A method to decompose a 3D object into simple parts starting from its curve skeleton is described. Branches of the curve skeleton are classified as meaningful and non-meaningful by using the notion of zone of influence of the points where skeleton branches meet. Meaningful branches are associated to subsets of the object, which are obtained by subtracting from the input object suitably expanded versions of the zones of influence, termed overlapping regions. Decision is taken on whether the overlapping regions should be individual decomposition components, or should be assigned to properly selected adjacent object’s subsets. The so obtained object’s decomposition components are subdivided into parts characterized by simple shape through the polygonal approximation of the corresponding skeleton branches.

Keywords: 3D object decomposition, Curve skeleton, Polygonal approximation, Distance information

1. Introduction

Recognition of 3D objects is one of the main tasks of computer vision. For objects having articulated shape, recognition can be facilitated if the object is subdivided into a number of parts, each of which characterized by shape simpler than the shape of the whole object. A graph, where nodes are the parts into which the object has been decomposed and edges account for the spatial relations among the parts, can then be used to accomplish the recognition task.

The main motivation for object decomposition is the way in which the human visual system performs. In this respect, early papers on this subject,[1–5], have been a stimulus to further investigations and different approaches have been proposed. For example, decomposition has been guided by skeleton partition [6–12], distance information [13,14], diffusion distance [15], spectral clustering [16], modal analysis [17], and triangle mesh object representation [18–21].

The skeleton is one of the shape descriptors most often employed to achieve structural analysis of the object it represents. In fact, the skeleton is characterized by features that make it profitably usable in various contexts. For example, thinness and homotopy are respectively useful for motion planning and shape retrieval. Moreover, the skeleton structure reflects geometrical features of the represented object. This latter property is due to the fact that skeleton points are symmetrically placed within the object.

Possible skeletal structure representations of 3D objects can be obtained by means of the Medial Axis Transform [22], or the computation of the Voronoi diagram [23], the Shock graph [24] and the Reeb graph [25]. In particular, the Medial Axis is a loss-less shape descriptor. In fact, the object can be recovered by the envelope of the balls centered on the Medial Axis points and having radii equal to the distances of these points from the boundary of the object.

Since object recoverability is an important feature for a shape descriptor, an object decomposition method via skeleton decomposition is fruitful only starting from a skeleton whose points are labeled with their distance from the complement of the object. In this way, the skeleton retains information also on the morphological features of the represented object.

A number of algorithms in the literature originate distance labeled skeletons. In this work we use the discrete skeletonization algorithm [26], which reasonably well approximates the Medial Axis and originates a skeleton characterized by good recoverability. The skeleton is obtained by working with the (3,4,5)-weighted distance transform of the object [27]. Each voxel in the distance transform is associated the value of its distance from the complement of the object. Such a value can be interpreted as the radius of the ball that, centered on the voxel at hand, touches the object boundary. In particular, balls associated to the voxels symmetrically placed within the object touch the boundary in at least two distinct parts. Among the symmetry points, a crucial role for skeletonization is played by the centers of maximal balls. These are associated to balls, termed maximal balls, having two properties: (i) a maximal ball is not completely overlapped by any other single ball in the object and (ii) the envelope of all the maximal balls coincides in shape and size with the object. The skeletonization algorithm
[26] is based on the inclusion in the skeleton of centers of maximal balls. Moreover, that skeletonization algorithm includes an effective pruning phase that makes the results to be not significantly biased by the presence of noise or by topological transformations applied to the input.

Two different kinds of skeleton can be computed: (i) the surface skeleton, consisting of the union of surfaces and curves, and (ii) the curve skeleton, exclusively consisting of curves. Since skeletonization is a topology preserving process, the curve skeleton can be obtained only for objects rid of cavities. In fact, if a cavity exists in the object a closed surface surrounding it will unavoidably exist in the skeleton. Moreover, the curve skeleton should not be computed in case of objects whose main symmetry axes are outside the objects themselves. In fact, when this is the case, the curve skeleton would consist of curves that do not adequately represent the input object. For example, the curve skeleton of a hollow cylinder would be a ring that does not ensure object's recoverability. In fact, the radii associated to the skeleton voxels account for the thickness of the hollow cylinder but not for its height.

Only objects that are perceived as consisting of elongated intersecting parts (snake-shaped objects) are reasonably well represented by the curve skeleton, so that their decomposition can be guided by a suitable decomposition of the corresponding skeletons. In this work, we use the curve skeleton and, accordingly, snake-shaped objects constitute our image domain.

We decompose the skeleton into branches, with the aim of obtaining a decomposition of the object into a number of disjoint components, possibly equal to the number of branches. Object components associated with branches meeting in common points, i.e., the branch points of the skeleton, overlap with each other. Thus, to obtain object decomposition into disjoint components, each overlapping region has to be individually analyzed. An overlapping region should be left in exactly one of the object components including it, or it should be suitably subdivided into portions, each of which is assigned to one of the object components including the overlapping region. One further possibility to obtain decomposition into disjoint components is to consider the overlapping region as an individual component of object decomposition. In this case the number of disjoint object components overcomes the number of skeleton branches.

Our decomposition scheme is the follow up of previous work that we have done to decompose 3D and 2D objects by means of skeleton decomposition [9–11,28,29]. In particular, the decomposition method presented in this paper has been inspired by the method in [11], with respect to which the main difference is the inclusion of an additional step of the process. This step is aimed at a sub-division of the decomposition components into parts characterized by simple shape through the polygonal approximation of the corresponding skeleton branches. The relevance of this additional step can be appreciated in the framework of object recognition. In fact, the decomposition before the additional step can be used to identify the class to which an object belongs, in terms of the number of components and of their spatial relationships. In turn, the decomposition resulting after the additional step can be used to distinguish objects in the same class, for example having different size or pose.

The rest of the paper is organized as follows. Preliminary notions are given in Section 2, the method is illustrated in Section 3, experimental results are discussed in Section 4, and a brief conclusion is given in Section 5.

2. Preliminaries

The snake-shaped objects considered in this paper are recorded in binary voxel images in cubic grids and are rid of cavities. To avoid topological paradoxes, two different metrics are necessary for the object A and the background B. We use the 26-connectedness for A and the 6-connectedness for B. Without losing generality, we suppose that A consists of only one component. The 3D object “dog” shown in Fig. 1 left is used as running example.

The 26 neighbors of a voxel p are the voxels sharing with p a face, an edge, or a vertex. They are respectively denoted as face-, edge- and vertex-neighbors.

The (3,4,5)-weighted distance between two voxels p and q is defined as the length of a minimal discrete path linking p to q, where the three integer weights w_f = 3, w_e = 4 and w_v = 5 are used to measure the unit moves from a voxel towards its face-, edge- and vertex-neighbors along the path, respectively. As shown in [27], the (3,4,5)-weighted distance provides a good approximation to the Euclidean distance.

The (3,4,5)-weighted distance transform DT of the object A is a multi-valued replica of A, where each voxel is assigned the value of its (3,4,5)-weighted distance from B.

The curve skeleton S of A is a subset of A homotopic to A and whose voxels are symmetrically placed within A and are associated with their distance from B. For the running example, the skeleton is shown in Fig. 1 middle.

A voxel p of S is an end point, a normal point, or a branch point depending on whether it has respectively one, two or more than two neighbors in S. End points, branch points and normal points are respectively shown in green, red and black in Fig. 1 middle.

A skeleton branch is a connected subset of S entirely consisting of normal points, except for the two delimiting voxels. These can be end points or branch points. If both extremes are branch points, the skeleton branch is called internal. Otherwise, the skeleton branch is called peripheral. For the running example, S includes ten branches, out of which eight are peripheral branches and two are internal branches.

The object can be recovered starting from its skeleton by computing the envelope of the balls associated to the skeleton voxels. For the running example, the recovered object is shown in Fig. 1 right. Object recovery can be obtained by applying to the skeleton the reverse distance transformation [30]. Actually, only partial object recovery from the curve skeleton is generally possible. Complete recovery is guaranteed only if the skeleton includes all the centers of maximal balls. Inclusion of almost all centers of maximal balls is likely to happen only for the image domain including objects consisting of elongated parts. For such an image domain, the symmetry points of the object, and hence also the centers of maximal balls, are mostly aligned along symmetry axes. For other image domains, symmetry points are placed along symmetry planes and symmetry axes and, hence, only a subset of the centers of maximal balls can be included in the curve skeleton. In any case, we remark that a different number of voxels generally constitute the input object A and the envelope A’ of the balls associated with the voxels of S (compare Fig. 1 left with Fig. 1 right).

3. The method

The decomposition method includes the following steps: (1) detection of the zones of influence, i.e., the envelopes of the balls associated to the branch points; (2) detection of the meaningful and non-meaningful skeleton branches; (3) detection of the overlapping regions, starting from the zones of influence; (4) detection of object subsets by subtracting the overlapping regions from the object; (5) analysis of the overlapping regions for their assignment to the adjacent object subsets; and (6) possible further subdivision

1 For interpretation of color in Figs. 1, 2 and 4, the reader is referred to the web version of this article.
of the obtained disjoint object components into parts having simpler shape.

3.1. Zones of influence and meaningful skeleton branches

The zones of influence are obtained by computing the reverse distance transform of the branch points of S. Each zone of influence is the envelope of the balls centered on the branch points included in the zone. In fact, the balls centered on branch points that are neighbors of each other or have a distance smaller than the sum of the corresponding radii partially overlap and constitute a connected component. Thus, the number of branch points of S is generally larger than the number of zones of influence. See Fig. 2 left, where a unique zone of influence, shown in gray, is obtained in correspondence with sufficiently close branch points, shown in red. We note that a zone of influence is generally a nearly convex set since the balls whose envelope identifies the zone are rather close to each other.

A key role of a zone of influence is to gather in a single branching configuration those branch points of S that in an ideal continuous skeleton would correspond to a unique branch point where the skeleton branches, actually meeting in different branch points inside the zone of influence, would join with each other.

The zones of influence are used to distinguish meaningful and non-meaningful skeleton branches. We regard as non-meaningful any skeleton branch that is completely included in a zone of influence (see Fig. 2 left, where an internal skeleton branch results to be completely included in the zone of influence). All other skeleton branches are meaningful and are the only branches playing a role in the object decomposition process. In Fig. 2 right, the three zones of influence resulting for the running example are shown. Different colors denote identity labels assigned to the meaningful branches and to the zones of influence. The ten meaningful branches are in correspondence with ears, muzzle, neck, torso, tail, front legs, and back legs of “dog”.

3.2. Overlapping regions and object subsets

Even if the image domain includes only snake-shaped objects, the number of connected object subsets obtained by subtracting the zones of influence from the input object A is likely to be smaller than the number of meaningful skeleton branches identified in S. See Fig. 3 left.

We note that if the zones of influence were subtracted from the recovered object $A'$, i.e., the object obtained by applying to S the reverse distance transformation, the number of so achieved connected subsets would be equal to the number of meaningful branches of S. See Fig. 3 right.

Obviously, any given zone of influence Z is by all means adjacent to the complement B of the input object. However, the part of the surface of Z adjacent to the background may be not wide enough to divide the remaining part of the surface of Z into a number of parts equal to the number of meaningful skeleton branches meeting in Z. Thus, the part of the surface of Z non-adjacent to the background may provide cutting surfaces that do not separate into disjoint object subsets all the regions of A, whose associated skeleton branches meet inside Z. In general, the number of achieved connected object subsets is smaller than expected, since the zones of influence have smaller size than the true overlapping regions. Thus, the zones of influence have to be suitably expanded, so as to identify the true overlapping regions that, subtracted from the input object, will originate the desired number of object subsets. In other words, the zones of influence have to be expanded enough to create cutting surfaces in correspondence with significant curvature minima along the boundary of A.

A suitable expansion of the zones of influence is obtained by exploiting distance information available in DT, where all object voxels are labeled with their $(3,4,5)$-weighted distance from B. We identify the true overlapping regions with the sets obtained by applying the reverse distance transformation to the distance labeled zones of influence. In this way, the expanded zones have a sufficiently large part of their surface that is adjacent to B, so that the desired number of disjoint object subsets can be achieved. Propagation of the identity labels assigned to the zones of influence is also accomplished while performing the reverse distance transformation, so as to individually identify the overlapping regions.

We remark that a topology preserving expansion of the zones of influence is actually taken into account. In fact, fusion of overlapping regions may cause some meaningful skeleton branch to have all its voxels covered by overlapping regions. Thus, not all object subsets obtained by subtracting the overlapping regions from the input would include at least a voxel of a meaningful skeleton branch. As a consequence, a correspondence between meaningful skeleton branches and object subsets could not be established. To perform topology preserving expansion, voxels that during the reverse distance transformation are at the same distance from more than one zone of influence are not assigned to any overlapping region.
In Fig. 4 left the overlapping regions are shown. Topology preserving expansion has been crucial to prevent fusion between the yellow and gray overlapping regions, so that at least one skeleton voxel of the green meaningful skeleton branch will remain in the corresponding object subset.

Once the object subsets are obtained, the skeleton voxels placed therein are used as sources from which identity label propagation is accomplished within the object subsets. In this way, object subsets are labeled with the identity labels of the corresponding skeleton branches.

In Fig. 4 right, the object subsets are shown. Colors of object subsets denote the identity labels of the corresponding meaningful skeleton branches. We note that in correspondence with significant curvature minima along the boundary of “dog”, natural cutting surfaces are obtained. In fact, the property of the zones of influence of being nearly convex sets is preserved when reverse distance transformation is applied to them. In particular, the overlapping regions do not extend beyond the curvature minima along the boundary of the input object.

For the sake of completeness, we point out that, due to the discrete nature of the digital 3D space, in the set difference between the input object and the overlapping regions some very small size components of object voxels may exist, which do not include any skeletal voxel. These components do not correspond to meaningful object parts and their voxels are assigned to the overlapping regions they are adjacent to.

3.3. Analysis and assignment of the overlapping regions

We aim at decomposing an object into a number of disjoint components. To this purpose, we analyze each overlapping region to decide whether it has to be taken as an individual decomposition component. If the overlapping region is not an individual component, decision has to be taken on its assignment to the object subsets adjacent to it.

Let us denote by \( OR_k \) any overlapping region and by \( P_k^1, P_k^2, \ldots, P_k^n \) the object subsets adjacent to \( OR_k \). Generally speaking, \( OR_k \) is an almost convex set. In turn, \( P_k^1, P_k^2, \ldots, P_k^n \) are characterized by a different degree of concavity in correspondence with the portions of their boundary adjacent to \( OR_k \).

Let \( n(P_k^i) \) denote the number of voxels of \( OR_k \) having at least one face-neighbor in \( P_k^i \). We consider \( n(P_k^i) \) as a rough evaluation of the intrusion of \( OR_k \) within the adjacent subset \( P_k^i \). We think that the more \( OR_k \) intrudes in a given adjacent subset \( P_k^i \), the better would be the shape characterizing the compound region obtained by ascribing \( OR_k \) to \( P_k^i \).

We observe that if the degree of intrusion of an overlapping region into a given object subset is large, the difference in thickness between the object subset and the overlapping region is small. Thus, the compound region obtained by ascribing the overlapping region to the object subset would have a boundary characterized by good continuity.

In turn, a small degree of intrusion corresponds to an object subset whose thickness is remarkably smaller that the thickness of the overlapping region. In such a case, the boundary of the compound region would be affected by evident concavities. Thus, if for each \( P_k^i \) the corresponding \( OR_k \) has a degree of intrusion smaller than a threshold \( \delta \), \( OR_k \) should be accepted as an individual part of the decomposition. Of course, this choice implies that the number of object decomposition components overcomes the number of meaningful skeleton branches.

To fix the value of the threshold \( \delta \) in a way somehow based on the mutual relations among any overlapping region \( OR_k \) and its adjacent subsets \( P_k^1, P_k^2, \ldots, P_k^n \), we should normalize the measure \( n(P_k^i) \). The ratio \( R_k^i = n(P_k^i)/M_k \) is used to this aim, \( M_k \) measures the surface of \( OR_k \) by counting the number of voxels of \( OR_k \) having
at least one face-neighbor outside OR³. We have experimentally found that at least 10% of the surface of OR⁴ should be adjacent to P_i in order (part of) OR⁵ can be ascribed to P_i. Thus, we use as default value for the threshold the value δ = 0.1.

If OR³ intrudes sufficiently into only one adjacent subset P_i, then OR³ is assigned to P_i. If more than one object subset exists into which OR⁴ significantly intrudes, OR⁴ should be split into a proper number of portions each of which has to be ascribed to the proper adjacent object subset. To this purpose, let n(P_i) be the maximum among the n(P_i) of the object subsets for which δ has been overcome. Then, the adjacent subsets to which portions of OR⁴ should be ascribed are selected as those for which the ratio n(P_i)/n(P_i⁻) is larger than a threshold τ. We have found experimentally that τ = 0.85 is a threshold value producing in the average good results as regards object decomposition into perceptually significant regions. To properly split OR⁴ and assign its portions to the selected adjacent subsets, the identity labels pertaining those subsets are propagated over OR⁴.

To perform analysis and assignment of the overlapping regions in a computationally convenient manner, we resort to the use of an adjacency matrix. The adjacency matrix has a number of rows equal to the number of overlapping regions and a number of columns equal to the number of object subsets plus one for the background. During one inspection of the image, the number of voxels of each overlapping region with at least a face-neighbor in an adjacent part (object subset or background) is computed and is recorded in the proper position of the matrix.

The adjacency matrix for the running example is given in Table 1, where the three overlapping regions OR¹, OR² and OR³ are respectively those colored in yellow, gray and light blue in Fig. 4 left. The disjoint object parts obtained after assignment of the overlapping regions are illustrated in Fig. 5. We observe that the two overlapping regions OR¹ and OR² are assigned to the same object’s subset, namely the torso. The remaining overlapping region OR³ is assigned to the neck.

### 3.4. Simple parts

At this stage of the process, the object is decomposed into disjoint components. In our opinion, components corresponding to overlapping regions that are taken as individual regions of the decomposition cannot be furthermore subdivided. In fact, overlapping regions are almost convex blobs. In turn, we think that components associated with skeleton branches may be interpreted as having articulated structure. Indeed, curvature and thickness changes may exist along any of these object components. Thus, a final processing step is accomplished aimed at dividing object components associated with skeleton branches into a number of smaller components characterized by simple shape.

We regard as simple the shape of a region whose boundary is rid of significant curvature changes and whose thickness is either constant or evolves along the region in a linearly increasing or decreasing manner. In other words, we aim at dividing an object component into parts whose shape is that of either a cylinder or a cone in the limits of the chosen approximation.

The above goal can be reached by dividing the meaningful skeleton branches into segments, each of which consisting of voxels aligned along straight lines and with distance values that are either equal or change in a linearly increasing or decreasing manner. In fact, curvature changes along the boundary of an object component are reflected by curvature changes along the associated skeleton branch, and thickness changes are reflected by changes in the distance values of the skeleton voxels. To take into account simultaneously changes in curvature and thickness, a representation of the skeleton branches is considered in a 4D space, where skeleton voxels are mapped into points whose four coordinates are the three Cartesian coordinates and the distance values of the skeleton voxels themselves. In this 4D space, polygonal approximation is accomplished.

We use the split type polygonal approximation algorithm [31]. The extremes of each meaningful skeleton branch are accepted as vertices. The Euclidean distance of the skeleton voxels from the straight line joining the two extremes is computed; the skeleton voxel with the largest distance is accepted as a vertex, provided that such a distance overcomes an a priori fixed threshold θ. Any new vertex divides the branch into two pieces, to each of which the split type polygonal approximation is applied. Splitting is repeated as far as new vertices are found.

To compute the Euclidean distance d of a voxel c of a meaningful skeleton branch of S from the straight line joining the two extremes v and w of the branch, the following expression is used:

\[ d^2 = ||P_{vc} - P_{vc}||^2 - (v_{vc} - P_{vc})^2 ||vw||^2 \]

where ||vw|| is the norm of the vector vw, and P_{vc} is the scalar product between vectors vw and vc.

We have experimentally found that by using a threshold value θ = 5, the obtained polygonal approximation is reasonably faithful to the original skeleton branches, without creating an excessive fragmentation.

The vertices detected in the 4D space are mapped back onto the 3D skeleton. Thus, each meaningful skeleton branch is divided into segments rid of curvature changes and characterized by distance values that either are constant or linearly change. Segments are assigned an identity label to distinguish them, while maintaining
information also on the skeleton branch to which they belong. See Fig. 6 left.

Consecutive segments along a skeleton branch share common vertices and the balls associated with them may be not enough to separate into disjoint parts the object component mapped into that skeleton branch. Thus, analogously to what done to expand the zones of influence (see Section 3.2), the balls associated to the vertices are expanded into larger regions, termed hinges, by means of a topology preserving process guided by distance information. The hinges are then subtracted from the object component to split it into disjoint subsets. The identity labels of the skeleton segments included in the disjoint subsets are used to label the subsets themselves. As for the hinges, their voxels are assigned to the disjoint subset to which they are closer.

We point out that for any skeleton branch whose associated object component does not include the overlapping region, some segments of the approximated branch may exist that do not correspond to any of the disjoint parts into which the object component is split. This is the case when vertices of the polygonal approximation are found along the meaningful skeleton branch in positions that are inside the overlapping region.

At the end of the process, object components associated to skeleton branches result to be possibly divided into a number of smaller parts characterized by simple shape. See Fig. 6 right.

4. Experimental results and discussion

We have evaluated the performance of our method on a number of 3D snake-shaped objects taken from repositories such as the Princeton Shape Benchmark [32] and the McGill 3D Shape Benchmark [33].

The decomposition method involves only three thresholds. The threshold $\delta$ is used to establish if an overlapping region should be regarded as an individual decomposition component. The threshold $\tau$ is used to select the object subsets to which overlapping regions are ascribed. The threshold $\theta$ is used during polygonal approximation of meaningful skeleton branches, so as to divide the associated object components into parts characterized by simple shape. The same threshold values, $\delta = 0.1$, $\tau = 0.85$ and $\theta = 5$, have been used during the experimental work. The obtained results can be regarded as generally satisfactory from the perceptual point of view. In Fig. 7 some examples are given showing the performance of the method.

The decomposition method originates mostly the same result if changes of pose or size characterize the input object. In this respect, refer to Fig. 8, where different poses of the object “horse” are shown. All the skeletons, shown in the second line of Fig. 8, have the same number of meaningful branches. This is due both to the good performance of the adopted skeletonization algorithm and to the use of the zones of influence to distinguish meaningful and non-meaningful skeleton branches. The object subsets and the overlapping regions are shown in the third line of Fig. 8. In all cases, the two overlapping regions are assigned to the same subset, so identifying the torso of “horse”, as it can be seen in the fourth line of Fig. 8. The effect of polygonal approximation on the meaningful skeleton branches is reflected by the subdivision of the object components into parts with simple shape. By looking at Fig. 8 bottom line, we may observe that the different poses of “horse” correspond to a different articulation into simple parts of the object components.

For most of the 3D objects we have examined, each overlapping region is always assigned to exactly one adjacent object subset obtaining a compound region whose boundary shows good continuity. The selected object subset was characterized by thickness close to the thickness of the overlapping region.

We have also found a few objects for which an overlapping region is divided into a number of portions to be ascribed to adjacent object subsets. An example is given in Fig. 9, showing from left to right the 3D object “sea star”, its skeleton, the unique overlapping region and the adjacent object subsets, and the decomposition where the overlapping region has been divided among the five adjacent object subsets. In fact, for “sea star”, the default values of $\delta$ and $\tau$ are overcome for each of the five adjacent object subsets.

We have not found examples where the overlapping regions should be retained as individual decomposition components. Thus, we resort to the description of an artificial example and, for the sake of simplicity, we suppose that only one overlapping region exists. Let us refer to a spiny object like a sea urchin, where the overlapping region coincides with the whole shell. When the overlapping region is subtracted from the input object, the obtained subsets are as many as the spines of the urchin. If any spine, say $P_s$, was selected to absorb the overlapping region OR, it can be argued that the decomposition would result rather unnatural. In fact, the boundary of the compound region (shell plus the selected spine) would present significant curvature minima. On the other hand, if the overlapping region was subdivided among all the spines, the perceptual meaning of the obtained components would be questionable, even if the components have boundary characterized by good continuity. Most possibly, a human observer would decompose the sea urchin into its shell (the overlapping region) and the spines (the object subsets adjacent to the overlapping region). We achieve this goal by taking into account that the overlapping region intrudes very little (less than $\delta$) within each spine, so that the overlapping region is taken as an individual decomposition component.

The stability of the decomposition method with respect to topological changes of the input depends on the stability of the adopted
skeletonization algorithm. The algorithm [26] used in this work is rather stable. However, small details of the object may not always correspond to skeleton branches. As an example, refer to the object “armadillo” shown in Fig. 10. The main parts of “armadillo” (torso, four limbs, tail, ears and muzzle) are detected in all cases as individual decomposition parts, while small peripheral parts (toes) are not always individually detected as object parts.

We point out that our method decomposes the skeleton provided that it includes points that can be interpreted as delimiting skeleton branches. The simplest case is when the skeleton consists of a single branch, which is delimited by two end points. Thanks to the 4D polygonal approximation, the branch can be sub-divided into rectilinear segments so that the object can be partitioned into simple parts. The general case is when the skeleton includes end points and branch points.

Tunnels may exist in snake-shaped objects. If the object is a ring whose skeleton is a circle, no decomposition is obtained. In fact, no end points or branch points are found in the skeleton. If the ring has variable thickness or boundary curvature and its decomposition is desired, some criterion to identify on the skeleton two initial vertices, possibly coinciding with each other, should be devised, so that polygonal approximation can be performed. Criteria based on the maximal curvature along the skeleton or on the minimal distance value of skeleton voxels could be used to this aim. If branch points exist in the skeleton, loops found in correspondence with tunnels can be interpreted as consisting of as many branches as many are the branch points along the loops and decomposition is accomplished by following the scheme already described. An example of snake-shaped object and its decomposition are given in Fig. 11. There, we show from left to right the object “hand”, its skeleton, the unique zone of influence, the decomposition before and after polygonal approximation. We observe that notwithstanding the presence of a number of branch points in the skeleton, they are all included in the same zone of influence, found in

Fig. 7. From top to bottom: input objects, their skeletons, object subsets and overlapping regions, decompositions after assignment of the overlapping regions, and final decompositions where object components are divided into simple parts.
correspondence with the palm, from which the five fingers and the wrist protrude. Actually, since in “hand” the thumb and the forefinger touch each other, a unique meaningful skeleton branch is found in correspondence to these touching fingers. Accordingly, a single component including thumb and forefinger is found before polygonal approximation. After polygonal approximation, the component
Fig. 10. The skeleton of “armadillo” in different poses/sizes, top. Decompositions before assignment of the overlapping regions, middle. Decompositions after assignment of the overlapping regions, bottom.

Fig. 11. From left to right, an object with a tunnel, its skeleton, the zone of influence superimposed onto the skeleton, the decomposition before and after polygonal approximation.
in correspondence with the two fingers is divided into parts and, in particular, a separation between thumb and forefinger is found in the proper position.

5. Conclusions

In this work we have described a method to decompose a 3D snake-shaped object through the decomposition of its curve skeleton. The branch points in the curve skeleton are used to identify the zones of influence, i.e., the regions where different skeleton branches meet. The role of the zones of influence is twofold: (i) to group branch points sufficiently close to each other, so as to identify a unique branching configuration, and (ii) to distinguish meaningful and non-meaningful skeleton branches.

Only meaningful skeleton branches are considered in the decomposition of the skeleton. Object components associated to meaningful skeleton branches partially overlap in overlapping regions that are identified starting from the zones of influence. By subtracting the overlapping regions from the input object, disjoint object subsets adjacent to the overlapping regions are identified. Overlapping regions are taken as individual components or are suitably assigned to the adjacent subsets.

A polygonal approximation of meaningful skeleton branches has also been taken into account to furthermore decompose object components mapped into those skeleton branches. The obtained simple parts are characterized by thickness that is constant or evolves in a linearly increasing/decreasing manner and by absence of significant curvature changes along the boundary.

Satisfactory results have been obtained in the average by using the default values for the three thresholds involved in the method.

References

[33] McGill 3D Shape Benchmark <http://www.cim.mcgill.ca/~shape/benchMark/>. Luca Serino received the doctoral degree in Computer Science from the University of Salerno in 1991. In the period 1991-2014, he has been working at the “Institute of Cybernetics E. Caianiello” of the National Research Council of Italy. In March 2014 he moved to the “Institute for High Performance Computing and Networking” of the National Research Council of Italy. Currently, his research mainly regards segmentation of grey-scale pictures as well as skeletonization and decomposition of 3D objects in binary images.

Carlo Arcelli received the doctoral degree in Physics from the University of Bologna, Italy, in 1969. He has been active in the low-level and mid-level areas of computer vision. In particular, his work has been mainly concerned with feature extraction, digital geometry, pattern representation and description. Currently, he is a research fellow at the “Institute of Cybernetics E. Caianiello” of the National Research Council of Italy. He has served in the Fellow and Education Committees of the International Association of Pattern Recognition (IAPR) and is IAPR Fellow.

Gabriella Sanniti di Baja received the doctoral degree in Physics from the University of Naples, Italy, and the PhD Honors Causa from the Uppsala University, Sweden. She is active in the field of image processing and pattern recognition at the “Institute of Cybernetics E. Caianiello” of the Italian National Research Council, where she is Director of Research. Her main research interests concern 2D and 3D shape representation and analysis. She published about 200 papers in international journals and conference proceedings, is Co-Editor-in-Chief of Pattern Recognition Letters, IAPR Fellow and Foreign Member of the Royal Society of Sciences, Uppsala, Sweden.