A complete behavioral model of radiation-pattern characteristics of phased array using a novel digital phase shifter

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Abstract — A complete and behavioral model for radiation-pattern characteristic of phased array has been presented. The model has many input parameters and it has a lot of features, such as parameter simulations with results analysis, unconventional two-dimensional color graph representation capability in order to show more clearly the results. It is an optimum instrument to investigate the performance of a beamformer in relation with an analogue phase shifter, ideal digital phase shifter, simulated real phase shifter and measured real phase shifter. Measured values of three different prototype of the proposed phase shifter architecture have been used in order to study the performances of the same architecture.

Index Terms — Behavioral model, phased array, phase shifter, Phase Locked Loop, antenna.

I. INTRODUCTION

Phased array antennas play an important role in many radar and airborne applications; their success is mainly due to high agility in reconfiguring pattern and quick steering capability. Many phased-array antennas are designed to use digital or digitalized phase shifters in which the phase shift varies in discrete steps rather than continuously; the latter is a peculiarity of analogue phase shift, which by definition is able to deliver the exact phase to any array element of a phased array to steer the beam in the desired direction. As a consequence of using digital phase shifter, the beam can be steered only in discrete steps and the granularity [1], defined as the finest realizable increment between adjacent beam positions, depends by the number of bits and number of antennas; furthermore the phase shifts produced by the phase shifter are not ideal, due to the fact that a phase shifter itself is affected by errors and non idealities and it implies that the radiation pattern characteristics are altered further. So, a complete model is required to predict antenna array performance in terms of beam shape, directivity, side lobe levels, main lobe deviation etc., and it has to take into consideration also the non-idealities above described. This is the aim of the presented paper which describes a behavioral model, based on basic antenna array theory, developed in MATLAB®.

The model presented has a lot of input parameters and options, i.e. the number of antennas, the inter-element spacing between the antennas, the range of angles for pattern calculation, the wanted scan angle or range of scan angles, the type of phase shifter (analogue, digital with ideal quantization, digital with real quantization and random errors, or finally, digital with real quantization obtained by measurements on a phase shifter) and weighed amplitude of the array elements. Another notable feature is the use of recent unconventional two-dimensional color graph representations in a Cartesian coordinate system [2] which gives more legibility to the graphical representation when a parametric simulation is performed. This tool is useful if the user wanted to predict the behavior of an antennas array using a measured phase shifter (the vector of the measurements done on the phase shifter could be inserted) or to study the effect of parameter changes on some relevant figure of merit of an antennas array, thanks to the possibility of parametric simulation that gives the opportunity to vary important parameters such as number of antennas, inter-element spacing, root mean square error of the digital phase shifter and the number of bits of the phase shifter.

The latter characteristic, for example, is the specific featured used to study and validate the novel proposed architecture of phase shifter recently introduced in [3]-[4]. The resultant architecture, as it will described in detail in Section II, can cover a wide frequency bandwidth with a good phase resolution (because it depends essentially on a digital circuit easily doable) without giving up the frequency sweep speed and low phase noise. In relation to the proposed architecture where, differently than in other architectures, it is relatively easy to obtain a large number of bits due to the fact that it is demanded to a low-frequency digital circuit, we are able to investigate the influence of the number of the bit on the beam shape. As another example, this model could be used to choose the right number of bit in a particular application where it is not required a fine granularity, etc.

Section II gives an overview of the phase shifter basic operation and the system architecture proposed. The model is described in Section III; the analysis of the proposed architecture is presented in Section IV, while in the Section V the conclusions are given.

II. PHASE SHIFTER AND SYSTEM OVERVIEW

In Fig. 1, it is shown the block schematic of the phase shifter proposed and described in [3]-[4]. The system is composed by a Phase Locked Loop (PLL) that synthesizes the clock used by the accumulator; an accumulator which drives the $2^k$ comparator, where $k$ is the number of bit of the phase shifter and $2^k$ is the number of possible phase shifts; $2^k$ PLLs
driven by the wanted mutually shifted signals generated by the comparators; a DSP or Microcontroller which imposes the phase control word to each digital comparator.

At the outputs of the Phase Control Block there are $2^k$ square waves that drive the PLLs with digital PFDs, mutually shifted depending on the various phase control word and the accumulator’s output.

![Fig. 1. Phase shifter architecture proposed.](image1)

Therefore such architecture is composed by a digital circuit called phase control block which provides the phase shifted reference for the PLLs, that are able to provide the output signal at the wanted frequency preserving the phase shift imposed at the input. The proposed architecture takes advantage of the wide band operation of the PLLs (the phase shifts are also frequency independent), and of the stable and reliable reference signal generated by the control phase block characterized by a fine resolution and fast switching.

So the great flexibility of the proposed architecture is that the frequency scalability is simply demanded at the use of suitable PLLs and the phase resolution is demanded to a very simple digital circuit which allows to obtain a digital phase shifter with great number of bits. Also this capability imposes the need to have a complete behavioral model helpful for system dimensioning and above all for further studies on the impact of the number of bits on the radiation-pattern in relation to the proposed architecture.

III. MODEL DESCRIPTION

The program used, as just said, is Matlab® from The Mathworks™ which is a high-level technical computing language and interactive environment for algorithm development, data visualization, data analysis, and numeric computation. Further details are provided in the following subsections.

A. Array configuration

The tool takes into account the array factors of linear arrays and it is based on basic antenna array theory. The array configuration is shown in Fig. 2 where the elements, considered as point sources, are positioned on the horizontal axis.

![Fig. 2. Array configuration.](image2)

The angle $\theta$ represents the elevation angle which is the angle of the beam with respect to the z axis. The scan angle is the elevation angle imposed.

B. Type of simulations

The model presented is very complete, in fact it accepts as input, according to the simulation type chosen, the range of elevation angles for pattern calculation, the number of antennas and the inter-element spacing between the antennas, the type of phase shifts fed at the antennas (ideal phase shift which simulates an ideal analog phase shifter, ideal quantization which simulates an ideal digital phase shifter, calculated real quantization which implements a simulated real digital phase shifter and finally real quantization obtained by inserting a vector of real digital phase shifter measurements and which implements a real measured phase shifter), the wanted elevation angle (scan angle) or range of scan angles.

The program is organized in menu; first of all the user can choose the type of simulation among the following types:

1) Variable number of antennas: it is a parametric simulation which has as parameter the number of the antennas; this type of simulation is important if the user want to study or analyze, for example, the main lobe aperture. The particular parameters required are the antenna start number, the antenna stop number and the antenna increment number. The other parameters that have to be inserted are the elevation start angle, the elevation stop angle, the elevation increment angle, the elevation start scan or start steering angle, the stop scan angle, the increment scan angle, the antenna spacing.

2) Variable value of rms error: it is a parametric simulation which varies the rms error added to the ideal phase shifter in order to simulate a real phase shifter and to analyze the impact of the rms phase error introduced by a non-ideal phase shifter, on the radiation-pattern characteristic. In this case the parameters required are the number of simulation and the rms value to be used in those simulations. This is one of the most interesting feature of the program because it gives the opportunity to investigate if there is a particular relation
between the rms phase error of the phase shifter and some parameter of the radiation-pattern [5]. The other parameters required are the elevation start angle, the elevation stop angle, the elevation increment angle, the elevation start scan, the stop scan angle, the increment scan angle, the number of the antennas, the inter-element antenna spacing.

3) Variable value of inter-element spacing: it is a parametric simulation which has as variable the distance, in terms of wavelength, between the antennas on a linear distribution. This type of simulation could be used to choose the antenna linear spacing in relation to certain radiation-pattern parameter. For example it is well known that the inter-element spacing would be chosen to be half a wavelength to avoid grating lobes; with this simulation the user is able to see the influence of this parameter on other figure of merit. The peculiar inputs required for this simulation are the inter-spacing start value, the stop value and the increment. The other parameters that have to be inserted are the elevation start angle, the elevation stop angle, the elevation increment angle, the elevation start scan, the stop scan angle, the increment scan angle, the number of antennas.

4) Variable number of bit: also in this case there is a parametric simulation which has as input the start number of bit, the stop number of bit and the increment. Such a simulation is the key point in those application where a great precision is required during the scan. In fact the more are the bits, the more is the scan precision till the ideal situation of analog phase shifter which could be assimilated to a phase shifter with infinite number of bit. So this type of simulation could be used as a dimension tool or as an instrument to study how effective is the increment of the number of bits and what is the impact on the radiation-pattern. For example in a system with a great number of antennas, that means a close main lobe, it could be important to dispose a significant number of bits. In this particular situation, the proposed architecture could be a good solution because, as said in Section II, the number of bit of the phase shifter is entrusted to a relative simple low frequency digital circuit. A complete study can be carried on, crossing the results of the variable number of bit simulation and the variable number of antennas simulation in order to set the right number of bits in relation with the number of antennas and precision required. The parameters required are the elevation start angle, the elevation stop angle, the elevation increment angle, the elevation start, the stop scan angle, the increment scan angle, the number of antennas and the inter-element antenna spacing.

5) Not parametric simulation: in this simulation there is no parameter and the simulation is carried out on a spread of desired scan angles. So the input parameters are the elevation start angle, the elevation stop angle, the elevation increment angle, the start scan angle, the stop scan angle, the increment scan angle, the number of antennas and the inter-element antenna spacing.

6) Single simulation: it is the same of the previous simulation type with the difference that the simulation is carried on only a single specific scan angle. The input variables are the elevation start angle, the elevation stop angle, the elevation increment angle, the elevation scan angle, the number of antennas and the inter-element antenna spacing.

C. Round-off techniques

For simulation type number 1, 3, 4, 5 and 6, the user can choose what kind of round-off technique he wants to use among the following ones:

1) Exact phase at each elements: this is the ideal situation represented by an ideal analog phase shifter. With this type of simulation the user can study the geometry of the array without worrying about the non idealities of the phase shifter.

2) Ideal quantization: most phase shifters are digital devices, or at least are digitally controlled. Therefore only discrete values of phase shift are allowed, and they may not be the precise values required; so this simulation takes into account the errors due to an ideal quantization. The situation is outlined in Fig. 3.

3) Real simulated quantization: the digital phase shifts really fed to each element are not ideal and they depend on the phase shifter adopted. A figure of merit is the rms error of the phase generation of the phase shifter. So, starting from the ideal quantization, a casual and random error distribution, with imposed rms, is added to the phase shifts in order to obtain the simulated allowable phase states as shown in Fig. 4.

4) Measured values: it is possible to insert the real measurements made on a phase shifter in order to test the behavior of own phase shifter and the resultant array pattern.
D. Radiation pattern plot types

When the simulations are finished, the user can plot the radiation-pattern obtained (or rather the radiation-patterns obtained) in various format: classical rectangular plot, classical polar plot or unconventional two-dimensional color graph representation in Cartesian coordinate system. In fact, conventionally, radiation patterns are represented in two dimension (2D), in Cartesian or polar coordinate systems. In the Cartesian coordinate system, the magnitude of the radiated field, usually in decibels (dB), is indicated on the Y axis, whereas the angular parameter, the elevation angle \( \theta \) or the quantity \( u = \sin \theta \), is on the X axis. In the polar coordinate system, the radius usually represents the magnitude of the radiated field and the angle represents the elevation angle. When comparison is required or a parametric simulation is carried out, several curves are plotted on the same graph or another dimension is added. If we uses another axis as the third dimension, the extrapolation of the data is not so easy, considering that the three dimensional representation is actually two dimensional on a planar surface [2]. An attractive representation is then the use of a color parameter as the third dimension, which represents the variable or the parameter which depends on the user’s interest. As a clarifying example consider the case of a not parametric simulation in which the user wants to study the array factor of a sixteen element array with the inter-element spacing equal to half a wavelength and exact phase at each element, over a range of scan angles; the elevation start angle is \(-90^\circ\), the elevation stop angle is \(90^\circ\), the elevation increment angle is \(0.2^\circ\), the start scan angle is \(0^\circ\), the stop scan angle is \(70^\circ\) and the increment scan angle is \(0.5^\circ\). Two different representations of the same results are shown in Fig. 5 where Fig. 5a is the classical representation with the overlap of all simulation in different color and Fig. 5b is the proposed unconventional plot with the color scale in dB. In the latter, the wanted elevation scan angle is the parameter on the Y axis, while on the X axis there is the elevation angle and finally the color indicates the magnitude in dB. So each horizontal line corresponds to the array factor calculated for the relative wanted elevation scan angle; in such a representation, it is easy to visualize the main lobe, sidelobe, nulls, and half-power-beamwidth (HPBW) variations when the scan angle changes and if there are some grating lobes [2]. Particularly, with the help of the black contour line at the -3dB level, the user can notice the widening of the HPBW with the increase in the scan angle. Referring to the Fig. 5a is quite impossible to extrapolate the results related to a single simulation.

![Fig. 4. Allowable phase states due to simulated real quantization.](image)

![Fig. 5. a) Overlapped simulations in a rectangular classical plot. b) Unconventional color polar plot.](image)
An example that demonstrates the simpleness and immediacy of the color graphical representation is shown in Fig. 6 where there are represented the different radiation patterns of an analogue phase shifter (Fig. 6a), an ideal four bit digital phase shifter (Fig. 6b) and a simulated four bit real phase shifter with a phase error of about 1.7°(Fig. 6c).

The differences are visible easily and they give the opportunity to underline the influence that the quantization effect has on the resultant radiation pattern and in particular the real quantization; in fact a Side Lobe Level (SLL) increment and also the not monotone trend of the HPBW are evident.

These are only some examples of the tool capabilities which depend on the characteristics the user wants to focus on.

IV. ANALYSIS OF THE PROPOSED ARCHITECTURE

The first utilization of the model has been the analysis of the performance of three prototypes of the proposed architecture explained in Section II, at three different frequencies. The prototypes are composed by two integer PLLs and a FPGA which in particular implements the phase control block. On the prototypes, ten measures of all phase shifts have been done with the help of an oscilloscope at the VCO center band and the mean of the ten measurements have been used to investigate the performances of the same prototypes.

In detail the characteristics are:

- First prototype: the center band of the VCO is 290MHz, the number of bits is 7, so the minimum ideal phase shift is \( \frac{360°}{2^7} \approx 2.8125° \); the measured rms phase error is about 0.436° while the maximum absolute phase error is about 0.7°.
- Second prototype: the center band of the VCO is 2.45GHz, the number of bits is 7 with a minimum phase shift equal to \( \frac{360°}{2^7} \approx 2.8125° \); the measured rms phase error is about 0.5056° while the maximum absolute phase error is about 0.8°.
- Third prototype: the center band of the VCO is 3.825GHz, the number of bits is 6 with a minimum phase shift of \( \frac{360°}{2^6} \approx 5.625° \); the measured rms phase error is about 0.3194° while the maximum absolute phase error is about 0.8°.

For briefness, the analysis presented are related to the second prototype, and in particular the measured phase shifts are used for the parametric simulation of changing the number of antennas and for the parametric simulation of changing the number of bits; the latter is easily obtained taking into consideration the right subset of the phase shifts available as the measured phase shifts for the program.

The simulation setup is shown in table I.
In the Fig. 7 there are shown the results of a parametric simulation with the variable number of antennas. In particular the Fig. 7a shows the maximum lobe deviation for each simulation over the entire range of scan angle in relation with the number of antennas; in the Fig. 7d shows there are results of the same simulation but using the color plot which gives the opportunity to show not only the maximum of the single simulation, but also all the values without losing any information and with a great interpretative easiness. The same considerations are valid for the Fig. 7b and Fig. 7e which represent respectively the maximum SLL value as a function of the number of antennas and all the SLL values as a function of the antennas and scan angle and for the Fig. 7c and Fig. 7f which show the maximum HPBW as function of the number of antennas and all the HPBW values as a function of the antennas and scan angle.

Furthermore in Fig. 8 there are shown the results of a parametric simulation with the variable number of bit. The Fig. 8a represents the maximum lobe deviation for each simulation over the entire range of scan angle in relation with the number of bits, the Fig. 8b represents the maximum SLL value for each simulation over the entire range of scan angle in relation with the number of bits, the Fig. 8c represents the maximum HPBW for each simulation over the entire range of scan angle and for the Fig. 8d and Fig. 8e which show the maximum HPBW as function of the number of antennas and all the HPBW values as a function of the number of bits and scan angle, the Fig. 8e shows all the SLL values as a function of the bits and scan angle and finally the Fig. 8f

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Variable number of antennas</th>
<th>Variable number of bits</th>
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</thead>
<tbody>
<tr>
<td>Elevation start angle (degree)</td>
<td>-90</td>
<td>-90</td>
</tr>
<tr>
<td>Elevation stop angle (degree)</td>
<td>90</td>
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</tr>
<tr>
<td>Elevation increment angle (degree)</td>
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<td>0.02</td>
</tr>
<tr>
<td>Scan start angle (degree)</td>
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<td>0</td>
</tr>
<tr>
<td>Scan stop angle (degree)</td>
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<td>70</td>
</tr>
<tr>
<td>Scan increment angle (degree)</td>
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<td>0.5</td>
</tr>
<tr>
<td>Number of antennas</td>
<td>Start: 5</td>
<td>Stop: 80</td>
</tr>
<tr>
<td></td>
<td>Increment: 5</td>
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<tr>
<td>Inter-element distance</td>
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<td>0.5</td>
</tr>
<tr>
<td>Number of bit</td>
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<td>Start: 3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Stop: 7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Increment: 1</td>
</tr>
</tbody>
</table>

The table above shows the details of the simulation setup with the variable number of antennas and bits. The simulations were performed with the specified parameters to analyze the performance of the system as a function of the number of antennas and bits.
Fig. 7. Simulation results obtained changing the number of antennas. a) Maximum lobe deviation. b) Maximum Side Lobe Level. c) Maximum Half Power Bandwidth. d) Lobe deviation. e) Side Lobe Level. f) Half Power Bandwidth.
Fig. 8. Simulation results obtained changing the number of bits.

By analyzing these results it is evident how such a tool is important in order to predict the influence of using a certain number of antennas or using a certain number of bits in certain system with a particular phase shifter; because, depending on system characteristics and requirements, it could happen that a wide number of antennas or bits don’t bring about a great improvement at the entire system or, even worse, it is the cause of a worsening.

V. CONCLUSIONS

In this paper we illustrate a complete and behavioral model that can be used to dimension and predict the behavior of whatever phase shifter, whether analogue phase shifter, or ideal digital one, or simulated real digital one or finally measured digital phase shifter. The model capabilities are very good in terms of possible type of simulations and clearness in data presentation. So the model is a great support as a basis for the user array pattern verification and for system measurement not only for the proposed architecture, but for whichever phase shift architecture.

REFERENCES