Modelling of Film Casting Manufacturing Process Longitudinal and Transverse Stretching

P. CARLONE, G. S. PALAZZO AND R. PASQUINO
Department of Mechanical Engineering, University of Salerno
Via Ponte Don Melillo 1, 84084 Fisciano (SA), Italy
gspalazzo@unisa.it

(Received November 2004; accepted December 2004)

Abstract—Film casting is a widely used manufacturing process used to shape polymers into high quality films for package or other industrial and commercial purposes. The relevant steps of the manufacturing process are the longitudinal and transverse stretching, in which the material assumes the required dimensions and mechanical properties. This paper deals with the modelling of longitudinal and transverse stretching of the film casting process of isotactic polypropylene. These models are based on the suitable analogy (confirmed by experimental data) between the behavior of the polypropylene and some metals and on the remarkable directionality of the mechanical properties of the final product. Modelling leads to evaluation of the profiles of pressure and speed and of normal and tangential stresses. © 2005 Elsevier Ltd. All rights reserved.

Keywords—Film casting, Longitudinal stretching, Transverse stretching, Modelling industrial processes.

1. INTRODUCTION

Film casting (Figure 1) is a widely used manufacturing process for polymeric films production; other processes are blow casting and extrusion, now generally used for the production of low quality films. A general overview of film casting and other analogous processes is given by Stoner [1] who also deals with manufacturing processes of different materials, as cellulose nitrate, polycarbonate, and polypropylene. An important advantage of film casting with respect other similar processes is the relatively faster and more uniform film cooling at the extrusion die exit, and the possibility to work at relatively higher temperatures, with good consequences on productivity, film thickness, and optical properties. All above manufacturing processes make use of an extruder (with plane extrusion die in film casting). The raw material, i.e., polymer with opportune additives is introduced in the extruder which is constituted by an empty cylinder in which a worm screw is placed (screw extruder).

Three different zones can be identified: an alimentation zone, a transition zone, and a material metering zone. At worm screw end, polymer across a regulation valve used to obtain the required pressure increase. Then, a filter constituted by a drilled plate is placed, melted polymer exit the extrusion die and the polymeric film is cooled by a roll at a constant temperature.
Figure 1. Film casting process.

(a). Non-stretched polymeric macromolecule.  
(b). Stretched polymeric macromolecule.

Figure 2.

Figure 3. Transverse stretch.

An air jet is blown on the polymer to obtain a good adhesion between roll and film and another air jet is used to make easier film release after lamination. Roll is dried to avoid gas pocket formation.

Polymeric film is now ready for the longitudinal and transverse stretching (spreading), which produce a preferential macromolecular orientation in the stretching direction; as a consequence, mechanical properties acquire a remarked directionality (Figures 2a and 2b)).

Longitudinal stretching is obtained with several couple of counter-rotating rolls kept at a constant temperature by an heating system used to maintain the polymer temperature in the right range during the elongational process. In this way better mechanical properties can be obtained due to the preferential direction (in stretching direction) of the polymeric molecules. Rolls couples velocity ratio is called "longitudinal stretch ratio".

Transverse stretching is obtained using "rameuse" working machines, constituted by two systems of chains and clamps which grip the film edges and make it spread. Transverse stretching is made up by different substeps (Figure 3): preheating (1), stretch (2), thermal stabilization (3), cooling (4).

The subsequent steps of the longitudinal and transverse stretching the polymeric film is subjected to a thickness measurement by a $\beta$ ray scanner device, as well as to the flame-corona treatment, to obtain the required increase of the film surface tension which lead up to a better film response to coloration, metallization or coating (in general meaning). The last steps are then the film recovering and winding.
Currently, a relevant technological problem refers to the analysis of the longitudinal and transverse stretching. Indeed, several researchers have recently studied film casting process, focusing their attention on only two aspects of the process: the stability (draw resonance) and the molten polymeric film extrusion. In particular, a stability analysis based on Giesekus rheological model have been realized by Iyengar et al. [2] and Pis-Lopez et al. [3,4]. Silagy et al. [5] developed 2D Newtonian and viscoelastic models in stationary and unstationary conditions for stability analysis. A non linear analysis has been developed by Ramos [6] and numerical modelling have been proposed by Lee et al. [7,8].

Therefore, this paper is mainly focussed with the modelling of the last part of the manufacturing process which is generally dealt with heuristically in the existing literature, which is focussed to the first steps of the manufacturing process, as documented in the above literature. In details, Section 2 deals with the modelling of the longitudinal stretching, Section 3 develops a similar analysis for the transverse stretching, while a critical analysis is proposed in Section 4.

2. LONGITUDINAL STRETCHING MODELLING

This section deals with the modelling of the longitudinal stretching of isotattic polypropylene film casting. An important feature of this model is that the longitudinal stretch is associated with the metal lamination process. This association is suggested by the analogy of the stress-strain curves of the isotattic polypropylene and metallic materials. Isotattic polypropylene shows the above characteristic up to melting temperature (which is considerably higher than the temperature of the process), although the increase of the temperature may cause elastomerization phenomena, intermolecular bond became weaker and weaker and strength falls. In this way, polymer can be only approximately considered as a viscous fluid.

Let us now consider, for simplicity, that the elongational process is obtained (Figure 4) with only two rolls (with parallel axis and rotating at the same velocity). During the stretching at rolls interface the polymer is subjected (in every point) to a force normally directed respect contact surface. Polymer reaction can be considered of the magnitude of the product between the contact surface and the material deformation strength.

![Figure 4. Longitudinal stretch.](image-url)
Considering that the flow is nonmonodimensional, mass flow rate writes,

\[ \dot{m} = \rho v A, \]

where

\[ v = \frac{1}{A} \int_A \vec{v} \cdot \hat{n} dA, \]

and where \( A \) is the cross section and \( n \) its normal direction.

Considering the material density as a constant, taking into account the mass flow rate conservation the polymer velocity in the longitudinal direction increases, as the cross section (polymer speed is lower than peripheral rolls speed in mouth section) decreases. The specific cross section, between mouth and exit sections, in which the polymer average speed is equal to the peripheral rolls speed, is called "inversion section". Experimental data show that transverse slip increases with rolls diameter and with polymer-roll friction and it can be neglected using roll with reduced diameter and lubricants additives for the material. In this way, the material width can be considered as a constant and the flow as bi-dimensional.

Assuming that the polymeric material behaves as a Newtonian fluid, the continuity equation writes,

\[ \frac{\partial \rho}{\partial t} + v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} = -\rho \left( \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \right). \] (2.1)

For a steady state and incompressible fluid one has

\[ \nabla \cdot \vec{v} = \left( \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \right) = 0, \] (2.2)

while Stokes model, for a bi-dimensional problem, writes,

\[ \sigma_{xx} = -p + 2\mu \frac{\partial v_x}{\partial x} + \lambda \nabla \cdot \vec{v}, \] (2.3a)
\[ \sigma_{yy} = -p + 2\mu \frac{\partial v_y}{\partial y} + \lambda \nabla \cdot \vec{v}, \] (2.3b)
\[ \tau_{xy} = \tau_{yx} = \mu \left( \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right). \] (2.3c)

The incompressibility assumption yields,

\[ \sigma_{xx} = -p + 2\mu \frac{\partial v_x}{\partial x}, \] (2.4a)
\[ \sigma_{yy} = -p + 2\mu \frac{\partial v_y}{\partial y}, \] (2.4b)
\[ \tau_{xy} = \tau_{yx} = \mu \left( \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right). \] (2.4c)

Consider now momentum conservation equations:

\[ \rho \left( v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} \right) = \rho g_x + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y}, \] (2.5a)
\[ \rho \left( v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} \right) = \rho g_y + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{xy}}{\partial x}. \] (2.5b)

Equations (2.4) can be rewritten as follows:

\[ \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} \right) + \mu \left( \frac{\partial}{\partial x} \left( \frac{\partial v_x}{\partial x} \right) + \frac{\partial}{\partial y} \right), \]
which, in the case of incompressible flow, writes,

\[
\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} \right).
\]

Proceeding in the same way, for the term \( \frac{\partial \sigma_{yx}}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} \) and neglecting inertial and weight, the momentum equation writes,

\[
\rho \left( v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} \right),
\]

\[
\rho \left( v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} \right).
\]

Considering that the calibration leak \( h \) is of a smaller order with respect to the rolls ray \( R \), the polymer flow between two rolls can be considered as a flow between two parallel and infinite surfaces and the "lubrication approximation" implies that \( v_y \) is of a smaller order with respect to \( v_x \) as well as \( \frac{\partial v}{\partial x} \) with respect to \( \frac{\partial v}{\partial y} \).

Combining (2.6) with (2.2) and taking into account the lubrication approximation yields,

\[
\frac{\partial v_x}{\partial x} = 0, \quad \frac{\partial p}{\partial x} = \mu \left( \frac{\partial^2 v_x}{\partial y^2} \right), \quad \frac{\partial p}{\partial y} = 0,
\]

\[
\sigma_{xx} = \sigma_{yy} = -p,
\]

and

\[
\tau_{xy} = \tau_{yx} = \mu \left( \frac{\partial v_x}{\partial y} \right).
\]

So that one finally has,

\[
v_x = v_x(y),
\]

\[
\frac{dp}{dx} = \mu \frac{d^2 v_x}{dy^2},
\]

\[
p = p(x).
\]

Supposing that the speed of the film-rolls contact points is equal to rolls peripheral speed and indicating with \( h(x) \) contact points ordinate (Figure 4), the previous boundary condition writes,

\[
v_x(\pm h) = U,
\]

being \( U = \omega R = 2\pi n R \) and with \( n \) rolls angular velocity.

Integrating (2.9b) two times yields,

\[
v_x(y) = \frac{y^2}{2\mu} \left( \frac{dp}{dx} \right) + C_1 y + C_2.
\]

Some manipulations of (2.11), taking into account the boundary condition (2.10) yield,

\[
C_1 = 0,
\]

and

\[
C_2 = U - \frac{h^2}{2\mu} \left( \frac{dp}{dx} \right).
\]

Finally,

\[
v_x(y) = U + \frac{y^2 - h^2}{2\mu} \left( \frac{dp}{dx} \right),
\]
considering that the flow is bi-dimensional, volumetric flow rate writes

\[ q = 2 \int_0^h v_x \, dy = 2h \left[ U - \frac{h^2}{3\mu} \left( \frac{dp}{dx} \right) \right]. \]  

(2.13)

Suppose now uniform material speed in the exit section (flat speed profile), the boundary condition (2.10) yields,

\[ v_x (y) = U. \]  

(2.14)

In the exit section,

\[ \int dy = v_y = \frac{dy}{dx} = 0 \rightarrow \frac{d^2 v_x}{dx^2} = \mu \left( \frac{d^2 v_x}{dy^2} \right). \]  

(2.15)

Equation (2.14) allows to evaluate volumetric flow rate in the exit section as follow,

\[ q = 2H_u U, \]  

(2.16)

where \( H_u = h(x_u) \) and \( x_u \) is the abscissa of the exit section. Taking into account (2.13),(2.16) pressure gradient writes,

\[ \frac{dp}{dx} = 3\mu U \left( 1 - \frac{H_u}{h} \right). \]  

(2.17)

This term is null for \( h = H_u \), i.e., considering symmetry, for \( x = \pm x_u \). Substituting (2.17) in (2.12), the speed profile writes,

\[ v_x (y) = U \left[ 1 + \frac{3}{2} \left( \frac{y}{h} \right)^2 - 1 \left( 1 - \frac{H_u}{h} \right) \right]. \]  

(2.18)

The speed profile results symmetric with respect to the lamination axis (for control volume in Figure 3) and, being \( C_1 = 0 \), only a minimum or a maximum point along \( y \) direction is admitted. Boundary condition (2.10) implies a flat speed profile in inversion section (with an average speed value equal to rolls peripheral speed). (Inversion section abscissa is indicated, qualitatively, with \( x^* \) in Figure 7).

The \( x_0 \) section is characterized (considering the conservation of the volumetric flow rate) by the maximum average speed value (major than rolls peripheral speed).

In the \( x_0 \) section the speed profile cannot be flat, and it is characterized by a convexity in the advancement direction. This fact means that Figure 4 is not in agreement with (2.14), which needs a flat speed profile in the exit section. It is necessary to refer to Figure 5 which is, anyhow, a more realistic one related to the specific considered process (Figure 6). In sections successive to \( x_0 \), the polymeric flow average speed decreases (i.e., speed profile tends to become flat again) till exit section in which speed profile is completely flat and the speed value is uniformly identical to rolls peripheral speed.

Taking into account (2.17) tangential stress writes,

\[ \tau_{xy} = \tau_{yx} = y \frac{3\mu U}{h^2} \left( 1 - \frac{H_u}{h} \right). \]  

(2.19)

Taking into account (2.9b) and (2.17) yields,

\[ \frac{d^2 v_x}{dy^2} = \frac{1}{\mu} \left( \frac{dp}{dx} \right) = \frac{3U}{h^2} \left( 1 - \frac{H_u}{h} \right) = 3U \left[ \frac{h - H_u}{h^3} \right]. \]  

(2.20)

The function between square parenthesis (with \( h = h(x), x \in [x_1; x_u] \) and \( h \in [H_0; H_u] \)), is null for \( h = H_u \), being negative for \( h \in [H_0; H_u] \): pressure gradient presents a maximum for \( h = H_0 \) and a minimum for \( h = H_u \). Considering that the relationship between tangential stress and speed gradient (Newtonian fluid) is linear, \( \tau \) assumes its maximum value in the calibration leak and it is equal to zero in the exit section, according to (2.15). The pressure profile is obtained integrating (2.17) and using a successively evidenced boundary condition.
Figure 5. Longitudinal stretch with flat speed profile in the exit section.

Figure 6. Multi-stage lamination.

Figure 7. Speed profiles.

\[ p(x) = \int \frac{3\mu U}{h^2} \left( 1 - \frac{H_u}{h} \right) dx = 3\mu U \int \frac{1}{h^2} dx - 3\mu U H_u \int \frac{1}{h^3} dx. \quad (2.21) \]

The profile evaluation is now related to the integral

\[ \int \frac{1}{h^n} dx, \quad (2.22) \]

with \( n = 2, 3 \).
Then, (2.22) writes
\[
\int_{x_j}^{x_u} \frac{\cos \theta}{(R_0 + H_0 - \sqrt{R^2 - X^2})^n} dx. \quad (2.23)
\]

Figure 8 shows the relationship \( h = h(x) \), where \( X^2 + [y - (R + H_0)]^2 = R^2 \), and \( h(x) = (R + H_0) - (R^2 - X^2)^{0.5} \).

In other words, considering that \( h = (R + H_0) - R \cdot \cos(\theta) \), and \( x = R \cdot \sin(\theta) \), (see Figure 9), the integral (2.22) writes,
\[
\frac{1}{R^{n-1}} \int_{\theta_c}^{\theta_d} \frac{\cos \theta}{[(R_0 + H_0)/R - \cos \theta]^n} d\theta, \quad (2.24)
\]
where \( \theta_c \) is the first contact angle and \( \theta_d \) the breakaway angle. Integration of (2.17), considering (2.24), gives the following expression of the pressure,
\[
p(\theta) = F(\theta) + K.
\]
The constant $K$ value is deducible from a boundary condition on hydrostatic pressure or, in the same way, on normal stress, (as expressed in (2.8@)),

$$\sigma_{xx} = \sigma_{yy} = -p. \quad (2.25)$$

The necessary boundary conditions derives by the supposition that hydrostatic pressure is null at the breakaway between the material and one of the two rolls, i.e., at the exit section ($x = x_u$, $\theta = \theta_d$).

The above model is consistent as we have derived a number of equations equal to the number of dependent variables.

### 3. TRANSVERSE STRETCHING MODELLING

Transverse stretching is the process in which the film width is increased by the use of “rameuse” working machines, constituted by two systems of chains and clamps which grip the film edges and make it spread. Transverse stretching is made up by four different processes visualized in Figure 3,

- preheating (1), characterized by a very small divergence of the chains, so that the polymeric film is put under tension without causing a real stretching;
- stretching (2), until a fixed stretching ratio is obtained using an opportune divergence of the chains;
- thermal stabilization (3): in this zone the chains move along parallel directions and the macromolecule of the polymeric film reach a stable configuration to avoid film shrinking;
- cooling (4), characterized by film cooling to room temperature.

This section deals with the modelling of the transverse stretching of film casting manufacturing process. The present model is developed considering the polymeric film as a series of thin fibres which move along the material advancement direction according with Navier Stokes model and momentum conservation equation. This approach to transverse stretching is characterized by a certain loss in material continuity, largely justified by the remarkable material orientation.

The modelling is based on the following assumptions which are largely adopted and in agreement with experimental data: steady-state regime and the polymer treated as a Newtonian incompressible flow. The polymer temperature, the divergence corner (called $\theta$), and the polymer velocity in axial direction (called $v_x$) are assumed as constants during the stretching. The present model does not take into account eventual edge effects due to the presence of the clamps. Results of this model are the polymer speed, the pressure field, and the tensional field. A very important aspect is the “beroullian” expression of pressure field.

\begin{align}
\rho \left( v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} \right) &= -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} + \frac{\partial^2 v_x}{\partial z^2} \right), \quad (3.1a) \\
\rho \left( v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} + v_z \frac{\partial v_y}{\partial z} \right) &= -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} + \frac{\partial^2 v_y}{\partial z^2} \right), \quad (3.1b) \\
\rho \left( v_x \frac{\partial v_z}{\partial x} + v_y \frac{\partial v_z}{\partial y} + v_z \frac{\partial v_z}{\partial z} \right) &= -\frac{\partial p}{\partial z} + \mu \left( \frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial y^2} + \frac{\partial^2 v_z}{\partial z^2} \right). \quad (3.1c)
\end{align}

Considering that $v_x$ is a constant, (3.1a) yields,

$$\frac{\partial p}{\partial x} = 0, \quad (3.2)$$

and then there are no hydrostatic pressure variations in the $x$-direction.

Supposing now that the speed profiles are perfectly flat in planes orthogonal to machine axe $x$, i.e., the film cross sections are always rectangular, one has,

$$\frac{\partial v_y}{\partial z} = \frac{\partial v_z}{\partial y} = 0. \quad (3.3)$$
Some manipulations of (3.3), taking into account (3.1b),(3.1c) yield,
\[
\rho \left( v_x \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial y^2} \right),
\]
\[
\rho \left( v_x \frac{\partial v_z}{\partial x} + v_z \frac{\partial v_z}{\partial z} \right) = -\frac{\partial p}{\partial z} + \mu \left( \frac{\partial^2 v_z}{\partial x^2} + \frac{\partial^2 v_z}{\partial z^2} \right).
\]

Steady state and incompressible assumptions yield
\[
\frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0.
\]

Consider now a fiber element whose x-shape is small enough so that,
\[
\frac{\partial v_y}{\partial x} = \frac{\partial v_z}{\partial x} = 0.
\]

Equations (3.4a),(3.4b), for the considered fiber element, write,
\[
\rho \left( v_y \frac{\partial v_y}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v_y}{\partial y^2} \right),
\]
\[
\rho \left( v_z \frac{\partial v_z}{\partial z} \right) = -\frac{\partial p}{\partial z} + \mu \left( \frac{\partial^2 v_z}{\partial z^2} \right).
\]

Some manipulations of (3.6), taking into account (3.5), yield,
\[
\frac{\partial^2 v_y}{\partial y^2} = -\frac{\partial}{\partial y} \left( \frac{\partial v_z}{\partial z} \right) = -\frac{\partial}{\partial z} \left( \frac{\partial v_z}{\partial y} \right).
\]

Proceeding in the same way for \(\frac{\partial^2 v_z}{\partial z^2} \), assumption (3.3) yields,
\[
\frac{\partial^2 v_y}{\partial y^2} = \frac{\partial^2 v_z}{\partial z^2} = 0.
\]

Considering that \(v_y = v_y(y)\) and \(v_z = v_z(z)\) and (3.6a),(3.6b) can be written as follows,
\[
\rho \left( v_y \frac{dv_y}{dy} \right) = -\frac{\partial p}{\partial y},
\]
\[
\rho \left( v_z \frac{dv_z}{dz} \right) = -\frac{\partial p}{\partial z}.
\]
Figure 11 shows the remarkable symmetry of the problem, it is possible and more convenient to consider only the positive space, where

- \( L_0 \) is the initial half-width \((t = t_0)\);
- \( L \) is the half-width at \( t \) time \((t = t_0 + dt)\);
- \( S_0 \) is the initial half-thickness \((t = t_0)\);
- \( S \) is the half-thickness at \( t \) time \((t = t_0 + dt)\).

Considering that

\[ L = L(x) = L_0 + x \cdot \tan(\theta), \]

the volume conservation (or the cross section conservation in this specific case) implies,

\[ S = S(x) = \frac{S_0 L_0}{L(x)} = \frac{S_0 L_0}{L_0 + x \tan \theta}. \]

Indicating the clamps speed when they move away as \( U \), yields,

\[ v_y(L) = U, \]

and, taking into account that \( v_y(0) = 0 \) and \( v_z(0) = 0 \), one has

\[ \frac{dL}{dt} = \frac{dx}{dt} \tan \theta = v_z \tan \theta = v_y(L) = U \]

and

\[ \frac{U}{L} = \frac{(dL/dt)}{L} = \frac{(dL/L)}{dt} = \frac{dx}{dt} = \dot{x}. \]
Equality between deformation rate and speed gradient writes,

\[
\frac{U}{L} = \dot{\varepsilon} = \frac{dv_y}{dy}.
\]

Therefore,

\[
v_y = \frac{U}{L} y + C.
\]

Considering that \(v_y(0) = 0\) yields \(C = 0\), one finally has,

\[
v_y(x, y) = \frac{U}{L_0 + x \cdot \tan \theta} y. \tag{3.8}
\]

In the same way, (3.5) yields,

\[
v_z(x, z) = -\frac{U}{L_0 + x \cdot \tan \theta} z. \tag{3.9}
\]

Taking into account the boundary condition \(v_z(0) = 0\), yields,

\[
v_z(s) = -\frac{S_0 L_0}{(L_0 + x \cdot \tan \theta)^2} U. \tag{3.10}
\]

Equations (3.7a),(3.7b) can be rewritten as follows,

\[
\begin{align*}
\frac{\partial p}{\partial y} &= -\left( \frac{\rho}{2} \right) \frac{\partial v_y^2}{\partial y}, \tag{3.10a} \\
\frac{\partial p}{\partial z} &= -\left( \frac{\rho}{2} \right) \frac{\partial v_z^2}{\partial z}. \tag{3.10b}
\end{align*}
\]

The reintroduction of the \(x\) dependence implies, in (3.7), a reshut of the previously totalized derivative. This artifice (on whose rightfulness some words will be spent in the following) allows to lead a surely more complex problem to a Bernoulli theorem particular case.

Combining (3.8),(3.9) with (3.10a),(3.10b) yields,

\[
\begin{align*}
\frac{\partial p}{\partial y} &= -\rho \left( \frac{U}{L} \right)^2 y, \tag{3.11a} \\
\frac{\partial p}{\partial z} &= -\rho \left( \frac{U}{L} \right)^2 z. \tag{3.11b}
\end{align*}
\]

Pressure expression is in the form,

\[
p(y, z) = p_y(y) + p_z(z) + C_{yz}. \tag{3.12}
\]

Equations (3.10a),(3.10b) imply that, at fixed \(x\), both partial derivative are null in the axis origin, which is a stationary point for \(p(y, z)\). The Hessian matrix is always positive, then the origin is a minimum or maximum point. Considering that \(\frac{\partial p_x}{\partial y} < 0\) and \(\frac{\partial p_x}{\partial z} < 0\) (for \(y > 0\) and \(z > 0\)), the pressure value is maximum in the cross section barycentre of the polymeric film.

Equations (3.11) yield,

\[
p(y, z) = -\rho \left( \frac{U}{L} \right)^2 [y^2 + z^2] + C_{yz}. \tag{3.13}
\]
Bernoulli theorem generalization implies that pressure decreases in the increasing speed direction; the speed value is maximum for $z = s$ in $z$ direction and for $y = L$ in $y$ directions and $(L, s)$ is a point of minimum pressure, which can be considered equal to zero.

The boundary condition $p(L, s) = 0$ allows the evaluation of $C_{yz}$ and finally one has,

$$p(y, z) = -\frac{\rho}{2} \left( \frac{U}{L} \right)^2 \left[ (L^2 - y^2) + (s^2 - z^2) \right].$$  

(3.14)

Stokes model write,

$$\tau_{yz} = \tau_{sy} = \mu \left( \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial z} \right) = 0, $$

(3.15a)

$$\sigma_{yy} = -p + 2\mu \frac{\partial v_y}{\partial y}, $$

(3.15b)

$$\sigma_{zz} = -p + 2\mu \frac{\partial v_z}{\partial z}. $$

(3.15c)

Considering (3.8), (3.9), and (3.14), it is verified that the model is selfconsistent.

4. CONCLUSIONS

The modelling of longitudinal and transverse stretching of polypropylene film casting process have been developed in this paper. Models are based on analogies (confirmed by experimental data) between polypropylene and some metals and on the remarkable mechanical properties directionality of the final product.

Longitudinal stretching modelling, using the above mentioned analogy, assimilates this process to metallic materials lamination and leads up to the evaluation of the profiles of pressure and speed and of normal and tangential stresses. In particular, an inversion section with a flat speed profile is defined and has been shown that the exit section cannot coincide with the calibration section according with the real process. In the transverse stretching modelling, the polymeric film is associated to transverse placed thin fibres which move along the material advancement direction, according with Navier-Stokes model and momentum conservation equation. The reduction of the cross section (consequence of stretching) has been related only to thickness reduction, neglecting any deformation along machine axis. This approach to transverse stretching is characterized by a loss in material continuity, which can be largely justified by the remarkable orientation of the processed material. In order to obtain a more accurate solution, material continuity implies that,

$$\frac{dl}{l} = -\frac{dA}{A},$$
where \( l \) is the fiber length and \( A \) its cross section. Results of this model are the profiles of speed and pressure and of the tensional field. A very important aspect is the "Bernoullian" expression of the pressure field.

Equation (3.15b) implies the following general relation,

\[
\frac{3}{2} (\sigma_{yy} + p) = \lambda \frac{\partial \upsilon_y}{\partial y},
\]

where \( \lambda \) is the so called "elongational viscosity" (in first approximation equal to three times the Newtonian one).

Of course, the model can be made relatively more accurate by removing the various simplifications which have, however, generated some useful analytic solutions. These, however approximated description of the real behavior can be, in the authors' opinion, used to optimize the manufacturing process.

Of course, an interesting hint for research perspectives is the analysis of the system with inner random properties. This means developing for this specific system methods of stochastic mechanics [9–11], developed to derive homogenized models corresponding to a distributed randomness. As an alternative one may deal directly with the stochastic system of partial differential equations [12]. In all cases the leading idea consist in describing by suitable mathematical terms complex industrial processes [13]. Simulations may support, reducing the costs, the related experimental investigations. Modelling, mathematical methods and scientific computing have then a common objective [14].

REFERENCES