Abstract— In the context of automatic image processing, noise is often the hardest problem to cope with. This is particularly true for images corrupted by multiplicative speckle noise, such as Synthetic Aperture Radar (SAR) images. To tackle such an issue, this paper proposes a novel anisotropic diffusion filter that manages to simultaneously fulfill competing requirements: reduce noise on homogeneous regions, preserve weak edges, and keeping hard targets intact. The capabilities of our filter were proved by comparing it with other state-of-the-art filters on a 1-look Cosmo-SkyMed (CSK) image.

Index Terms— Image denoising, anisotropic filters, nonlinear filters, speckle, synthetic aperture radar (SAR) images.

I. INTRODUCTION

MANY imaging technologies operate coherently to acquire images and, as such, are subject to speckling effects which greatly reduce observable details. For example, in synthetic aperture radar (SAR) imagery, speckle generally reduces the performance of automatic scene segmentation and classification algorithms. Presupposing the multiplicative speckle model, many despeckling filters have been designed to remove this kind of noise. Classic filters such as Lee, Kuan, Frost and Gamma MAP (Maximum A Posteriori) [1] exploit the pixel values inside a small window centered at a given pixel in order to make inference and to reconstruct its true value. In particular, in addition to multiplicative noise assumption, such filters presuppose the image to be as an ergodic process where statistical means can be substituted by spatial means. Even though these classic filters manage to reduce speckle on homogeneous areas, they completely fail to reconstruct the scene whenever an edge is in the local sliding window. In order to avoid such behavior, a sharp transition inside the local window has to be treated in a different way. Nevertheless, edge detectors relying on a classical gradient operator produce signal-dependent results, and finer statistical edge detectors have to be used for this scope [1].

More recently, denoising filters based on the non-local means paradigm [3] have been designed to remove multiplicative speckle noise in SAR images [4]. In the design of the probabilistic patch based (PPB) filter in [4], Nakagami distribution of the SAR image amplitude is presupposed. Furthermore, a Bayesian framework is used to compute both a similarity measure among patches and the weighted mean used for pixel reconstruction. Thanks to its impressive visual results and since its performance outperforms even more sophisticated wavelet-based methods [5], PPB presents itself as a state-of-the-art reference for despeckling algorithms. Whereas effort has been dedicated to adapting the filters in [4] and [5] from the additive to the multiplicative noise model, denoising filters in [6] do not require this adaptation. In fact, contrary to the aforementioned despeckling filters, these are non-linear diffusion (NLD) filters that do not need any prior assumption, i.e. they can be directly applied to any type of noise-corrupted image. In addition, since the final result is a solution of a Partial Differential Equation (PDE), many theorems and properties hold for such a solution (e.g. invariance to grey level shift, invariance to reverse contrast, invariance to image translation and rotation, preservation of average grey level, it respects the maximum-minimum principle, etc.) [7]. Nevertheless, even though several contributions and improvements (see [8]-[11]) have been made, little is known about the use of such filters for speckle removal. Furthermore, none of the filters described so far considers the edge and target preservation as a primary goal.

In this paper, a novel NLD denoising filter aimed at edge and target preservation is devised. Moreover, to validate our approach, a comparison with other state-of-art despeckling filters has been performed on a real 1-look CSK image.

The remainder of the paper is organized as follows. Sec. II provides the mathematical formalization of NLD paradigm together with three common NLD filters. Sec. III defines our filter. In Sec. IV results are discussed and compared to other state-of-the-art despeckling filters. The final conclusions are drawn in Sec. V.

II. NLD FILTERS

A. Theoretical Background

Some very powerful classes of despeckling filters are those that compute the solution of a PDE applied to the image. As an example, filtering an image with a Gaussian kernel $G_\sigma$, where $\sigma$ indicates the Gaussian standard deviation, it is equivalent to consider each grey level in the original image as a temperature measurement and let the system evolve until time $t = \sigma^2/2$ [1]. Formally, each value of the grey level image $I$,
with a scalar function corresponding eigenvalues. Considering is high on edges. Moreover, the eigenvector with this problem, authors in [1] propose replacing the matrix of the system leads to a completely blurred image. To solve Clearly, without any modification to (2), the natural evolution effect due to near opposite gradient directions. Once

**Related Works**

Related Works

Clearly, without any modification to (2), the natural evolution of the system leads to a completely blurred image. To solve this problem, authors in [1] propose replacing the matrix \( D \) with a scalar function \( g(\nabla I) \) reverse proportional to the image gradient module. In particular, they suggest a function of the type \( g(\nabla I) = e^{-|\nabla I|/h^2} \) with the parameter \( h \) acting as a threshold. It should be noted that inverse proportionality between \( g(\nabla I) \) and \( |\nabla I| \) enables the filter to stop diffusion (smoothing) on edges (i.e., points with high value of \( |\nabla I| \)). The Perona-Malik (PM) article [1] has been the basis of many proposed evolutions and an interested reader can find a detailed overview in [10]. Nevertheless, even though these filters are often referred to as anisotropic diffusion (AD) techniques, the first real anisotropic diffusion approach was proposed in [7], where the Coherence Enhancing Diffusion (CED) was devised. Practically speaking, in CED the diffusion matrix \( D \) is not substituted by a scalar function, instead its elements are retrieved from the image structure tensor \( S(\nabla I_o) \) computed as:

\[
S(\nabla I_o) = \nabla I_o \cdot \nabla I_o^T
\]

where \( \nabla I_o \) indicates the image \( I \) smoothed by a Gaussian kernel \( G_o \). Roughly speaking, \( S(\nabla I_o) \) holds the same information that can be retrieved from the gradient \( \nabla I_o \), with the difference that \( S(\nabla I_o) \) can be smoothed element-wise without any cancellation effect due to near opposite gradient directions. Once \( S(\nabla I_o) \) is smoothed element-wise by \( G_o \) to reduce the noise, the resultant matrix \( S_p \) (symmetric and positive semi-definite) can be decomposed to find the image principal components:

\[
S_p = G_p \ast S(\nabla I_o) = VAV^T = [v_1, v_2] \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} [v_1^T \ v_2^T] (4)
\]

where \( v_1, v_2 \) indicate respectively the eigenvectors parallel and orthogonal to the image gradient, with \( \lambda_1, \lambda_2 \) the corresponding eigenvalues. Considering \( \lambda_2 \leq \lambda_1 \), the eigenvector \( v_1 \) corresponds to the gradient direction and its eigenvalue \( \lambda_1 \) is high on edges. Moreover, the eigenvector \( v_2 \) is orthogonal to the gradient direction and its eigenvalue \( \lambda_2 \) is high only on object corners and singularities. In [7], to prefer smoothing along the direction \( v_2 \) and consequently link interrupted lines, the diffusion matrix \( D \) is derived from \( S_p \) as:

\[
D = [v_1, v_2] \begin{bmatrix} g_{\lambda_1}(\lambda_1, \lambda_2) & 0 \\ 0 & g_{\lambda_2}(\lambda_1, \lambda_2) \end{bmatrix} [v_1^T \ v_2^T] (5)
\]

with:

\[
\begin{align*}
& g_{\lambda_1}(\lambda_1, \lambda_2) = \alpha \\
& g_{\lambda_2}(\lambda_1, \lambda_2) = \alpha + (1 - \alpha)e^{-\frac{C_m}{(\lambda_1 - \lambda_2)^2}}
\end{align*}
\]

where \( \alpha \) permits a small diffusivity (usually \( \alpha = 0.05 \)) even when no preferential direction exists and \( k \) acts as a threshold to \( (\lambda_1 - \lambda_2)^2 \) value. Moreover, the positive constant \( C_m \) is introduced to correct the bias in the original Perona-Malik diffusivity function [7]. Since the aim of [7] was not to devise a denoise filter, applying CED on a speckle-corrupted image yields results with completely distorted details. However, as suggested in [10], an edge enhancing diffusion requires modification to the diffusivity functions \( g_{\lambda_1}(\lambda_1, \lambda_2) \) and \( g_{\lambda_2}(\lambda_1, \lambda_2) \) according to the following relation:

\[
\lim_{|\nabla I| \to \infty} g_{\lambda_1}(\lambda_1, \lambda_2)/g_{\lambda_2}(\lambda_1, \lambda_2) = 0 (7)
\]

**III. IMPROVED EDGE ENHANCING DIFFUSION (IEED)**

We propose the following modifications to the CED schema to keep the real anisotropic behavior of CED while respecting the relation in (7):

\[
\begin{align*}
& g_{\lambda_1}(\lambda_1) = 1 - e^{-\frac{-C_m}{(\lambda_1)^2}} \\
& g_{\lambda_2}(\lambda_2) = 1 - e^{-\frac{-C_m}{(\lambda_2)^2}}
\end{align*}
\]

In fact, remembering that \( \lambda_1 \) is in direct proportion to \( |\nabla I| \) whereas \( \lambda_2 \) is not, the property (7) holds (see Fig. 1). The choice in (8) can be better appreciated exploiting a convenient decomposition of the flux \( J \). Indicating the eigenvalues of the diffusion matrix \( D \) as \( \mu_1 = g_{\lambda_1}(\lambda_1), \mu_2 = g_{\lambda_2}(\lambda_2) \), and the image derivatives along \( v_1, v_2 \) respectively with \( I_{v_1} \) and \( I_{v_2} \), the flux \( J \) can be decomposed as (see Appendix):

\[
J = \mu_1 I_{v_1} v_1 + \mu_2 I_{v_2} v_2 (9)
\]

Therefore, using the diffusivities in (8), three different filter behaviors are highlighted by the novel IEED filter:

1) On homogeneous areas both \( \lambda_1 \) and \( \lambda_2 \) are low. Therefore, \( \mu_1 \) and \( \mu_2 \) are near 1, i.e. an isotropic diffusion case (speckle removal).

2) On edges \( \lambda_1 \) is high and \( \lambda_2 \) is low. Consequently, \( \mu_1 \) is near 0 and \( \mu_2 \) is near 1, i.e. an anisotropic diffusion case with smoothing along \( v_2 \) (edge preservation).

3) On corners both \( \lambda_1 \) and \( \lambda_2 \) are high. As a consequence, \( \mu_1 \) and \( \mu_2 \) are near 0, i.e. a no diffusion case (corner preservation).

It should be noted that the last behavior avoids shape distortion of objects and hard targets smoothing. In fact, not only corners and singularities, but also small objects with high RCS are characterized by a high value of both \( \lambda_1 \) and \( \lambda_2 \), thus
causing diffusion stop. Summarizing, with the diffusivity in (5) we manage to effectively combine the respective advantages of the PM and CED filter. In the PM schema, the speckle on homogeneous regions is removed but there is no diffusion along edges, so they remain noisy in the final result. Conversely, the CED filter provides little speckle removal on homogeneous areas, but edges are correctly reconstructed.

A. Algorithm Description

Usually, the edge detection problem in noisy images is tackled by directly or indirectly using information related to the image gradient. Nevertheless, in the SAR community, the image coefficients of variation is more commonly used to face the same issue in despeckling filter implementation [8]. Even though the multiplicative speckle assumption justifies this choice [2], a simple logarithmic transformation solves the previous problem allowing a gradient operator to be used for edge detection in SAR images. As an example, the SRAD filter in [8] and its anisotropic extension in [11] use to avoid biased gradient operations. However, they both use numerical implementation where differences between near values are computed, thus nullifying the advantages. For these reasons, in our filter implementation, a logarithmic transformation is applied to the input image and a reverse exponential function is operated to the end. Furthermore, the structure tensor matrix in (4), which relies on the gradient information, is utilized as edge detector.

With regards to unknown variables, both diffusivities in (8) depend on two parameters, namely the threshold and the exponent , with . In order to have the same behavior in both directions, the exponent has been set equal for both diffusivities ( ) and a value of has been selected to have a fast transition of the diffusivity function from to around the threshold . Instead, exploiting a homogeneous area of the processing image, each threshold is estimated by the corresponding quantile - set by the user as input parameter - of the experimental cumulative density function (ecdf) of the respective eigenvalue . In this way, the thresholds are automatically adjusted at each step by the degree of despeckling reached at that point. It should be noted that to avoid user selection of a homogeneous area, the index of the original noisy image can be computed and thresholded to yield a binary mask. In this way, only the pixels belonging to this mask are used in the ecdf computation.

Another improvement of the IEED performance with respect to the filters presented in [1]-[9] and [11] is the use of an appropriate numerical schema [12] to compute the solution of the PDE in (2). In fact, the schema was optimized for rotational invariance, which is a fundamental property when the gradient computed along the main axes is used for directional derivatives. Moreover, the explicit numerical schema proposed in [12] combines the advantages of classical (Euler-forward) explicit schemas of yielding a small error in final solution computation, with a higher efficiency (up to 4 times) typical of implicit or semi-implicit techniques. Indeed, a time step up to can be used without compromising the numerical stability. Finally, the numerical computation in [12] can be directly implemented by simple convolution operations. In fact, in [12] all derivatives are computed by means of a simple Sobel-like 3x3 derivative mask at the place of the classical stencil-like schemas [10].

IV. EXPERIMENTAL RESULTS AND ANALYSIS

This section describes the despeckling experiments performed on an actual SAR image (see Fig. 2(a)) to illustrate the effectiveness of the proposed method. The IEED filter was compared to NLD filters such as Perona-Malik (PM) [1] and SRAD [8]. Furthermore, the promising Probabilistic Patch Based (PPB) filter [4] (with and without iterations) was considered as state-of-the-art reference. The SRAD filter was implemented with the classical Euler-forward schema. Moreover, a time step and iterations were selected to obtain stable results. Finally, a homogeneous region of the image was passed as input to enable the filter to compute the instantaneous coefficient of variation [8]. The same homogeneous region was also passed in input to PM and IEED to estimate the respective thresholds. PM was implemented with the same numerical schema as IEED and the threshold was estimated by the quantile of the gradient module ecdf. For the IEED filter, the edge-controlling quantile was set to and the corner-controlling quantile was set to . Note that having the ecdf computed on a homogeneous area, the value is an estimate of the false alarm probability of the respective edge ( ) and corner ( ) on it. In fact, the previous values of indicates that on a noisy homogeneous area, 5% of the occurrences have values comparable to the ones obtained on edges. The same applies for corners where has been set to 1 presupposing no corners or singularities within the homogeneous region in input. Therefore, the quantile may be lowered for a stronger corner (target) preservation and the quantile can be raised if only strong edges are concerned. Moreover, a Gaussian kernel with standard deviation was applied before filtering (in order to make the initial image differentiable), and we selected to minimize the loss in resolution. Then, at each step, a Gaussian kernel with was applied as regularization [9]. Finally, a Gaussian filter with was selected to have good noise suppression on gradient direction estimation by structure tensor components in (4). The non-iterative PPB filter parameters were set according to author suggestions [4], i.e. number of looks , search windows , similarity window , -parameter quantile . The same applies for the iterative PPB filter, where the variable window size optimization was used [4] with , and 25 iterations. The results of the evaluated despeckling algorithms
are shown in Fig. 2 for a CSK image acquired over Tucson (Arizona) with the following characteristics: product L1A, Spotlight 2 acquisition mode, polarization HH, incidence angle 24° and number of looks . The images are bit deep and they are shown after a logarithmic transformation . Then, the same linear mapping of dynamic range between and was applied to each result to further improve visualization and comparison. It should be pointed out that the result yielded by CED is reported only for the sake of visual comparison, since CED does not deal with noise removal but it aims to complete interrupted lines and enhance flow-like structures (e.g. enhancement of fingerprint images). As can be seen from the results, application of the SRAD filter loses even some large details. Moreover, targets are distorted and observed as bright blobs. The PM filter reveals small radar cross section (RCS) variations, however the region boundaries remain noisy. The PPB filter introduces some artificial texture on homogeneous areas and some edges and targets are blurred. Instead, the iterative PPB filter retrieves target sharpness but some edges and fine details are not retrieved by iterations. Furthermore, the PPB filter increases the dynamic range at each iteration, i.e. low values are lowered and high values are increased. Clearly, this behavior tends to generate some artifacts, such as dark spots besides brighter areas. Differently, the IEED filter preserves even small details and weak edges. In addition, dynamic range extension is theoretically avoided since the maximum-minimum principle has to be respected by the PDE solution. The considerations can be further appreciated for PM, iterative PPB and IEED filters by the magnifications in Fig. 3 and Fig. 4. As can be clearly seen from these figures, only the IEED filter manages to fully remove speckle without smoothing the finest details (e.g. see the small roads in Fig. 4). Moreover, to support visual impressions by absolute measurements, the following quality indexes were evaluated on the filtered images described above:

\[
\text{ENL} = \ln \frac{\text{mean}(\text{filtered})}{\text{mean}(\text{noise})},
\]

where indicates the despeckled image, , are the mean and variance of on a homogeneous region, and is the mean of the ratio noise computed as . Clearly, the ENL index gives an indication of despeckling capacity on homogeneous regions, whereas indicates the degree of RCS preservation in that area. In fact, considering a multiplicative noise model, in a homogeneous area, image can be expressed as , where is the RCS of the area (constant or with texture) and is the speckle noise. If an ideal despeckle is concerned, is equal to the “true” RCS , therefore is exactly equal to . Thus, the nearer is to the ideal , the better preserved the radiometric data are. As an example, for the image of Tucson, with theoretical , the ideal value of is . TABLE I summarizes the ENL and values computed on Region I of the Tucson SAR image and the computational time for a image on a PC with GHz Intel Core2 Duo and GB RAM. From TABLE I, the despeckling performance indexes indicate that IEED outperforms the other filters even on homogeneous areas, both in terms of noise removal and radiometric preservation. Finally, even the computational time of IEED is lower than the iterative PPB filter.

**TABLE I. PERFORMANCE INDEXES OF VARIOUS DESPECKLING FILTERS ON REGION I AREA OF TUCSON SAR IMAGE**

<table>
<thead>
<tr>
<th>Filter</th>
<th>ENL</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orig.</td>
<td>0.94</td>
<td>-</td>
</tr>
<tr>
<td>SRAD</td>
<td>249.01</td>
<td>63s</td>
</tr>
<tr>
<td>PM</td>
<td>184.43</td>
<td>169s</td>
</tr>
<tr>
<td>PPB</td>
<td>117.14</td>
<td>153s</td>
</tr>
<tr>
<td>It. PPB</td>
<td>144.22</td>
<td>357s</td>
</tr>
<tr>
<td>IEED</td>
<td>702.37</td>
<td>238s</td>
</tr>
</tbody>
</table>

where indicates the despeckled image, , are the mean and variance of on a homogeneous region, and is the mean of the ratio noise computed as . Clearly, the ENL index gives an indication of despeckling capacity on homogeneous regions, whereas indicates the degree of RCS preservation in that area. In fact, considering a multiplicative noise model, in a homogeneous area, image can be expressed as , where is the RCS of the area (constant or with texture) and is the speckle noise. If an ideal despeckle is concerned, is equal to the “true” RCS , therefore is exactly equal to . Thus, the nearer is to the ideal , the better preserved the radiometric data are. As an example, for the image of Tucson, with theoretical , the ideal value of is . TABLE I summarizes the ENL and values computed on Region I of the Tucson SAR image and the computational time for a image on a PC with GHz Intel Core2 Duo and GB RAM. From TABLE I, the despeckling performance indexes indicate that IEED outperforms the other filters even on homogeneous areas, both in terms of noise removal and radiometric preservation. Finally, even the computational time of IEED is lower than the iterative PPB filter.
V. CONCLUSIONS

In this paper, we have presented a novel anisotropic diffusion filter that manages to combine normally contrasting requirements: reducing noise on homogeneous regions, preserving weak edges, and keeping corners and targets intact (maintaining them as seen in the original image). Moreover, since IEED is a PDE-based filter, no noise model is presupposed so that, in principle, it can be applied to any noise type. We want to emphasize that this last property has a strong impact on its possible application. In fact, no mathematical modeling effort is required (e.g. statistical modelling of both noise and radar reflectivity) to change sensor or data type (e.g. intensity or amplitude). In addition, since the filtered image is a solution of a PDE, many theorems and properties hold for such a solution. Finally, visual impressions and performance indexes confirm that IEED outperforms state-of-the-art filters for SAR image despeckling.

APPENDIX

The diffusion tensor can be decomposed as:

\[ \text{diffusion tensor} = \begin{pmatrix} a & b \\ b & a \end{pmatrix} \]

Then, indicating with the directional derivative of along , the flux can be written as:

\[ \text{flux} = \left( \frac{\partial u}{\partial x} \right)_n = \left( \frac{\partial u}{\partial x} \right)_n + \left( \frac{\partial u}{\partial y} \right)_n \]

REFERENCES