3D PIV MEASUREMENTS IN TWO PHASE FLOW AND ROPE PARAMETRICAL MODELING

Gabriel Dan CIOCAN
Associate Professor, Dr. Ing., Laval University, LAMH,
Pavillon Adrien-Pouliot, QC, G1K 7P4, Canada,
tel.+33630378998, mail gabriel.ciocan@gmc.ulaval.ca

Monica Sanda ILIESCU
Researcher, Dr. Ing., Laval University, LAMH,
Pavillon Adrien-Pouliot, QC, G1K 7P4, Canada,
tel. +1(418)656-2131 / 13147, mail Monica.Iliescu@gmc.ulaval.ca

ABSTRACT

Partial flow rate operation of hydro turbines with constant pitch blades causes complex unstable cavitating flow in the diffuser cone. A PIV system allows investigating the flow velocity field in the case of a developing cavitation vortex, the so-called vortex rope, at the outlet of a Francis turbine runner. The synchronization of the PIV flow survey with the rope precession, allows, applying the ensemble averaging by phase technique, to extract both the periodic velocity components and the rope shape. Starting from the measurement results, a parametrical model of the vortex rope is proposed. Two sigma values are explored and the results are presented.

KEY WORD: Francis turbine, 2 phase 3D PIV, partial flow operation, rope, cavitation
INTRODUCTION

Hydropower is a clean form of power generation, which uses a renewable source of energy: water. Moreover, its storage capacity and flexible generation makes hydropower the quasi-ideal form of power generation to meet the variable demand of the electricity market. The drawback is the need of extending the range of the possible operation of the turbines towards lower flow rates, for adapting to the grid necessities imposed by the liberal energy market. For an extensive use of the turbine far from the optimum flow conditions, it follows on damages by material fatigue due to the associated pressure fluctuations and cavitation erosion.

In particular, at part load operating conditions, turbines with fixed-pitch runners show a strong vortex at the runner outlet, in the draft tube’s cone. The vortex rope, a helical vortex that is cavitating in its core, appears and the cavity volume varies with the under-pressure level. The rope frequency depends on the \( \sigma \) level and if it comes close to one of the eigen frequencies of the turbine or associated circuit, the resonance phenomenon may occur. In this case, the unsteady fluctuations can be amplified and lead to important damage. Jacob [1] presents a review of the effects of this operating regime for power plant operation. For all these reasons, the study of the cavitating vortex in partial load conditions is a current concern in the turbomachinery field.

A first approach is to model the rope phenomenology. Different models are proposed. Arndt [2] makes a synthesis of the classical approaches by defining a swirl parameter to characterize the vortex rope – called the hub vortex. Qualitative correlations between the swirl number and flow visualization or pressure measurements seem to be in good agreement.

Alekseenko et al. [3], present experiments of axisymmetrical vortices in a vertical vortex chamber with tangential supply of liquid through turning nozzles. They give an analytical solution of an elementary cylindrical vortex structure considered like an infinitely thin filament, accounting for the helical shape of the vortex lines. Different laws of vorticity distribution and finally a formula for the calculation of the self-induced velocity of helical vortex rotation (precession) in a cylindrical tube is given. Okulov [4] extends this approach to a conical vortex, for a small opening angle. Validations are not available for this approach and the cone angle limitation is a strong constraint to applying this model to turbine vortex ropes.

Wang et al. [5] propose two mathematical models: a partially rolled-up vortex for predicting the rope frequency and a “spiral cone cavity” for predicting the cavity volume in the draft tube. The results are compared with the wall pressure measurements and qualitatively with rope visualizations. Philibert and Couston [6], propose a hydro-acoustic model, the rope is represented by a pipe characterized by its length, wave celerity and a coefficient related to the rope radius variation with the discharge. The \( \sigma \) influence is neglected and experimental data are needed for the model calibration.

Hocevar et al. [7] use radial basis neural networks to predict the vortex rope dynamics in a Francis turbine. The pressure spectrum is well predicted and the void fraction corresponds qualitatively to the experimental estimation. However, the method depends on the learning set and can not predict the unstable behavior of the rope.

Zhang et al. [8], starting from 3D RANS numerical simulations, and Susan-Resiga et al. [9], starting from a theoretical analysis of experimental data, show that the rope origin is the absolute instability of the swirling flow at the cone inlet of the turbine draft tube. However, the rope evolution in the cone is predicted only from the point of view of its stability.

A second approach, made possible by the increase of computers’ power and by the ability of CFD codes to simulate the complex flow behavior, is the numerical simulation of the rope phenomenology. The state of the art in the numerical computation for the cavitation-free configuration is presented in Ciocan et al. [10]. They show, for the rope configuration in cavitation-free conditions that the RANS calculation can give very accurate results, but a detailed validation is necessary before using the CFD codes for design purposes. More complex numerical investigations are presented in Paik et al. [11], comparing 3D URANS and 3D DES numerical simulations in a hydraulic turbine draft tube. Globally the phenomenology is well predicted, but discrepancies
persist in the turbulence and vortex structures in the straight diffuser downstream the elbow. Once more, the need for detailed flow measurements to improve the draft tube phenomenology modeling is mentioned by the authors.

These approaches are based on a series of hypotheses for the rope phenomenology, which are not yet verified experimentally. Therefore, detailed measurements of the rope volume and the associated velocity fields, in addition to classical wall pressure and torque measurements are of prime importance for quantitative validation. An extension of the investigation is necessary in order to demonstrate the σ influence and/or the turbine-circuit interaction influence on the rope phenomenology.

PIV is a well-established measurement technique - see Adrian [12]. For increasing the accuracy of the rope contour detection and for obtaining simultaneously the 3D velocity field, a new PIV measurement method is proposed - see Iliescu [14]. The vector field detection is based on the classical 3D-PIV technique, and the strong scattering in the laser wavelength at the rope boundary is overcome by using fluorescent particles and cut-off filters on the cameras. A light background is used for enhancing the rope shape detection. The development of an image-processing algorithm allowed discriminating on a same image the rope contour from the particles used for the velocity measurement. Based on the rope measurements, a parametrical model of the rope is proposed.

**SCALE MODEL OF THE FRANCIS TURBINE**

In a Francis turbine, the flow incoming from the feed pipe is uniformly distributed along the runner inlet circumference by a spiral casing with gradually decreasing cross-section. A first row of stay vanes insures the mechanical resistance of the assembly, while a second row, of adjustable guide vanes, provides the optimum incidence angle to the flow at the runner's periphery and adjusts the turbine flow rate.

The kinetic energy of the water is then transformed into rotational energy by the runner and transmitted through the shaft to the power generator, which produces electrical energy. The residual kinetic energy at the runner outlet is transformed into pressure energy in the draft tube, composed of the cone, elbow and diffuser, and then the flow is released in the downstream reservoir.

The investigated case corresponds to the scale model of the Francis turbines of high specific speed, \( n_s = 88 \) of a hydropower plant built in 1926, owned by ALCAN. The 4.1 m diameter runners of the machines were upgraded in the late '80s. The elbow draft tube is especially designed for the purpose of the Flow Investigation in Draft Tubes, FLINDT research project EUREKA 1625, see Avellan [15].

The scale model is installed on the third test rig of the EPFL Laboratory for Hydraulic Machines and the performances tests are carried out according to the IEC 60193 standards [16].

A single operating point is selected in partial flow rate operating conditions: for \( \psi = 1.18 \) and \( \phi = 0.26 \), which corresponds to about 70% \( Q_{std} \). Two cavitation numbers are considered: \( \sigma = 1.18 \) — cavitation-free condition, and \( \sigma = 0.38 \) — maximum cavity volume. For these values of \( \sigma \), 9 phases are acquired in order to reconstruct the rope shape by the ensemble averaging by phase technique.

**TWO PHASE – 3D PARTICLE IMAGE VELOCIMETRY MEASUREMENTS**

**3D PIV System for 2 Phase Measurements**

The PIV measurements have been performed with a Dantec M.T. system. The setup is a classical 3D PIV one, but to be able to capture the rope behavior, 2 adaptations are used: florescent particles for enhancement and a backward illumination to can discriminate the rope contour.

The pulsed light sheet with a thickness of 3 mm is generated by two double-cavity Nd-YAG lasers delivering 60 mJ per pulse. An optical high power light guide is used for the positioning of the light sheet in the diffuser cone. Two CCD HiSense PIV cameras are used to visualize the illuminated zone, acquiring series of paired images speared by 150 - 200 µs time delay. The camera resolution is 1’280x1’024 pixels\(^2\) for a 0.20x0.14 m\(^2\) spatial domain. The measurement zone is located at the runner outlet, in the cone of the turbine.
For these measurements, rhodamine (RhB) particles of 1-10µm diameter, receiving 532nm wavelength and emitting at 594nm wavelength are introduced in the flow. The cameras use a cut-off filter (>570 nm) on the emission wavelength of fluorescent particles. The two cameras are synchronized with the luminous flashes and then simultaneously exposed.

The backward illumination is insured by two panels of LEDs, Osram OS-LM01A-Y, placed in front of each camera in the opposite part of the cone – see Figure 1. Arrays of 22x24 LEDs wired in parallel are mounted on two plates and connected to the same power supply system and synchronization board. A diffusing screen is placed in front of the LED panels.

The vector processing is performed with a FlowMap 2200 PIV specific processor, based on an 8 bit resolution cross-correlation technique. The shape of the vortex rope is determined through image processing on one frame of the same images.

**PIV Calibration**

From the measuring field to the camera, the optical path encounters 3 media of different optical indices: water, PMMA (Polymethyl methacrylate) and air. For minimizing the optical distortion on the image, the diffuser cone walls are manufactured with flat external surfaces, in front of the cameras and the incident laser light sheet. However, due to the conical internal surface of the draft tube cone, the image distortions remain important.

The calibration consists in defining the coefficients of a transfer function, either linear or nonlinear, that correlates the spatial coordinates in the object plane with the corresponding positions in the recording plane. This transformation integrates the geometrical and optical characteristics of the camera setup, the perspective distortion, lens flaws and the different media refractive indices, see Soloff et al. [17]. By acquiring images of a target with markers of well-specified spatial position, their corresponding positions in the image plane are known and thus, the geometrical transformation matrix coefficients can be determined through a least squares fitting algorithm. The present measurement configuration uses a third-order polynomial function.

The calibration was performed by placing a 2D target of 200x200 mm$^2$ with 40x40 black dots on a white background in the measurement plane position. The test section is then filled with water for reproducing the optical configuration during measurements. The cameras are focused on the target, the calibration images are acquired and the laser sheet is aligned with the target surface. The target is removed taking care of not modifying the optical setup. The accuracy of the optical arrangement after the target removal is verified by checking that the particles do not appear blurred on the image.

The estimated uncertainty for the velocity field measurement is less than 3% and it has been checked by comparing the velocity measurement obtained with both PIV and LDV, see Iliescu [14].
PIV Synchronization
The rope has a 3D helical shape. A conditional sampling of the image acquisition is necessary for reconstructing the spatial position of the rope. The frequency of rope precession is influenced by the $\sigma$ value, and may change during a revolution in certain operating conditions. Therefore, the triggering system cannot be based on the runner rotation.

The detection the rope precession is based on the measurement of the pressure pulsation generated at the cone’s wall by the vortex passage in front of the sensor. It has the advantage of working even for cavitation-free conditions when the rope is no longer visible – see Ciocan et al. [10]. For this reason, it has been selected as trigger for the PIV data acquisition.

DATA PROCESSING

Image processing
The images – see Figure 2 are processed separately to obtain the 3D velocity fields and to extract the rope characteristics (position and diameter).

The gray level distribution on the raw image histogram, Figure 3, shows three separate peaks corresponding to the rope shadow, background light and particles, respectively.

For the image processing the first objective is to extract the rope shape from the recorded images, and transform it into a binary mask used for outliers’ removal from the vector field. The second one is to detect the rope diameter at its intersection with the laser plane within the measurement area limits.

The distortion correction is performed with the calibration transform prior to the image processing sequence. Using a polynomial optical transfer function, the deformed raw image is mapped onto the real measurement position. Each pixel on the image depends now linearly on the real coordinates through the scale factor.

A series of image processing steps, summarized in Figure 4, are applied on both cameras’ raw images, to obtain the rope contour.

Tree main steps are needed in the image processing – all details of the image processing parameterization are exposed in Iliescu [14].

The first step is an improvement of the image quality reported to the final objective: the rope characterization. The rope zone is extracted and the particles, considered as noise, are filtered on the image.

The second step concerns the rope contour detection. An adaptive threshold is applied on the gray-level distribution for separating the rope area from the background pixels. In the next step, spurious spots are removed from the white background and bright zones corresponding to reflections from the rope boundary facing the camera are removed from the rope foreground, giving
a smooth rope area. This binary image will be used as mask on the corresponding velocity field obtained by cross-correlation from the same raw image and its pair. The image is filtered to extract the rope contour only.

The last step concerns the rope diameter calculation. The rope has a 3D spiral structure: the rope passing in front of the laser sheet performs a shadow zone and fluorescent particles present in the laser plane, behind the rope, do not appear on the image. This zone is used to detect the intersection of the rope with the laser sheet. The particle density is then calculated on horizontal slices. The particle density distribution is given in Figure 5. An adaptive threshold for the minimum number of particles present in a slice gives the intersection limit of the rope with the laser sheet – horizontal line.

The boundary of the rope is recovered from black-white transitions in the binary mask. A linear fit is applied on the median line in the proximity of the horizontal limit determined previously.

Finally, the rope diameter is calculated within the rope boundary limits in the direction perpendicular on the linear fit – Figure 6, and the center is considered at their intersection. For each image, the intersection of the laser plane with the rope is detected and the values of the rope centre and diameter are transformed into real coordinates through the scale factor. Considering the mean values for each phase, the rope volume is reconstructed by the phase averaging technique.

The aberrant images are filtered on criteria of minimum/maximum dimension of the rope area and rope diameter. The rate of validated images is 95%.

The processing of the two images, corresponding to a same instantaneous acquisition, show for the rope center position and the rope diameter differences inferiors of the 2% $R_{te}$ - the global error on the rope diameter. The image procession is a time consuming processes and for the analysis only one image are considered.
Velocity fields processing

For the velocity field calculation, the raw vectors maps are processed by cross-correlation of the two frames from each camera. The raw values are filtered based on range (vectors four times higher than the mean value are rejected) and peak (the relative height of the highest cross-correlation peak compared to the second highest is chosen 1.2) validation criteria. The distortions of position coordinates and particles displacements are corrected with the calibration transform. In order to eliminate the aberrant vectors in the region of the rope or due to the shadow produced by bubbles, the binary mask obtained previously by image processing is applied on the vector maps by multiplication. The statistical convergence is achieved with 1200 velocity fields for 3% uncertainty. The results are filtered by range (vectors four times higher than the mean value are rejected) and peak (the relative height of the highest cross-correlation peak compared to the second highest is chosen 1.2) validation criteria.

Vortex Center Line for the Non-Cavitating Configuration

By ensemble averaging on phase of the instantaneous velocity fields, the mean velocity field is obtained relatively to the phase of the rope precession and for different volumes of the rope corresponding to \( \sigma \) values 0.380 and 1.180. Assuming that the vortex rope and the corresponding flow field remain constant over one revolution, the spatial velocity field can be reconstructed accordingly.

For the cavitation-free configuration for \( \sigma = 1.180 \) - the vortex center is detected starting from the velocity field. For the vortex centre-tracking algorithm two criteria are used: the minimum of velocity and the streamline curvature centers density – see Ciocan et al [10]. The vortex center is obtained as the average position of the curvature centers of streamlines released in the regions with minimum velocity magnitude. An instantaneous result, obtained by these two methods shows a very good coherence of the center position – within the measurement resolution. The validation rate on instantaneous fields, based on coherent position criteria, is higher than 99%.

Rope Contour Phase Average

Starting from each image processing result, for each rope precession phase, the ensemble averaging by phase of the rope diameter is performed, and thus the rope characteristics (position and diameter) in the measurement plane are determined.

Assuming that the rope angular velocity is constant over the rope revolution, which is equivalent to assuming a solid body rotation of the rope, the rope precession temporal phase is transposed into angular position. In this way the spatial position of the rope core is reconstructed in the turbine cone. Adding the measured diameters, in a plane normal to the rope core, leads to the restoration of the rope volume.

For the same \( \sigma \) value, the rope center position and the mean value of the rope diameter are calculated and the corresponding standard deviations are estimated for the entire measurement zone.

VORTEX FILAMENT AND ROPE MODELING

The vortex is caused by the residual circumferential velocity component and the low axial velocity of the flow discharged from the runner in the hub region. The rope turns in the same direction as the runner rotates, but the rope has the form of helix that winds on the backflow/stagnation region towards the opposite direction. A particle, which is released in on the starting point of the vortex rope, will have its velocity decelerated under the influence of the velocity field outgoing from the runner.

Applying the vortex-core detection method for all phases, the vortex center position is determined in the cone – see Figure 7.

A representation of the vortex filament shape can be a conical helix, which is the trajectory of a particle moving with constant velocity on a line steadily rotating about an axis. The tangent to this curve makes a constant angle with the axis.
The vortex filament rounds a conical surface, and the angle of inclination of this cone can be obtained by fitting a linear equation – Eq. 1 - on the vortex core coordinates in the meridian section for all the phase shifts $\tau$ – see Figure 8.

$$\gamma : z = r / \tan \beta$$  \hspace{1cm} (Equation 1)

In this case, the vortex filament attaches the inferior part of the hub, so it can be set as upstream limit for the starting point of coordinates $(\theta_o, r_o, z_o)$. The axial evolution of the vortex pattern is then given by Eq. 2.

$$z = z_o - (r - r_o) / \tan \beta$$  \hspace{1cm} (Equation 2)

The linear fit on the vortex center coordinates gives a mean inclination angle $\beta$ of 17°, twice the cone’s inclination – Figure 8. The angle is slightly different in the upstream and downstream directions, due to the pressure gradient effect given by the elbow. These vortex centerlines enclose the stagnation zone visible on the mean velocity fields. The deformation of the velocity field is visible on the radial and tangential components, which have lower magnitude than the axial one.

The common assumption in the literature is that the vortex path follows the cone wall slope, and for small angles of the cone, the vortex opening angle is approximated with the value of the cone angle. The tangent of this angle is also approximated with the angle value. For the present case the cone and vortex opening angles are much larger and dissimilar, thus the general assumption cannot be accepted.

Fifteen phase shift values are considered for recovering a complete rotation of the vortex. Although, the measurement plane position limits the detected vortex cores to spatial positions. Projecting the centers positions in cross-section, they recover only half rotation, the centers on left and right side of the measurement plane being shifted with $\sim 2\pi$.

$$k r_a b \theta$$  \hspace{1cm} (Equation 3)

In cross-section, the radius change with the angular position $\theta$ follows a logarithmic spiral given by the Eq. 3.
In the present case, the best fit on the center positions is given by Eq. 4.

\[ r = r_\theta b^{\theta/2\pi} \]  

(Equation 4)

where \( b \) represents the rate of radial growth for a complete rotation \( \theta = 0:2\pi \).

Combining the two equations, the 3D path described by the vortex core is obtained under the following parametric representation Eq. 5 and Eq. 6.

\[ \vec{r} = \vec{r}(\theta) = x(\theta) \hat{i} + y(\theta) \hat{j} + z(\theta) \hat{k} \]  

(Equation 5)

\[
\begin{align*}
  x &= r_\theta b^{\theta/2\pi} \cos \theta \\
y &= r_\theta b^{\theta/2\pi} \sin \theta \\
z &= z_\theta - r_\theta(b^{\theta/2\pi} - 1)/\tan \beta 
\end{align*}
\]  

(Equation 6)

The vortex centers, made dimensionless with the runner outlet radius \( R_{\text{ref}} \), and the fitted conical helix are presented in Figure 9. The intrinsic parameters are in this case:

- radius of the vortex starting point at the hub \( r_\theta \);
- axial position at the hub \( z_\theta \);
- rate of radial growth \( b \);
- opening angle of the conical supporting surface: \( \beta \).

The same equations as for the conical helical vortex have been used for modeling the center of the cavitation vortex rope. The comparison vortex filament model for two \( \sigma \) values – 1.18 for cavitation-free conditions and 0.38 for vapors core vortex - is shown in Figure 10 and Table 1.

There are two noticeable differences:

- a higher radial growth rate, from 3.2 to 4, which indicates a higher frequency of the vortex precession in cavitation conditions, tendency confirmed by the power spectra from wall pressure signals;
- the cone supporting the vapors-core vortex center has a larger opening than the vortex filament in cavitation-free conditions, from 17° to 25.5°, which means a larger stagnation region enclosed by the vortex; as the flow rate is the same in both cases, the flow is constrained to accelerate near to the wall due to the smaller discharge cross-section.

<table>
<thead>
<tr>
<th></th>
<th>Cavitation-free vortex</th>
<th>Vapors-core vortex</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial radius</td>
<td>( r_\theta ) 0.09</td>
<td>0.15</td>
</tr>
<tr>
<td>Initial depth</td>
<td>( z_\theta ) -0.615</td>
<td>-1.2</td>
</tr>
<tr>
<td>Rate of radial growth</td>
<td>( b ) 3.2</td>
<td>4</td>
</tr>
<tr>
<td>Cone angle</td>
<td>( \beta ) 17°</td>
<td>25.5°</td>
</tr>
</tbody>
</table>

Table 1 Conical helical vortex model parameters

This model gives the same characteristics like the cavitation-free configuration: both the curvature radius and the torsion radius of the conical helix depend linearly of the local radius of the curve. The length of the conical helix – \( s(z) \), has a linear variation with the cone depth (z).
Fitting a linear equation to the vapors-core vortex radius in the measured positions, added to the vortex filament model, allows reconstructing the rope volume in the draft tube - Figure 11. In this way, starting to the experimental results, the rope is completely parameterized.

The void volume of the vortex core is calculated by integrating its boundary over the length of the vortex filament. For this calculation 1.9 helix rotations are considered. The rope volume is \( V = 0.0018 \) m\(^3\).

**CONCLUSIONS**

The PIV system gives the opportunity to survey the 3D flow velocity field in the diffuser cone of a scale model of a Francis turbine in part load operating conditions with a precessing cavitation rope. The complete description and quantification of the 3D velocity, simultaneously with the rope boundary behavior have been obtained from the experimental results, first, for different cavitation conditions.

The synchronization of the PIV acquisition with the rope position allows, by ensemble averaging, the reconstruction of the rope volume in correlation with the corresponding velocity field, in the cone of the turbine model. Image processing provides an estimation of the rope diameter and the positions of the vortex center in the measuring zone.

A parametrical model of the rope is proposed. The rope is modeled as a conical helix and its geometrical parameters are obtained. The behavior of these parameters is studied for two sigma values. Based on the model, the volume of the rope obtained.

An experimental database has been built and is now available for future validation of modeling and numerical simulation of draft tube flows.

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