Optimal Parameter Setting of Performance Based Regulation with Reward and Penalty*  
Nuo XU, Fushuan WEN, Minxiang HUANG, Zhaoyang DONG

Abstract—The employment of performance based regulation (PBR) in distribution systems could provide some incentive for improving operating efficiency and reducing electricity prices. However, if the PBR mechanism is not properly designed, the enforcement of the PBR may have a negative effect on the supply reliability. In this paper, a mathematical model for optimally setting the parameters of the PBR with a reward/penalty structure is presented, with the minimization of the costs associated with the enforcement of the PBR as the objective and the required reliability level for the distribution system operation as the constraint. Finally, the well-known genetic algorithm is employed for solving the optimization problem. The effectiveness of the approach is demonstrated on a sample example.

Index Terms—electricity market; distribution company; performance based regulation; reliability; reward/penalty mechanism

I. INTRODUCTION

The traditional power industry had the vertically integrated management organization, where government or regulated organization employed, widely, integrated regulation models based on cost of service (COS) or rate of return (ROR). Along with the reform of electricity industry, various sectors of a power system, i.e. generation, transmission, distribution and retail, are separated. Different sectors should be enforced different regulation methods according to their features. Generation and retail would be deregulated by introducing competition. While transmission and distribution should still be regulated strictly, the approach of regulation could be reformed, since traditional COS/ROR regulation has many drawbacks such as the problem of excessive investment[1,2].

Performance Based Regulation (PBR) is introduced into many countries’ power industry especially in the distribution sector in order to improve efficiency and reduce costs[3,4]. If PBR is introduced in the distribution system, it is not difficult to improve the efficiency of distribution companies by applying some measures to reduce costs, such as cutting down or deferring capital projects, reducing the number of staff members, increasing maintenance intervals[5]. These measures may reduce the distribution system reliability. It is obvious that PBR may negatively affect the reliability of distribution companies. Thus in order to guarantee the reliability, when introducing PBR, the regulation organization should consider the effects on the reliability, and adopt incentive (reward/penalty) measures accordingly. In [6], the outlines of distribution companies’ PBR and the incentives to guarantee the reliability in Sweden, Norway, Spain and the Dutch were compared. Ref. [7,8] analyzed and compared the practical implementations of power quality regulations with PBR mechanisms for distribution systems in England and Wales, Argentina, Norway, California, and Spain. Ref. [5,9-11] illustrated an approach of introducing a reward/penalty structure into the common PBR. According to given reliability standard, distribution companies with better operational condition will be rewarded and those unable to reach the standard will be punished. This will discourage distribution companies from sacrificing reliability for reducing the costs. This mode is applied in Ontario Canada, Italy and some utilities in America[5,9-11].

Up to now, about the PBR with a reward/penalty structure designed for distribution companies, the researches have emphasized on the risk and management problems of the distribution companies faced under this regulation mechanism. However, how to set the parameters of the PBR with a reward/penalty structure to the distribution companies in maintaining the required reliability level, while minimizing the associated costs is still an important problem of research. This is also the central problem researched of this paper.

In this paper, a mathematical model for optimally setting the parameters of the PBR with a reward/penalty structure is presented, with the minimization of the costs associated with the enforcement of the PBR as the objective and the required reliability level for the distribution system operation as the constraint. Finally, the well-known genetic algorithm is employed for solving the optimization problem. The proposed approach is demonstrated on a sample example.

II. PBR WITH A REWARD/PENALTY STRUCTURE

For the PBR with a reward/penalty structure, the reward/penalty mode illustrated in Fig.1 can be adopted. Eqn (1) is the functional form of the curve in Fig.1.
Total Interrupted Supply Energy per Unit Time

The penalty payment increases as

condition, including the adjustment of the zero zone (\( \mu + \sigma \)), the parameters such as \( P_{\text{max}} \) and slopes of the two biases in

\[
P(x, \alpha, P_{\text{max}}) = \begin{cases} 
P_{\text{max}} & x < \mu - \alpha \\
P_{\text{max}} - \frac{x - \mu}{\sigma - \alpha} (\mu - \sigma) & \mu - \alpha < x < \mu - \sigma \\
0 & \mu - \sigma < x < \mu + \sigma \\
P_{\text{max}} - \frac{x - \mu}{\sigma + \alpha} (\mu + \sigma) & \mu + \sigma < x < \mu + \alpha \\
-P_{\text{max}} & x > \mu + \alpha 
\end{cases}
\]

Where \( x = \frac{\text{Total Interrupted Supply Energy per Unit Time}}{\text{Total Supply Energy per Unit Time}} \), and \( x \), which is termed as interrupted supply rate, is simulated as a random variable. The reliability index of the distribution company is then defined as \( 1 - x \).

\( P(x, \alpha, P_{\text{max}}) \) is the reward/penalty payment of the distribution company for the different value of \( x \). The positive value of it means reward, and the negative value means penalty. \( P_{\text{max}} \) expresses the maximum reward/penalty payment. \( P(x, \alpha, P_{\text{max}}) \) is a section function and it has a zero zone (i.e. in \( \mu + \sigma \)). \( \mu \) and \( \sigma \) are set by regulation organization according to performance conditions of distribution companies. \( P_{\text{max}} \) and \( \alpha \) are the main parameters regard to the reward/penalty structure.

When \( x \) is larger than the upper limit of the zero zone \( \mu + \sigma \), a penalty payment is accessed to the distribution company. The penalty payment increases as \( x \) increases and is capped at \( P_{\text{max}} \) when \( x > \mu + \alpha \). Similarly, when \( x \) is smaller than the lower limit of the zero zone \( \mu - \sigma \), a reward payment is accessed to the distribution company. The reward payment increases as \( x \) decreases and is capped at \( P_{\text{max}} \) when \( x < \mu - \alpha \).

The regulation organization will assess the reliability index of the regulated distribution company for a given period (for example one year), in order to calculate the reward/penalty payments. After a given regulation period (for example three to five year), the regulation organization may adjust the reward/penalty structure according to the set condition, including the adjustment of the zero zone (\( \mu + \sigma \)), the parameters such as \( P_{\text{max}} \) and slopes of the two biases in

Fig.1, in order to incentive the distribution company to improve the reliability index to the level expected by the regulation organization. \( P_{\text{max}} \) and \( \alpha \) are the parameters needed to set optimally by the regulation organization.

III. PARAMETER OPTIMIZATION MODEL

A. Objective Function

The objective of the regulation organization is to minimize the costs associated with the enforcement of the PBR mechanism by setting the parameters of the reward/penalty structure. The associated costs include the customer interruption loss and reward/penalty payments provided for distribution companies.

1) Customer Interruption Loss

The customer interruption loss includes the direct and overhead losses. The direct loss includes the loss of productions and raw materials, the damages of the customer belongings and the inconvenience in life because of interruption, etc. The overhead loss includes the cancellation of order form, the increase of committing rate, and the move of office and factory because of interruption. Obviously, the customer interruption loss can be reduced by supply reliability improvement. The customer interruption loss is expressed as \( L_{\text{cgt}}(\mu_g) \), where \( \mu_g \) is the mean value of \( x \).

2) Reward/penalty payments provided for distribution companies

In a given reward/penalty structure, reward/penalty payments issued by the regulation organization are not only related to the mean value of \( x \) but also to the distribution of \( x \). For example: provided that the mean value of \( x \) of one distribution company is 3.3%, the zero zone of the reward/penalty structure adopted by the regulation organization is (3.0±0.5%), which doesn’t mean that this distribution company will not be rewarded or punished by the regulation organization, because 3.3% is only the mean value of \( x \). Difference of the distribution of \( x \) will cause difference of reward/penalty payments. Provided that a regulation period is three years, the regulation organization will assess the distribution company every year. The following will analyze two situations.

a) The \( x \) changes in a smaller range. The mean values of \( x \) in the three years regulation period are 3.2%, 3.3% and 3.4% respectively. Thus the average value of \( x \) in the three years is 3.3%. The mean values of \( x \) in the three years are all in the zero zone, so the regulation organization will reward this distribution company.

b) The \( x \) changes in a larger range. The mean values of \( x \) in the three years regulation period are 3.6%, 2.6% and 3.7% respectively. Thus the average value of \( x \) in the three years is also 3.3%. However, the regulation organization will punish this distribution company on the first year and third year.

It is obvious that the reward/penalty payment assessed by the regulation organization is related to the probability distribution of \( x \). This probability distribution can be
obtained according to the system structures of distribution companies, historical data, the reliability evaluation measures, and so on [5,9-11]. Because \( P(x, \alpha, P_{\text{max}}) \) is a function of \( x \), and \( x \) is simulated as a random variable, \( P(x, \alpha, P_{\text{max}}) \) is also a random variable.

Thus, the reward/penalty payment assessed by the regulation organization can be expressed as the following equation.

\[
U(\alpha, P_{\text{max}}) = \int P(x, \alpha, P_{\text{max}}) \cdot f(x, \mu_x, \sigma_x) dx \tag{2}
\]

Where, \( f(x, \mu_x, \sigma_x) \) is the probability density function of \( x \). \( \mu_x \) is the mean value of \( x \), and \( \sigma_x \) is the standard deviation of \( x \). It is supposed that regulation organization can obtain these data by analyzing the historical probability data of distribution companies.

Thus, the objective function for optimal parameters setting of PBR can be expressed as:

\[
\min Y(P_{\text{max}}, \alpha) = L_{\text{tot}}(\mu_e) + U(\alpha, P_{\text{max}}) = L_{\text{tot}}(\mu_e) + \int P(x, \alpha, P_{\text{max}}) \cdot f(x, \mu_x, \sigma_x) dx \tag{3}
\]

Where, \( Y(P_{\text{max}}, \alpha) \) is the related cost after the PBR is adopted. \( \mu_e \) and \( \sigma_e \) are the mean value and the standard deviation of \( x \) that the regulation organization expects the distribution company to reach. \( \mu_e \) and \( \sigma_e \) are associated with \( \mu \) and \( \sigma \) in Fig.1. The regulation organization will set \( \mu \) and \( \sigma \) according to \( \mu_e \) and \( \sigma_e \), as well as other related parameters.

**B. Constrain**

In order to provide enough incentive for the distribution companies, it should be guaranteed that when mean values of \( x \) reach the requirement, reward payments to distribution companies are larger than, or at least equal to, the increase costs used to improve the reliability, i.e.

\[
U(\alpha, P_{\text{max}}) \geq Q(\mu_e) \tag{4}
\]

Where, \( Q(\mu_e) \) is the increase cost in each year when the distribution company reduces \( x \) from \( \mu_0 \) to \( \mu_e \).

Provided that the investment cost in one lump of reducing the mean value of \( x \) from \( \mu_0 \) to \( \mu_e \) is \( C(\mu_e) \). The increase expense in each year of maintenance, depreciation of equipment and so on is \( M(\mu_e) \). It is assumed that the distribution company expects to recover investment cost in one regulation period (Given \( n \) years).

When considering the time value of capital, the income and expenditure occurred on different time should be converted on the same time point by capital equivalent calculation. The equality-payment capital recovery equation is adopted to calculate the reliability investment cost, as illustrated in Fig.2. The equality-payment capitals \( A \) from the first year to the year \( n \) are equivalent to the investment capital in one lump i.e. \( C \). Where, \( i \) is the rate of return on investment.

\[
A = C(\mu_e) \left[ \frac{i(1+i)^n}{(1+i)^n-1} \right] \tag{5}
\]

![Fig. 2. Cash flow of uniform series](image)

Thus, the total increased cost in each year of the distribution company to reduce \( x \) from \( \mu_0 \) to \( \mu_e \) is

\[
Q(\mu_e) = A(\mu_e) + M(\mu_e) = C(\mu_e) \left[ \frac{i(1+i)^n}{(1+i)^n-1} \right] + M(\mu_e) \tag{6}
\]

**C. Parameter Optimization Model**

Thus, the parametric optimizing model for the reward/penalty structure of PBR is:

\[
\min Y(P_{\text{max}}, \alpha) = L_{\text{tot}}(\mu_e) + U(\alpha, P_{\text{max}}) + \int P(x, P_{\text{max}}, \alpha) \cdot f(x, \mu_x, \sigma_x) dx \tag{7}
\]

s.t. \( U(\alpha, P_{\text{max}}) \geq Q(\mu_e) \)

**D. Solution Procedure**

The optimization model of (7) is a constrained nonlinear optimization problem, and could be solved by the well-known genetic algorithm (GA) as detailed below:

a) Coding. The binary system coding is employed. And the coding structure could be expressed as \( [0 1 0...010 1 0] \), with both the variable \( \alpha \) and \( P_{\text{max}} \) included.

b) Initializing. An initialize population should be produced random in this step.

c) Fitness calculation. The genetic algorithm maximizes the fitness function, which is different from the objective of the optimal model of (7). So the objective function is changed to fitness function using fitness=C-obj, where \( C \) is a given number to guarantee the fitness to get positive value.
And in order to deal with the constrains, the penalty function method is applied. 

d) Reproduction operation. Reproduction is a process in which individuals are copied according to their fitness function. The individual with higher fitness value has a higher probability of contributing one or more offspring in the next generation.

e) Crossover operation. First, members of the newly reproduced strings in the mating pool are mated at random; second, each pair will exchange partial codings between random positions.

f) Mutation operation. In the GA, the mutation is needed to avoid convergence to local optimal by introducing diversity and variability. First, a farther individual is random selected; second the code on a random bids will change from ‘0’ to ‘1’; or from ‘1’ to ‘0’. The mutation operation will occur on a certain rate.

Repeat the above step (c) to (f) until the algorithm converges or the terminating condition given is met.

IV AN SAMPLE EXAMPLE

A. The Model of Assessing Customer Interruption Loss

The customer interruption loss is affected by many factors such as types of customers, interruption time, interruption frequency, interruption duration. There are many methods to assess the customer interruption loss, such as average electricity price converting multiple, ratio of output value to unit electric energy consumption, total owning cost\(^{12}\). In the following example, the average electricity price converting multiple is adopted. In this method, the interruption loss is assessed via multiplying average electricity price by converting multiple:

\[
L_{loss}(\mu_d) = \mu_d * E_{total} * m * A_{price}
\]

(8)

Where, \(E_{total}\) is the total supply energy, \(m\) is the converting multiple, \(A_{price}\) is the average electricity price.

In England, the test result shows that for each 1 kWh energy interruption, the loss is 50 times of average electricity price, and in France and China, they are 100 and 25 times respectively. Namely the \(m\) is 50, 100 and 25 respectively in England, France and China.

B. The Probability Density Function of Interrupted Supply Rate \(x\)

Provided that \(x\) is subjected to normal distribution, i.e.

\[
f(x, \mu_d, \sigma_d) \sim N(\mu_d, \sigma_d)
\]

(9)

\(\mu_d\) and \(\sigma_d\) are the mean value and standard deviation of \(x\), respectively. It is obvious that the probability that \(x\) is smaller than 0 is zero, and the probability that the normal distribution function is in the out of \(\pm 3\sigma_d\) is very small, and can be regarded as zero approximately. Thus it can be considered that \(\mu_d = 3\sigma_d\) approximately.

According to equation (8), when the mean value of \(x\) is known, the customer interruption loss is constant, and will not affect the optimal solution. Thus, when solving the optimal problem described in equation (7), only the reward/penalty payment should be considered in the objective function. For the given probability distribution of \(x\) expressed as equation (9), the objective function i.e. the reward/penalty payment can be simplified as equation (10).

\[
U(\alpha, P_{\text{max}}) = P_{\text{max}} * \int_{-\alpha}^{\alpha} f(x, \mu_d) dx
\]

\[
+ \int_{-\alpha}^{\alpha} f(x, \mu_d) \frac{P_{\text{max}}}{\sigma} * x - \frac{P_{\text{max}}}{\sigma} (\mu - \sigma) dx
\]

\[
+ 0 * \int_{-\alpha}^{\alpha} f(x, \mu_d) \frac{P_{\text{max}}}{\sigma} * x - \frac{P_{\text{max}}}{\sigma} (\mu + \sigma) dx
\]

\[
- P_{\text{max}} * \int_{-\alpha}^{\alpha} f(x, \mu_d) dx
\]

(10)

The present mean value and standard deviation of interruption supply rate \(x\) for the distribution system considered here are specified to be \(\mu_x = 3.3\%\), \(\sigma_x = 1.1\%\) respectively. The regulation organization hopes that the mean value of \(x\) would be reduced to \(\mu_x = 3.1\%\) by enforcement of the PBR with a reward/penalty mechanism. Provided the cost that the distribution company would need to pay for this reduction is \(2*10^5\) (\$) and the rate of return on investment in equation (5) is 10%. The solutions solved by genetic algorithm are: \(\alpha = 0.032\); \(P_{\text{max}} = 7.7294*10^6\) (\$).

C. The Change of Associated PBR Costs under Different Mean Values of \(x\)

It is obvious that if the mean values \(\mu_x\) of interruption supply rate \(x\) that the regulation organization hopes the distribution company to reach are different, the optimal solutions and corresponding objective values obtained from the optimal model described in equation (7) will be different. Fig. 3 illustrates the change of associated PBR costs for different \(\mu_x\). When calculating the results depicted in Fig. 3, the parameters related to interruption costs are \(E_{\text{total}} = 2.5*10^8\), \(m=25\), \(A_{\text{price}} = 0.53\). With reduction of \(\mu_x\), the reliability level requirement to distribution companies is improved, and the costs needed to pay are larger. The regulation organization should balance the requirement of economic and reliability and select the corresponding interruption supply rate index, and then determine the PBR parameters.
V CONCLUSION

A mathematical model for optimally setting the parameters of the PBR with a reward/penalty structure is developed taking the minimization of the costs associated with the enforcement of the PBR as the objective and the required reliability level for the distribution system operation as the constraint. Finally, the well-known genetic algorithm is employed for solving the optimization problem, and a sample example served for demonstration.

It should be pointed out that the work done in this paper is very preliminary, since the assessment of customer interruption loss and the relationship between the interruption supply rate and investment costs of distribution companies are both simplified.

REFERENCES


BIOGRAPHIES

Nuo XU received her BE and ME degrees in electrical engineer from Zhejiang University, China. She is now a lecturer and Ph.D. candidate in Zhejiang University. Her research interest is in electricity market regulation and power system planning.

Fushuan WEN received his BE and ME degrees from Tianjin University, China, in 1985 and 1988, respectively, and PhD from Zhejiang University, China, in 1991, all in electrical engineering. He joined the faculty of Zhejiang University, China, in 1991, and has been a full professor there since 1997. Since 2004, he has been a University Distinguished Professor in South China University of Technology. His current research interests are in power industry restructuring, power system fault diagnosis and restoration strategies, as well as artificial intelligence applications in power systems.

Minxiang HUANG received his ME in electrical engineering from Zhejiang University, China, in 1983. He is now a professor in Zhejiang University. His current research interests are electricity market and power system planning.

Zhaoyang DONG (M’99–SM’06) received the Ph.D. degree from The University of Sydney, Sydney, Australia, in 1999. He is now a Senior Lecturer at the School of Information Technology and Electrical Engineering, The University of Queensland, Queensland, Australia. His research interest includes power system security assessment and enhancement, electricity market, artificial intelligence and its application in electric power engineering, power system planning and management, and power system stability and control.