Receiver only Optimized Semi-Hard Decision VQ
For Noisy Channels

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Abstract—This paper proposes a new receiver optimized semi-hard-decision vector quantization (SHDVQ) for noisy channels, as a technique to alleviate the drastic increase in distortion incurred when the output of the classical source optimized vector quantizer (SOVQ) is sent over a noisy channel. The advantages of the proposed method are that it is computationally simple, requires minimal extra storage, and can be implemented solely at the receiver; thus allowing the encoder to be independent of channel conditions. Another advantage is that it can be used in conjunction with an index assignment to obtain the benefits of index assignment (IA). The decoder considers errors and erasures based on thresholding the log-likelihood ratio (LLR) of the bits comprising the transmitted index. Then, the proposed decoder computes the output as a linear combination of the codebook vectors based on the erasure bit locations and the IA. A novel performance analysis is presented, where the overall distortion is expressed as a convex combination of the distortion with an ideal IA and the distortion with random IA. The analysis is used to find the erasure threshold that minimizes the overall distortion. Finally, Monte-Carlo simulation results are presented to corroborate the derived theoretical expressions.

I. INTRODUCTION

Vector quantization (VQ) is a lossy source compression technique where encoder compresses the source (vector) signals using a pre-determined number of candidate vectors such that expected distortion is minimized [1]. The output of a source encoder is the index of the vector in the code book that is closest to the source instantiation. This index is transmitted to the receiver (Rx) over a noisy channel, and hence could be received in error. The decoder at the Rx picks the codeword corresponding to the received index as the estimate of the transmitted vector. Due to the channel induced index errors, the distortion incurred at the Rx with classical source optimized VQ (SOVQ) can be considerably higher than the distortion due to the source compression [2], [3] (i.e., for error free channels). There are essentially two kinds of memoryless noisy channels: discrete and continuous. In discrete channels, the Rx first detects the index (which could be in error) and maps it to an estimate of source instantiation. With conventional SOVQ, the Rx simply outputs the codeword corresponding to the received index. In continuous channels, a soft-metric (such as bit log-likelihood ratio (LLR)) is first computed at the receiver, and then soft-information is mapped to an estimate of the source instantiation. This operation however, is often computationally very expensive.

Discrete memoryless channels have been studied extensively in the literature, and two dominant approaches to mitigate the effect of channel have emerged: optimum index assignment (IA) [4], [5], [6], and channel optimized VQ (COVQ) [7], [8]. The former involves mapping codewords to Tx indices such that the most probable error events result in codewords which are close to the codeword corresponding to the correct index. In COVQ, the distortion metric is changed to be the expected distortion after accounting for possible index errors and a new set of codewords and encoding regions is picked so that overall expected distortion is minimized. It is well-known that optimum IA is an NP complete problem and several sub-optimal methods are known [2] - [9]. IA and COVQ are techniques that work well for discrete memoryless channels, and can also be used simultaneously to obtain robustness. Their natural extension for continuous channels (such as additive white Gaussian noise (AWGN) channel) has also been explored, and is termed as soft-decision VQ (SDVQ). Here, a soft-metric (such as the bit log-likelihood ratio (LLR)) is used to estimate the source instantiation, and the expected distortion after averaging over the noise statistics is used to define a new set of encoding regions at the Tx [10], [11].

On the analytical side, high rate analysis of VQ for noiseless channels has been done by many authors (e.g., [2], [12], [13] and the references therein). Analysis of the distortion in noisy channels in the presence of channel mismatch was presented in [3]. Bounds on the performance of SOVQ under noisy channel conditions are given in [14]. The high rate analysis has been extended to the noisy symmetric error channel in [15]. Rx only optimized VQ using linear filtering to minimize the average distortion using random IA was proposed in [16].

Most of the aforementioned methods suffer from two major drawbacks. First, they require channel knowledge at the Tx, as both encoding and decoding operations are changed. Acquiring channel knowledge at the Tx is realized via periodic feedback from Rx to Tx, which requires additional bandwidth and complexity in the uplink direction. Also, in applications such as recording audio/video data on storage media or deep space communications, the channel error rate may not be known at the time of encoding. Second, they are computationally expensive to re-optimize when the channel is time varying. Hence,
it is desirable to look for computationally simple techniques to reduce the average distortion, that can be implemented only at the receiver.

In this paper, a new method, which we term semi-hard-decision vector quantization (SHDVQ), is proposed and analyzed. This method uses codebook optimization at the receiver only. One of the main differences in the proposed method compared to the past work in [10], [11] is that, the soft-metric is not used to recompute a soft output codeword (also known as an estimation based decoder, as against a detection based decoder) for every received symbol. Instead, the Rx first performs semi-hard-decision decoding and declares some bits as erasures (the other bits may be received correctly or in error). The resulting index is then mapped to a codeword chosen based on IA and the erasure bit locations. The proposed method works for both discrete channels (with errors and erasures) and continuous channels (by applying a threshold based hard decision on the LLRs). To keep the receiver simple, a threshold is employed (on the LLRs) to declare erasures and the decoder outputs the linear combinations of encoder centroids based the erasure location and IA. The erasure threshold that minimizes the expected distortion after accounting for channel errors and erasures is computed from the analytical expressions derived in the sequel.

On the analytical side, a novel performance analysis of the average MSE distortion performance of SHDVQ is presented. The total distortion is expressed as a convex combination of the distortion with ideal IA and random IA. The specific IA employed determines the factor used in the convex combination, and needs to be numerically evaluated only once for a given IA and the number of quantization bits B. The analytical framework presented in this paper forms a powerful tool for studying the high rate performance of VQ with a specific IA for noisy channels. Simulation results confirm that such an approach works well in practice.

The rest of this paper is organized as follows. Section II describes the problem that is addressed. Section III provides the high rate analysis of the average distortion for the binary symmetric channel (with error and erasure) case with mean squared error (MSE) as distortion measure. Section IV describes a method to choose optimum threshold for declaring erasures. Section V provides the simulation results to demonstrate the gain in the proposed method.

II. PROBLEM SETUP AND PROPOSED RECEIVER

In this work, an n-dimensional stationary vector source with zero mean and source distribution described by the marginal probability density function (PDF) \( f_X(x) \) is considered and quantized using \( 2^B \) vectors. The encoder at the Tx picks the index corresponding to the codeword \( x_i \) (from the codebook) which is closest to the source instantiation \( x \). The distortion

\[ d(x, x_i) = \|x - x_i\|^2 \]

\( d(x, x_i) \) incurred in representing the source instantiation \( x \) as \( x_i \) is taken to be the mean square error (MSE) distortion, i.e.,

\[ d(x, x_i) = \|x - x_i\|^2 \]

The index \( i \) is transmitted over an additive white Gaussian noise (AWGN) channel using binary phase shift keying (BPSK) modulation\(^2\). The channel signal to noise ratio (SNR) at the Rx is assumed to be known.

At the receiver, LLRs are computed for the bits comprising the received index and compared against a threshold. This threshold will be optimized later for the given channel SNR and \( B \). When the magnitude of the LLRs are below the threshold, the corresponding bits are declared as erasures before passing them to the source decoder. Thus, the AWGN channel is converted into a binary symmetric error and erasure channel, where a bit is erased with probability \( \rho \) and flipped with probability \( \alpha \). For a \( B \) bit transmission, probability of correct reception is \( (1 - \alpha - \rho)^B \). For example, the channel transition probability matrix \( P \) \(^3\) (for \( B = 2 \)) can be written as

\[
\begin{bmatrix}
00 & 01 & 10 & 11 \\
00 & (1 - \alpha - \rho)^2 & \alpha(1 - \alpha - \rho) & \alpha(1 - \alpha - \rho) \\
01 & \alpha(1 - \alpha - \rho) & \alpha(1 - \alpha - \rho) & \alpha(1 - \alpha - \rho) \\
10 & \alpha^2 & \alpha^2 & \alpha^2 \\
11 & \alpha^2 & \alpha^2 & \alpha^2 \\
\end{bmatrix}
\]

where \( E \) denotes a bit erasure.

Depending on the decoded and erased bit positions, the decoder chooses one of the codewords (which are precomputed depending on the erasure location and IA) as the estimate of the transmitted vector. Moreover, to keep the storage requirements minimal, additional centroids are stored only for all 1 bit erasure cases. Hence, there are \( B \) \( 2^{B-1} \) additional centroids at the Rx apart from the \( 2^B \) centroids used for no-erasure cases. The following section describes the design of new centroids used at the receiver.

A. New Centroids

It is known that, for MSE distortion, the optimal centroids at the Rx for a discrete error and erasure channel, described by the \( P \) matrix, can be expressed as a linear combination of the codewords \( x_i \) at the encoder as \([2]\)

\[ x_j = \frac{\sum_{i=1}^{2^B} p_{ij} x_i}{\sum_{i=1}^{2^B} p_{ij}}, \quad 1 \leq j \leq 2^B \]

where there are \( 2^B \) centroids at the Rx because each of \( B \) bits could be received correctly, erased, or flipped. For a continuous

\(^2\)Uncoded BPSK is used here for illustration purpose only; in general, any modulation scheme and channel code can be used. The only requirement is that the receiver be able to compute (experimentally or theoretically) the index transition probability matrix or the soft-decision metric.

\(^3p_{ij}, \) the element in the \( i \)th row and \( j \)th column element of \( P \) gives the probability of receiving index \( j \) when \( i \) is transmitted.
channel, the optimum decoder output for SDVQ, given the channel statistics $Pr(R = r | I = i)$, can be written as [11]

$$x_R = \sum_{i=1}^{2^B} Pr(I = i | R = r) x_i, \quad R \in D_R,$$

where $D_R$ is the domain of the received signal vector. For example, if BPSK transmission is used for transmitting the indices, then $D_R = \mathbb{R}^B$. However, the above definitions of the centroids at the receiver, enforce re-computation of the output codewords when the channel statistics change or soft-metrics change. Here, a new, sub-optimal set of centroids (codewords) for the given $P$ matrix are proposed. The new centroids can be pre-computed based on the channel SNR condition and given IA. That is, in the proposed SHDVQ, the following code points are output by the decoder:

(i) when an index $j$, $1 \leq j \leq 2^B$ is received, $x_j = x_j$ (i.e., the codeword employed at the transmitter corresponding to index $j$),

(ii) when a single bit is erased, i.e., for $2^B + 1 \leq j \leq 2^B + B 2^{B-1}$, $x_j = \frac{x_{j,0} + x_{j,1}}{2}$ where $x_{j,0}$ (similarly $x_{j,1}$) is the codeword at the Tx with index $j$, but the erasure bit location is replaced by a bit 0 (similarly bit 1), and finally

(iii) when multiple bits are erased, the decoder outputs the all zero codeword, i.e., for $2^B + B 2^{B-1} + 1 \leq j \leq 3^B$, $x_j = 0$.

Since the receiver uses semi-hard decisions on LLRs (for declaring erasures), the technique is termed semi-hard-decision VQ. The following section analytically computes the distortion for such a receiver.

III. AVERAGE DISTORTION FOR MSE DISTANCE

The average distortion of a $B$ bit quantizer, for an $n$-dimensional source with source distribution $f_X(x)$ and an error free channel, can be written as

$$E_{d}^{SO} = \sum_{i=1}^{2^B} \int_{x \in R_i} f_X(x) d(x, x_i) \, dx,$$

where $x_i$ is the $i$th code word (or representative vector of the region $R_i$), and $d(x, y)$ is the distortion measure. For the SOVQ under the MSE distortion metric, it is known that [17]

$$E_{d}^{SO} = \frac{n}{n + 2 \kappa_n} \int_{x \in \{x\}} \lambda \frac{x^2}{2} f_X(x) \, dx,$$

where $\lambda(x)$ is the so called “point density function” [1] (loosely speaking, $2^B \lambda(x_i) dx$ is the number of code points in a small volume $dx$ around $x_i$). The source optimized point density function $\lambda(x) = c \frac{x^2}{2} f_X(x)$ where $c$ is a normalization constant [2]. When the index is transmitted via an error free channel, any IA is optimum. However, if the channel can be modeled as a noisy discrete memoryless channel with index transition probability matrix $P$, then the average distortion can be written as

$$E_d = \sum_{i=1}^{2^B} f_X(x_i) \sum_{j=1}^{2^B} p_{\pi(i)\pi(j)} \int_{x \in R_i} \|x - x_j\|_2^2 \, dx,$$

where $\pi(i)$ represents the IA, which is a bijective map $\pi : [1 : 2^B] \to [1 : 2^B]$ and $x_i$ represents the centroid vectors used in the Rx decoder. As mentioned earlier, IA is an NP complete problem and often sub-optimal methods such as simulated annealing [8], Hadamard transform tool based mapping [9], etc. are used to arrive at a good IA. Thus, in the rest of this paper, the notation $\pi(i)$ is replaced by $i$ for simplicity under the assumption that a good IA has been chosen.

A. Average distortion with 1 bit erasure and ideal IA

Consider that only 1 bit erasures occur and ideal IA is done. Then, (4) can be simplified by splitting the contribution from the correct reception and incorrect reception with 1 bit erasures. Let $\phi_E = (1 - \rho)^B$ represent the probability of correct index reception. Then,

$$E_{d,1}^{\phi_E} = \phi_E E_{d}^{SO} + \frac{1 - \phi_E}{B} \sum_{i=1}^{2^B} f_X(x_i) \int_{x \in R_i} \|x - x_i\|_2^2 \, dx,$$

where the subscript ‘$I$’ is used to emphasize that “ideal” IA is assumed. $E_{d}^{SO}$ is the average distortion due to source quantization (given by (3)), and $S(i)$ is the set of $B$ indices with 1 bit erasures which have a probability $(1 - \phi_E)/B$ in $i^{th}$ row of $P$.

Assuming ideal IA $^4$, the closest centroids differ in their indices by 1 bit. For high rate coding, when the shape of the source Voronoi regions are similar, the distortion between the new centroids $\tilde{x}_i$ and the source vectors in region $R_i$ can be upper bounded by sum of distortion between the centroid of $R_i$ and an offset vector $\tilde{\varepsilon}$. That is,

$$d(x, \tilde{x}_i) \leq d(x, x_i) + \tilde{\varepsilon}_{i,j},$$

where $\tilde{\varepsilon}_{i,j} = d(x_i, \tilde{x}_j)$. Now, $\frac{1}{B} \sum_{j \in S(i)} \tilde{\varepsilon}_{i,j}$ can be approximated by the average distortion between the centroid $x_i$ and the boundary of the hyper-ellipsoid$^5$. With high rate quantization, it is known that the Voronoi regions $R_i$ can be well approximated by using hyper-ellipsoids [12], with the values of $x$ satisfying

$$\frac{(x - x_i)^T (x - x_i)}{\kappa_n} \leq \left( \frac{v_i}{\kappa_n^{\frac{1}{2}}} \right)^{\frac{1}{2}},$$

where $\kappa_n$ is the volume of an $n$-dimensional sphere of unit radius, $x_i$ is the centroid of the region and $v_i$ is the volume of the region $R_i$. With the above approximation,

$$\tilde{\varepsilon}_i \triangleq \frac{1}{B} \sum_{j \in S(i)} \tilde{\varepsilon}_{i,j} \approx \left( \frac{v_i}{\kappa_n^{\frac{1}{2}}} \right)^{\frac{1}{2}}.$$

Substituting (8) and (6) in (5), it can be shown that

$$E_{d,1}^{\phi_E} \approx E_{d}^{SO} + (1 - \phi_E) \frac{2}{\kappa_n^{\frac{1}{2}}} \sum_{i=1}^{2^B} f_X(x_i) v_i^{\frac{1}{2}} v_i,$$

$^4$It is not guaranteed that such an index assignment is possible for all values of $B$ and dimension $n$. However, for any given values of $B$ and $n$, this can be satisfied for a fraction of the centroids.

$^5$since the new centroids for the erasure case can be expected to lie on the boundary of the hyper-ellipsoidal region.
where \(v_i\) is the volume of the Voronoi region, which can be approximated as \(v_i \approx 1/(2^B \lambda(x_i))\). Hence,
\[
E_{d,l}^{E} \approx E_{d}^{SO} + \frac{(1 - \phi_E) \sigma_n^2}{2 \frac{2n}{n+2}} \sum_{i=1}^{2^B} f_{x}(x_i) \lambda \frac{x_i}{\lambda} (x_i).
\]

Using the Monte-Carlo integration formula,
\[
\frac{1}{N} \sum_{i=1}^{N} \beta(y_i) \approx \int \beta(y) f_Y(y) dy,
\]
\[(10)\]
(where \(\hat{=}\) represents the asymptotic equality when \(N \rightarrow \infty\), \(E_{d,l}^{E}\) can be written as
\[
E_{d,l}^{E} \approx E_{d}^{SO} + \frac{(1 - \phi_E) \sigma_n^2}{2 \frac{2n}{n+2}} \int_{x} f_{x}(x) \lambda \frac{x}{\lambda} (x) \ dx. \quad (11)
\]

Also, recognizing that the integral term in (11) is proportional to the \(E_{d}^{SO}\) in (4),
\[
E_{d,l}^{E} \approx E_{d}^{SO} \left[ 1 + (1 - \phi_E) \left( \frac{n + 2}{n} \right) \right]. \quad (12)
\]

For an \(n\)-dimensional stationary Gaussian distributed independent random variables (with zero mean and unit variance in each dimension), average distortion can be shown to be
\[
E_{d,l}^{E} \approx \left[ 1 + (1 - \phi_E) \left( \frac{n + 2}{n} \right) \right] \left( \frac{2 \pi \kappa_n}{2 \pi} \right)^{\frac{n}{2}} \left( \frac{n + 2}{n} \right)^{\frac{n}{2}} \quad (13)
\]

The significance of the above equation is that it shows that the average distortion with ideal IA decreases at the same rate\(^6\) \(2^{-\frac{n}{2}}\) as \(B\) increases, albeit with larger coefficient. This is in contrast with the high rate distortion for random IA, which is discussed next.

**B. Average distortion with 1 bit erasure and random IA**

When the IA is random, the above development will not work, as 1 bit erasures need not result in codewords corresponding to neighbouring cells being received. In this case, the average MSE distortion can be derived from (4) as follows. Note that, the codewords at the decoder corresponding to indices with a single bit erasure and random IA when index \(i\) is sent, are computed by the Rx as \(\hat{x}_j = \frac{x_i + x_l}{2}\) where \(l\) is some random index. Then, it follows that
\[
E_{d,R}^{E} = E_{d}^{SO} + \frac{1}{B} \sum_{i=1}^{2^B} f_{x}(x_i) \sum_{j \in S'_i} \frac{1 - \phi_E}{B} ||x_i - \hat{x}_j||^2 v_i,
\]

where the subscript ‘\(R\)’ is used to emphasize that random IA is assumed. Also, \(j > 2B\), and for these values of \(j \in S'_i\), \(\hat{x}_j = \frac{x_i + x_l}{2}\) (for some random \(l\)). That is, \(S'_i\) contains \(B\) indices with codewords of the form \(\frac{x_i + x_l}{2}\), where the indices \(l\) are randomly chosen. The above expression simplifies to
\[
E_{d,R}^{E} = E_{d}^{SO} + \frac{1 - \phi_E}{4} \sum_{i=1}^{2^B} f_{x}(x_i) \left[ \frac{1}{B} \sum_{j \in S'_i} ||x_i - \hat{x}_j||^2 \right] v_i.
\]

**C. Effect of index assignment**

In this work, average distortion analysis is done in two parts. For the first part, an ideal IA is assumed to be employed by the encoder. With ideal IA, single bit errors are assumed to correspond to centroids in neighbouring cells. In the second part, a random IA is assumed, where index errors lead to a random vectors in the codbook. Then, the overall distortion of any particular IA is modeled as a convex combination of the distortion with the ideal IA and random IA. The convex combination factor can be seen as a single parameter that measures the “goodness” of the IA, and can be obtained experimentally using simple measurements. Thus, the expression for total distortion for IID Gaussian source due to 1 bit erasure and a given IA can be modified as
\[
E_{d,c}^{E} \approx E_{d}^{SO} \left[ 1 + \eta_l (1 - \phi_E) \left( \frac{n^2 + 2}{n} \right) \right] \quad (15)
\]
\[
+ (1 - \eta_l) \left( \frac{1 - \phi_E}{2} \right) \left( \frac{n + 1}{n} \right) \sigma_x^2
\]

where \(\eta_l \in (0, 1]\) depends on \(n, B\) and the IA. Simulation results show that 60% of the centroids meet the neighbourhood condition for most of the IA. This will be demonstrated in the simulation results.

**D. Average distortion with zero output for erasures**

Consider the case when the decoder outputs zero centroid (the mean of the source) whenever an erasure occurs. In the above analysis, if we replace \(x_j\) with all zero centroid \(0\), we can show that
\[
E_{d,zero}^{E} \approx E_{d}^{SO} + (1 - \phi_E) \sigma_x^2 \quad (16)
\]

Thus, whenever an erasure occurs, the decoder introduces an maximum error equal to the variance of the source. Comparing, (14) and (16), it can be concluded that using the average between the two centroids always pays-off in reducing the total distortion even when the IA is random. That is, scheme in (14) is better than the scheme in (16) in terms of total distortion.

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\(^6\)same rate as for the average distortion of SOVQ with error-free channels.
E. Average distortion for 1 bit errors

On similar lines of the analysis for 1 bit erasures, the 1 bit error case can also be analyzed under the assumption of ideal IA and random IA. It can be shown that

\[ E_{d,c}^{1e} \approx E_d^{SO} [1 + 4 \eta_2 (1 - \phi_c) (\frac{n^2 + 2}{n})] + 2(1 - \eta_2) (1 - \phi_c) (\frac{n^2 + 1}{n}) \sigma_x^2, \]

where \( \phi_c \) is the probability of correct reception and \( \eta_2 = \eta_1 \) since the IA is the same for both cases.

F. Average distortion for the proposed receiver

For a given channel, there is a non-zero probability that multi-bit erasures and errors can occur. In order to consider all possible multi-bit erasures, 3D centroid vectors must be stored/computed which is expensive when \( B \) is large. On the other hand, for reasonable SNRs single bit errors and erasures are the most likely events compared to multiple bit errors or erasures. Hence one can consider a decoder that outputs the receiver optimized codewords described in Section II-A whenever a single bit erasure occurs and outputs the all-zero codeword whenever multi-bit erasures occur (when multi-bit errors occur with no erasures or 1 erasure, the receiver outputs the codeword described in Section II-A). Thus, the total distortion can be written as

\[ E_d^{tot} = p_0 E_d^{SO} + p_{1e} E_{d,c}^{1e} + p_{1E} E_{d,c}^{1E} + p_{rest} E_d^{allzero}, \]

where \( p_{rest} = 1 - (p_0 + p_{1e} + p_{1E}) \), \( p_0 \) is the probability that no error has occurred, \( p_{1e} \) is the probability that 1 bit error has occurred and \( p_{1E} \) is the probability of 1 bit erasure. In the above, the (small) probability of receiving multiple bits in error with zero or one erasure and the associated distortion has been approximated by including it in the \( p_{rest} \) term. For the given SNR and erasure threshold \( \gamma \), one can compute these probabilities easily using the Gaussian error function. In the following section, we pick the optimum \( \gamma^* \) which minimizes the total distortion.

IV. OPTIMUM THRESHOLD SELECTION

First, it is clear that there is an optimum threshold for declaring erasures, since the average distortion due to errors is larger than that due to erasures, but both are larger than the distortion without any errors. That is, if a bit is likely to be in error, it is better to declare it as an erasure, but if it is likely to be received correctly, it is better accept the hard-decisions. So, if the threshold is too high, there will be many erasures (and even bits that could be decoded correctly will be declared as erasures), leading to a larger average distortion. A similar argument applies when the threshold is too low, leading to an optimal threshold.

For a Log-Likelihood Ratio (LLR) based erasure detector, the probability of correct reception with \( \gamma \) as the detection threshold is given by

\[ 1 - \alpha - \rho = Q \left( \frac{\gamma - 2SNR}{2 \sqrt{SNR}} \right), \]

where \( Q(\cdot) \) is the standard function for computing Gaussian tail probability. On similar lines, probability of receiving with 1 bit error is given by

\[ \alpha = Q \left( \frac{\gamma + 2SNR}{2 \sqrt{SNR}} \right). \]

From the above equations, \( \rho \) can be found and used for computing the \( \mathbf{P} \) matrix. The optimum \( \gamma^* \) can be found by differentiating \( E_d^{tot} \) with respect to \( \gamma \) and equating to zero. However, since this is mathematically intractable, numerical computation is used to find the optimum \( \gamma^* \) for the given values of \( SNR, B \) and \( n \). Figure 1 shows the value optimum \( \gamma \) for various values of \( SNR \) for 2-D Gaussian source with \( B = 6 \). It can be noticed that \( \gamma \) goes to zero as the \( SNR \) increases. This is as expected, since, as the channel approaches an error free channel, the Rx should declare erasures less frequently.

V. SIMULATION RESULTS

For validation of the proposed method, a multi-dimensional Gaussian source with random IID entries with zero mean and unit variance is used. The performance of SHDVQ is plotted for various number values of \( B \) for a fixed source dimension \( n \). Comparison is done between the average distortion for SOVQ..
The optimum threshold values are chosen by numerically optimizing the random but fixed IA and the optimum threshold value. The values are used to declare erasure bit locations, depending on the values of the bits comprising the received indices. The LLR is used to compute the SNR, which is used to compute the LLR. The optimum threshold values are chosen by numerically minimizing \( E_{\text{tot}}^2 \) in (18).

Figure 2 shows the value of total distortion as a function of SNR for the optimum threshold. It also compares the performance of COVQ [15] and Rx filtering [16] under random IA assumption. It can be observed that the proposed method performs as good as or better than COVQ and Rx filtering method at all SNRs. This is due to the optimal threshold selection and usage of the new centroids. Figures 3 and 4 compare the performance of SHDVQ for various number of quantization bits. For these simulations, a (randomly chosen) fixed IA is used. Figures 3 and 4 also demonstrate the match between the simulation and the theoretical expression in (15) with \( \eta_1 = 0.6 \). Moreover, it has been observed through simulations that, the same value of \( \eta_1 \) works for wide range of values of \( B \) and SNR.

To conclude, in this paper, a receive-only optimized SHDVQ method was proposed for a noisy channel where errors and erasures are considered. It was shown that distortion can be reduced by adding \( B 2^{B-1} \) centroids to the Rx codebook. These new centroids are computed for the given IA and are independent of channel statistics, which provides a great computational advantage when compared to existing SDVQ methods. A novel method for the high rate analysis of average distortion was presented, by approximating the distortion as a convex combination of distortion due to ideal IA and random IA. A new decoder was proposed where 1 bit erasures result in picking vectors from an expanded set of centroids depending on the erasure location. The threshold used to declare erasure was obtained numerically by minimizing the overall distortion for the proposed receiver. Simulation results were provided to demonstrate the performance improvement using SHDVQ, and it was shown that SHDVQ performs as well as COVQ with random IA for the same SNR.

**References**


