EFFECTS OF QUANTIZATION, DELAY AND INTERNAL RESISTANCES IN DIGITALLY ZAD-CONTROLLED BUCK CONVERTER

FREDY EDIMER HOYOS*, DANIEL BURBANO†, FABIOILA ANGULO‡, GERARD OLIVAR§ and NICOLAS TORO¶
Department of Electrical and Electronics Engineering & Computer Sciences, Universidad Nacional de Colombia Sede Manizales, Campus La Nubia, Manizales, Colombia
*fehoyosv@unal.edu.co
†daburbanol@unal.edu.co
‡fangulog@unal.edu.co
§golivart@unal.edu.co
¶ntoroga@unal.edu.co

JOHN ALEXANDER TABORDA
Faculty of Engineering, Electronics Engineering, Universidad del Magdalena, Santa Marta, Magdalena, Colombia
jtaborda@unimagdalena.edu.co

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Zero Average Dynamics (ZAD) strategy has been reported in the last decade as an alternative control technique for power converters, and a lot of work has been devoted to analyze it. From a theoretical point of view, this technique has the advantage that it guarantees fixed switching frequency, low output error and robustness, however, no high correspondence between numerical and experimental results has been obtained. These differences are basically due to model assumptions; in particular, all elements in the circuit were modeled as ideal elements and simulations and conclusions about steady state stability and transitions to chaos have been carried out with this ideal model. Regarding the practical point of view and the digital implementation, we include in this paper internal resistances, quantization effects and 1-period delay to the model. This paper shows in an experimental and numerical way the effects of these elements to the model and their incidence in the results. Now, experimental and numerical analyses fully agree.

Keywords: ZAD strategy; experimental bifurcation diagrams; ZAD-controlled DC-DC converter.

1. Introduction

A buck power converter can be modeled as a piecewise linear system with three topologies. A complete introduction to power converters can be found in [Mohan et al., 1995; Ang & Oliva, 2005]. Since switching power converters are variable structure systems, they are a source of nonlinear phenomena.
change in the bifurcation diagrams and system behavior.

In the last decade, ZAD strategy has been developed for controlling DC-DC buck power converters. The controller was proposed by Fossas and co-workers in 2001 [Fossas et al., 2001]. This technique uses a sliding surface which is defined as a linear combination of the output error and its derivative. This surface is forced to have zero average for each sampling period. The first implementation results have been presented in [Ramos et al., 2003]; however, numerical and analytical results do not highly agree with experimental setups. The main hypothesis regarding the discrepancy between real and simulated results is that the differences lie in the quantization process, internal resistances, delays and other components or dynamics, which are usually neglected in the modeling stage. Previous works have analyzed the ZAD-controlled buck converter using an ideal mathematical model [Angulo, 2004; Angulo et al., 2005b, 2008a; Fossas et al., 2009]. In these works the authors have studied the bifurcation diagrams, the transition to chaos and the stability limit.

On the other hand, digital Pulse Width Modulator (PWM) has its application increased due to a number of potential advantages which include: lower power consumption, less sensitivity to analog components and parameter variations, potentially faster design process, and so on. Practical prototypes of digital control for DC-DC converters can be found in [Minag & Dinavahi, 2011; Sreenivasappa et al., 2010; Zane et al., 2004; Maksimovic et al., 2003]. Nevertheless, the digital PWM has disadvantages which are mainly related to quantization effects and delays in the control loop [Peng et al., 2006; Petrievich & Sanders, 2003].

In this paper we propose to control the buck converter with ZAD-strategy and a digital pulse width modulator. Our main aim in this work is to validate that a ZAD-controlled DC-DC buck converter needs a more precise mathematical model for obtaining good agreement between experiments and simulation. For studying the digital ZAD-strategy applied to buck power converter, we introduce a more precise system model including explicitly MOSFET resistance, inductor internal resistance, measurement resistance, power source resistance, diode voltage and the quantization of the states and duty cycle due to Analog to Digital Converter (ADC). The paper is organized as follows:

Section 2 presents the mathematical framework to analyze and control the buck converter. Numerical and experimental results are shown in Sec. 3. These results are highly coincident. Section 4 presents the conclusions of the work.

2. Mathematical Model

A simplified diagram of the buck power converter is shown in Fig. 1. $E$ is the power source, $L$ the inductor, $C$ the capacitor, $R$ the load, $i_L$ is the inductor current, $v_c$ is the voltage capacitor, and $V_o$ is the output voltage. Note that $v_c = V_o$.

We define $x_1 = v_c$ and $x_2 = i_L$. The system is described by the following differential equations,

\[
\begin{bmatrix}
\frac{dx_1}{dt} \\
\frac{dx_2}{dt}
\end{bmatrix} = \begin{bmatrix}
\frac{1}{RC} & \frac{1}{C} \\
-\frac{1}{L} & 0
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
0 \\
\frac{E}{L}
\end{bmatrix} u \quad (1)
\]

$u = 1$ corresponds to the ON switch position and $u = 0$ corresponds to the OFF switch position.

![Fig. 1. Simplified diagram of the buck converter.](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>40.006 V</td>
</tr>
<tr>
<td>$R$</td>
<td>39.3 Ω</td>
</tr>
<tr>
<td>$r_s$</td>
<td>0.3087 Ω</td>
</tr>
<tr>
<td>$r_{MOSFET}$</td>
<td>0.3 Ω</td>
</tr>
<tr>
<td>$r_{Meas}$</td>
<td>1.007 Ω</td>
</tr>
<tr>
<td>$r_L$</td>
<td>0.338 Ω</td>
</tr>
<tr>
<td>$x_{Ref}$</td>
<td>32 V</td>
</tr>
<tr>
<td>$L$</td>
<td>2.473 mH</td>
</tr>
<tr>
<td>$C$</td>
<td>46.27 μF</td>
</tr>
<tr>
<td>$m$</td>
<td>1</td>
</tr>
<tr>
<td>$F_s$</td>
<td>10 kHz</td>
</tr>
<tr>
<td>$T_s$</td>
<td>10 kHz</td>
</tr>
<tr>
<td>$T_s$</td>
<td>300 μs</td>
</tr>
<tr>
<td>$V_{ref}$</td>
<td>1.1 V</td>
</tr>
</tbody>
</table>

Table 1. Parameter values.
R, L, C and the sampling time $T$ are given in Table 1. The dynamical equations can be expressed in a compact form as $\dot{x} = Ax + Bu$, where $\dot{x} = [\dot{x}_1, \dot{x}_2]^T := [\frac{dx_1}{dt}, \frac{dx_2}{dt}]^T$. A and $B$ are defined accordingly to Eq. (1) and $u \in \{0, 1\}$. Using an ON–OFF control, the supplied control signal can be defined through Eq. (2). This scheme is known as centered PWM because the control signal is symmetric with regards to half the sampling time $T$.

$$
u = \begin{cases} 
1 & \text{if } kT < t < kT + \frac{dT}{2} \\
0 & \text{if } kT + \frac{dT}{2} \leq t \leq kT + T - \frac{dT}{2} \\
1 & \text{if } kT + T - \frac{dT}{2} < t < kT + T 
\end{cases}$$

(2)

$s_{pw}(x(kT - T)) = \begin{cases} 
\sigma_1 + (t - kT)\sigma_1 & \text{if } kT \leq t \leq kT + T\frac{d\tau}{2} \\
\sigma_2 + (t - kT - T\frac{d\tau}{2})\sigma_2 & \text{if } kT + T\frac{d\tau}{2} < t < kT + T - T\frac{d\tau}{2} \\
\sigma_3 + (t - kT - T + T\frac{d\tau}{2})\sigma_1 & \text{if } kT + T - T\frac{d\tau}{2} \leq (k + 1)T 
\end{cases}$

(3)

$s_1, \sigma_1, \sigma_2, \sigma_3$ and $s_3$ are given by Eq. (4). In the following, we only show the first equality since the others can be worked in the same way. $K_s$ is a positive constant that can be thought as a control parameter. This constant gives more or less weight to the error derivative.

$$s_1 = (x_1 - x_{1\text{ref}} + K_s\dot{x}_1)|_{x = x(kT - T), n = 1}$$

$$s_1 := x_1(kT - T) - x_{1\text{ref}} + K_s\left(\frac{x_2(kT - T)}{(RC)} + \frac{x_2(kT - T)}{C}\right)$$

$$\dot{s}_1 = (\dot{x}_1 + K_s\dot{x}_1)|_{x = x(kT - T), n = 1}$$

$$s_2 = T\frac{d\tau}{2} + s_1$$

$$\dot{s}_2 = (\dot{x}_1 + K_s\dot{x}_1)|_{x = x(kT - T), n = 0}$$

$$s_3 = s_2 + T(1 - d_k)s_2$$

Therefore, the zero average condition is

$$\int_{\frac{d\tau}{2T}}^{\frac{(d+1)T}{2}} s_{pw}(x(kT - T))dt = 0.$$  

(5)

$d$ is the duty cycle and corresponds to the time when the switch is ON ($t_{ON}$) divided by the sampling time ($T$), thus $d = t_{ON}/T$. The objective of the ZAD-control technique is to find the value of $d$ in each iteration achieving zero average of a preestablished function in every T-cycle. This function is taken from an earlier definition given by [Carpita et al., 1988]: $s(x) = x_1 - x_{1\text{ref}} + K_s(x_1 - x_{1\text{ref}})$. Carpita proposed a sliding function as a combination of the output error and its derivative. The output error is the difference between the reference and the output values. As it was reported in [Fossas et al., 2001; Angulo, 2004; Angulo et al., 2008a, 2008b], we choose a piecewise-linear function which is noted as $s_{pw}(x(kT - T))$ [see Eq. (3)]. In this case, we use the data of the state variables for computing the initial value and the slopes of each linear part. We explicitly introduce the notation $d_k, k = 0, 1, \ldots$ to make clear that $d$ is updated in each iteration.

$s_{pw}(x(kT - T)) = \begin{cases} 
\sigma_1 + (t - kT)\sigma_1 & \text{if } kT \leq t \leq kT + T\frac{d\tau}{2} \\
\sigma_2 + (t - kT - T\frac{d\tau}{2})\sigma_2 & \text{if } kT + T\frac{d\tau}{2} < t < kT + T - T\frac{d\tau}{2} \\
\sigma_3 + (t - kT - T + T\frac{d\tau}{2})\sigma_1 & \text{if } kT + T - T\frac{d\tau}{2} \leq (k + 1)T 
\end{cases}$

(5)

In order to find $d_k$, we solve Eq. (5) and we obtain

$$d_k = \frac{2s_1 + Ts_2}{T(s_2 - s_1)}$$

(6)

Finally the duty cycle is given by

$$d = \begin{cases} 
1 & \text{if } d_k \geq 1 \\
d_k & \text{if } 0 < d_k < 1 \\
0 & \text{if } d_k \leq 0 
\end{cases}$$

Using the model defined by Eqs. (1), (2) and (7), numerical simulations not included in this paper show that the system is unstable for all values of $K_s$. Solving this problem we use another controller. This new controller (called Fixed Point Inducting Controller, FPIC) forces the system to evolve to a fixed point. A complete description about FPIC technique can be found in [Angulo et al., 2005a, 2007; Taborda et al., 2009; El Azouli et al., 2009].
With this controller Eq. (6) is changed to
\[
d_k = \frac{2s_1 + T \dot{s}_2 + \tau d_{ss}}{m + 1}
\]
where \( m \) is a real number different from zero, and \( d_{ss} \) is given by
\[
d_{ss} = \frac{x_{1ref} E}{E - \frac{\tau}{L} E}
\]
The duty cycle applied to the system is given by Eq. (7), where \( d_k \) is computed from Eq. (8). Since the value of \( K_s \) is very low (\( K_s \in (0, 0.00369) \)) the results are presented as a function of the normalized parameter \( K_s \) (\( K_s = K_s / \sqrt{LC} \)). Using \( K_s \) as bifurcation parameter, we obtain Fig. 2.

For \( 1.2 < K_s < 2.15 \) and \( 4.5 < K_s < 5 \) the system evolves to one-band chaos. For \( 2.15 < K_s < 4.5 \) the system evolves to four-band chaos. These behaviors in those ranges were never found in the experimental set up. For this reason, we proceeded to define a more precise model in such a way that more characteristics were included in it. Figure 3 shows a detailed model of the experimental set up and in Eqs. (10) and (11) the corresponding mathematical model is detailed.

The device used for the switching action is a MOSFET IRFP350, \( V_{fd} \) is the diode voltage, \( r_s, r_M, r_L, \) and \( r_{Med} \) are the internal power source resistance, MOSFET resistance, internal inductor resistance and a resistance for measurement of inductor current. Parameter values are given in Table 1. When the switch is ON the mathematical model is
\[
\dot{x} = \begin{cases} 
\frac{1}{RC} \frac{1}{C} \frac{-1}{L} \left( r_s + r_M + r_{Med} + r_L \right) \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\
\begin{bmatrix} 0 \\ \frac{E}{L} \end{bmatrix} 
\end{cases}
\]
On the contrary, when the switch is OFF the differential equations are
\[
\dot{x} = \begin{cases} 
\frac{1}{RC} \frac{1}{C} \frac{-1}{L} \left( r_{Med} + r_L \right) \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\
\begin{bmatrix} 0 \\ \frac{V_{fd}}{L} \end{bmatrix} 
\end{cases}
\]
In compact form, these equations can be written as
\[
\dot{x} = \begin{cases} 
A_1 x + B_1 & \text{if } u = 1 \\
A_2 x + B_2 & \text{if } u = 0 
\end{cases}
\]
where $A_1$, $B_1$, $A_2$, and $B_2$ are selected accordingly to Eqs. (10) and (11). Hence, when the system is controlled by the centered PWM the dynamical equations are
\[
\dot{x} = \begin{cases} 
A_1 x + B_1 & \text{if } kT \leq t \leq kT + \frac{dT}{2} \\
A_2 x + B_2 & \text{if } kT + \frac{dT}{2} < t < kT + T - \frac{dT}{2} \\
A_1 x + B_1 & \text{if } kT + T - \frac{dT}{2} < t < kT + T.
\end{cases}
\]
(13)

The ZAD technique is applied to this model. Nevertheless, the values given by Eq. (4) change since $\dot{x}_1$ and $\dot{x}_2$ have changed too. Then the value of $d_k$ is different in this case. Besides, the steady state for the duty cycle also changes according to Eq. (14)
\[
d_{ss} = \left( x_{1\text{ref}} \left( 1 + \frac{\med + \RL}{R} \right) + V_{f_d} \right) - x_{1\text{ref}}^* \left( \rs + \RM R \right) + E + V_{f_d}
\]
(14)

With this model we obtained a better agreement between experiments and numerical simulations. In the following, some experimental and numerical results are shown.

3. Numerical and Experimental Results

Figure 4 shows a numerically-computed bifurcation diagram ($K_s$ is the bifurcation parameter) according to Eqs. (3), (8), (13) and (14). In this case, for...