasing function f(x) with  $f'(x) \leq 0$ , and an increasing function g(x) with  $g'(x) \geq 0$ . If both f(x) and g(x) are convex, then f(x)g(x) is convex.

Since  $E_m$  is the product of  $\Delta_1$  and  $\Delta_2$ , we can prove that  $E_{\rm m}$  is convex in  $\gamma_{\rm b}$  if  $\Delta_1$  and  $\Delta_2$  satisfy the conditions stated in Theorem 1. The results are stated as follows.

*Proposition 1:* For the FER given in (11), the normalized energy per information bit,  $E_{\rm m}$ , in (10) is convex in  $\gamma_{\rm b}$ .

*Proof:* The first derivative of  $\Delta_1$  with respect to  $\gamma_b$  is

$$\frac{\partial \Delta_1}{\partial \gamma_b} = -\frac{2\gamma_\omega}{\gamma_b^2} \exp\left(\frac{2\gamma_\omega}{\gamma_b}\right) \le 0.$$
(13)

Therefore  $\Delta_1$  is a decreasing function in  $\gamma_b$ .

The second derivative of  $\Delta_1$  with respect to  $\gamma_b$  is

$$\frac{\partial^2 \Delta_1}{\partial \gamma_b^2} = \frac{2\text{FER}''(\gamma_b)(1 - \text{FER}) + 6[\text{FER}'(\gamma_b)^2]}{(1 - \text{FER})^4}.$$
 (14)

From (11), we have

$$\text{FER}'(\gamma_{\rm b}) = -\frac{\gamma_{\omega}}{\gamma_{\rm b}^2} \exp\left(-\frac{\gamma_{\omega}}{\gamma_{\rm b}}\right), \qquad (15a)$$

$$\text{FER}''(\gamma_b) = \frac{\gamma_\omega}{\gamma_b^3} \left(2 - \frac{\gamma_\omega}{\gamma_b}\right) \exp\left(-\frac{\gamma_\omega}{\gamma_b}\right). \quad (15b)$$

Substituting (15) into (14) and simplifying lead to

$$\frac{\partial^2 \Delta_1}{\partial \gamma_b^2} = \frac{4\gamma_\omega}{\gamma_b^4} (\gamma_b + \gamma_\omega) \exp\left(\frac{2\gamma_\omega}{\gamma_b}\right) \ge 0.$$
(16)

Therefore  $\Delta_1$  is convex in  $\gamma_b$ .

It is straightforward that  $\Delta_2$  is a linear increasing function in  $\gamma_b$ . Based on Theorem 1,  $E_m = \Delta_1 \Delta_2$  is convex in  $\gamma_b$ .

Once the convexity of  $E_{\rm m}$  with respect to  $\gamma_{\rm b}$  is established, the optimum  $\gamma_b$  can be obtained through standard convex optimization method as follows.

Proposition 2: In a quasi-static Rayleigh fading channel, if the FER is given in (11), then the optimum  $\gamma_b$  that minimizes  $E_{\rm m}$  is

$$\hat{\gamma}_{\rm b} = \gamma_{\omega} + \sqrt{\gamma_{\omega}^2 + 2\gamma_{\omega} \frac{B}{A} \frac{L}{L + L_0}}.$$
(17)

where  $A = \frac{\xi_M N_0 G_d}{\eta}$ , and  $B = \frac{\beta}{R_b}$ . *Proof:* Since  $E_m$  is convex in  $\gamma_b$ , the optimum  $\gamma_b$  that minimizes  $E_m$  can be obtained by solving  $\frac{\partial E_m}{\partial \gamma_b} = 0$  as,

$$\frac{\partial \Delta_1}{\partial \gamma_{\mathbf{b}}} \Delta_2 + \frac{\partial \Delta_2}{\partial \gamma_{\mathbf{b}}} \Delta_1 = 0.$$
(18)

We have  $\frac{\partial \Delta_2}{\partial \gamma_b} = \left(\frac{L+L_0}{L}\right)^2 A \eta_M$ . Substituting (13) into (18)

$$\gamma_{\rm b}^2 - 2\gamma_\omega\gamma_{\rm b} - 2\gamma_\omega\frac{B}{A}\frac{L}{L+L_0} = 0.$$
<sup>(19)</sup>

The result in (17) can be obtained by solving (19).

It should be mentioned here that the optimum  $\gamma_b$  is the average  $E_{\rm b}/N_0$  at the receiver. Correspondingly, the optimum energy per symbol required at the transmitter is

$$\dot{E}_{\rm s} = \hat{\gamma}_{\rm b} \times N_0 \times r \times \log_2 M \times G_d,\tag{20}$$

where  $\hat{\gamma}_{b}$  is the optimum value calculated from (17).

#### B. Optimum L

The optimum length of information bits, L, that minimizes  $E_{\rm m}$  is studied in this subsection.

Similar to the results in Proposition 2, the optimum solution of L relies on the convexity of  $E_{\rm m}$ . However, the direct proof of the convexity of  $E_m$  with respect to L is quite tedious. To simplify analysis, we can show that  $E_m$  is convex in  $\xi = \log(L + L_0)$ . Therefore, we can solve the optimum L by minimizing  $E_{\rm m}$  with respect to  $\xi$ .

Proposition 3: For the FER given in (11), the normalized energy per information bit  $E_{\rm m}$  in (10) is convex in  $\xi = \log(L +$  $L_{0}).$ 

*Proof:* From the definition of  $\Delta_1$ , we have

$$\frac{\partial^2 \Delta_1}{\partial \xi^2} = \frac{4k_M^2}{\gamma_b^2} \exp\left(\frac{2\gamma_\omega}{\gamma_b}\right) \ge 0.$$
(21)

Therefore  $\Delta_1$  is convex in  $\xi$ . It is also straightforward to show that  $\Delta_1$  is an increasing function in  $\xi$ .

In addition,  $\Delta_2$  can be alternatively expressed as

$$\Delta_2 = \left(1 + \frac{L_0}{e^{\xi} - L_0}\right) \left[A\gamma_{\mathsf{b}}\left(1 + \frac{L_0}{e^{\xi} - L_0}\right) + B\right]\eta_{\scriptscriptstyle M}, \tag{22}$$

and  

$$\frac{\partial^2 \Delta_2}{\partial \xi^2} = \eta_{\scriptscriptstyle M} L_0 \frac{e^{\xi} \left[ 2 \gamma_{\rm b} A e^{\xi} (2e^{\xi} + L_0 - 1) + B(e^{2\xi} - L_0^2) \right]}{(e^{\xi} - L_0)^4} \ge 0$$

Therefore  $\Delta_2$  is convex in  $\xi$ . It is straightforward that  $\Delta_2$  is a decreasing function in  $\xi$ . The convexity of  $E_m$  with respect to  $\xi$  is then proved by using Theorem 1.

Once the convexity in  $\xi = \log(L + L_0)$  is established, the optimum L can be solved as stated in the following proposition.

*Proposition 4:* In a quasi-static Rayleigh fading channel, if the FER is given in (11), then the optimum L that minimize  $E_{\rm m}$  is

$$\hat{L} = \frac{2A\gamma_{\rm b} - 2Ak_{_M} + B}{4k_{_M}(A\gamma_{\rm b} + B)}\gamma_{\rm b}L_0 + \frac{\sqrt{4A^2k_{_M}^2 + 4Ak_{_M}(2A\gamma_{\rm b} + 3B) + (2A\gamma_{\rm b} + B)^2}}{4k_{_M}(A\gamma_{\rm b} + B)}\gamma_{\rm b}L_0.$$
(23)

where  $A = \frac{\xi_M N_0 G_d}{\eta}$ , and  $B = \frac{\beta}{R_b}$ . *Proof:* The optimum L is obtained by solving  $\frac{\partial E_m}{\partial \xi} = 0$ ,

$$\frac{\partial \Delta_1}{\partial \xi} \Delta_2 + \frac{\partial \Delta_2}{\partial \xi} \Delta_1 = 0.$$
(24)

Since

$$\frac{\partial \Delta_1}{\partial \xi} = \frac{2k_M}{\gamma_b} \exp\left(\frac{2\gamma_\omega}{\gamma_b}\right), \qquad (25a)$$

$$\frac{\partial \Delta_2}{\partial \xi} = -\frac{\eta_M L_0 e^{\xi}}{(e^{\xi} - L_0)^2} \left(\frac{2A\gamma_b e^{\xi}}{e^{\xi} - L_0} + B\right). \quad (25b)$$

Substituting the above results into (24) leads to

$$2k_{_{M}}(A\gamma_{\mathbf{b}}+B)L^{2}+\gamma_{\mathbf{b}}L_{0}(2Ak_{_{M}}-2A\gamma_{\mathbf{b}}-B)L-2A\gamma_{\mathbf{b}}^{2}L_{0}^{2}=0.$$

The result in (23) is obtained by solving the above equation.

It is worth pointing out that even though the result in Proposition 4 is obtained through  $\frac{\partial E_m}{\partial \xi} = 0$ , it is exactly the same as solving  $\frac{\partial E_m}{\partial L} = 0$  because  $\frac{\partial \xi}{\partial L} = \frac{1}{L+L_0} \neq 0$ .

# C. Joint Optimum $\gamma_b$ and L

In (17) and (23), the optimum value of  $\gamma_b$  is expressed as a function of L and vice versa. The global optimum operation point can be achieved by jointly optimizing  $\gamma_b$  and L [14].

The joint optimum values can be obtained by treating (17) and (23) as a system of two equations with two variables in  $\gamma_b$  and *L*.

Alternatively, the joint optimum values can be numerically calculated by iteratively invoking (17) and (23). Given an initial value L, we can calculate the optimum  $\gamma_b$  by using (17), the output of which is then used to update the value of L with (23). This procedure can be performed iteratively, and it will efficiently converge to the joint optimum values of  $\gamma_b$  and L.

Fig. 1 visualizes the convergence of the iterative procedure described above by plotting (17) and (23) in the same figure. The intersection of the two curves is the optimum value. In the figure, M = 4 and d = 100 m. Given initial value of  $L + L_0 = 10^4$  bits, the iteration follows the trace and eventually converges to the optimum value. The figure shows that the calculation converges in 3 iterations.



Fig. 1. Convergence to obtain joint optimum  $\gamma_b$  and L.

## IV. NUMERICAL RESULTS

Numerical results are presented in this section to verify the optimum system design. Most of the simulation parameters followed those in [1] with  $\eta = 3.5$ ,  $\beta = 310.014$  mw,  $N_0/2 = -174$  dBm/Hz,  $G_1 = 30$  dB,  $\kappa = 3.5$ , and  $M_1 = 40$  dB. The rest of the parameters are  $L_0 = 48$ ,  $\alpha = 0.22$ , and a fixed bit rate  $R_b = 300$  kbps. The channel code is a rate  $r = \frac{1}{2}$  convolutional code with the generator polynomial  $[5, 7]_8$  and a constraint length 3.

In Fig. 2,  $E_m$  is plotted as a function of  $L + L_0$  under various values of  $\gamma_b$ . The distance is d = 100 m. The optimum values of  $\hat{L}$  for different  $\gamma_b$  are calculated from (23), and are marked in the figure. As expected,  $E_m$  is convex in  $\log(L + L_0)$ . Excellent agreement is observed between the analytical optimum operation points and the simulation results. When  $\gamma_b$  is small, increasing L after its optimum operation point leads to a fast performance degradation. On the other hand, when  $\gamma_b$  is large, e.g.,  $\gamma_b = 8$  dB, increasing L after its optimum operation point has a much smaller impact, because the FER only degrades slightly with L at high  $\gamma_b$ . Therefore, system operates at low  $\gamma_b$  is more sensitive to the frame length.

In Fig. 3 and Fig. 4, we compare the  $E_t$  and SE of systems employing two different design metrics: one minimizes  $E_m$ , and the other one minimizes  $E_t$  [8]. The minimum  $E_m$  and minimum  $E_t$  are calculated from the joint optimization of  $\gamma_b$ and L. Minimize  $E_t$  achieves the optimum EE, but at the cost of SE. It can be seen from the figures that, compared to the  $E_t$ minimizing system, the proposed  $E_m$ -minimizing system can almost double the SE with very small loss in EE. At d = 100 m, the SE of the proposed scheme outperforms its  $E_t$ -minimizing counterparts by 70%, 80%, and 85% for systems with QPSK, 16QAM, and 64QAM, respectively. The corresponding loss in  $E_t$  is only 0.98 dB, 0.99 dB, and 1 dB, respectively.

## V. CONCLUSIONS

An optimum transmission scheme was proposed for coded Type-I ARQ systems operating in quasi-static Rayleigh fading



Fig. 2. Normalized energy per information bit  $E_{\rm m}$  v.s. number of bits per frame  $L + L_0$ .



Fig. 3. Energy consumption of minimum  $E_t$  and minimum  $E_m$  v.s. distance d.



Fig. 4. Spectral efficiency of minimum  $E_t$  and minimum  $E_m$  v.s. distance d.

channel. The scheme was designed to balance the EE-SE tradeoff by minimizing a new design metric, the NEE. Under the constraints of fixed modulation level and data rate, the NEE was minimized by identifying the optimum transmission energy in each transmission round and the optimum frame length, which were expressed as closed-form expressions of a wide range of practical system parameters, such as circuit power, modulation, coding, protocol overhead, etc. Both analytical and simulation results demonstrated that 1) systems operating in the low SNR regime is more sensitive to the frame length; 2) optimizing with respect to NEE instead of EE can almost double the SE with less than 1 dB loss in energy consumption per bit.

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