Using Competencies to Search for Suitable Exercises

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Abstract

With the massive use of the Web, numerous sites proposing exercises have appeared. The great difficulty for teachers and students is to find suitable exercises for the discovery of a new subject, training, revision or evaluation. On the basis of the teachers’ reflections, on how they determine the exercises which they need, we extracted the characteristics which seemed to us the most important and we propose modalities of exercise description. These descriptions are based on a taxonomy supplemented by a graph of competencies and sub-competencies. We conceived tools to create a base of exercises implementing these descriptions. We developed interfaces allowing the teachers to store exercises, to find them by following several ways of searching for exercises and to create interactive worksheets.

1. Introduction

On the web, many sites propose digital exercises to solve. School publishers and groups of teachers generally update and maintain these sites. In Mathematics, in France, we can quote Mathenpoche [7] (a site created and enriched by math teachers), Aventuremath (a site of the publishing houses of Pole and of Odile Jacob Multimedia) [2]. Within the framework of the ANR, our team has undertaken to create an editorial chain of exercise patterns. An exercise pattern is an exercise with constrained parameters so that the pattern, once instantiated, is of the level planned by the designer. IMS-QTI specification has been chosen to represent these patterns, to ensure the interoperability between the various educational platforms. We extended it to allow the description of interdependent parameters [1]. An editor of exercise patterns for teacher authors has been created. We also created a generator of web pages which, from extended IMS-QTI files, creates dynamic jsp and php pages so that the students can train on most of the Web servers. Now hundreds of patterns of mathematical exercises in schools are available, and our concern is the following one: how can we ensure that among these exercise patterns the various players (teachers and students) can find quickly those they need.

The expectations of the teachers and the students are not the identical, even if they are similar. Generally, the students search for the exercises by following the chapters of their course. Their difficulty in a totally free search context is that it is hard for them to know beforehand if they have the necessary pre-requisite knowledge to do an exercise. Furthermore, if they don't succeed, they become discouraged and abandon. It is thus essential that they be guided in their search, for example by supplying them with worksheets or learning scenarios.

We, thus, privileged in this study the expectations of the teachers as creators of exercises or users of a base of already created exercises. We questioned several math teachers by asking them to explain to us what they expected from an exercise base. Based on the results of this survey, we defined a system of classification based on competencies and we implemented it. The term “competency” has been chosen according to the discussions in [8][9].

2. How to choose an exercise? The questions that the teachers ask themselves

We sent to teachers of mathematics a questionnaire concerning on one hand the way they wish to search for an exercise in a base, on the other hand the way they wish that the result of the search, the exercises and their characteristics, be presented to them.

The official programs, in France, define for all the classes of secondary school an organization of educational content for four main objectives: - organization and management of data, functions; - numbers and calculation; - geometry; - quantities and measurement. For each of these objectives the content
is then specified according to the school level (sixth grade, seventh grade, etc.). To each item are associated the competencies or capacities\(^2\) to be acquired by the students and examples of activities to be implemented to obtain them [3]. The teachers agree that their first search criterion is school level. Textbooks closely follow the programs and we find the content and the competencies in the form of chapters and sub-chapters. The teachers search for the exercises by following the same criteria, but this is not enough for them. Indeed, in the same sub-chapter, it is necessary for them to discriminate between the exercises according to more specific criteria.

Concerning content, the teachers would like to find details on the prerequisite and the objectives of the exercise. They want to be able to answer the question: "given the state of progress of the course may I give this exercise?" without needing to resolve it. As an example, if the competency to be acquired is "to simplify an expression which contains square roots", numerous types of exercises correspond, containing expressions with one or several square roots the parameters of which are more or less complex. Table 1 shows that the competency can be refined so as to allow a more precise choice.

To be able to choose more quickly the exercises, the teachers would like to find directly supplementary information about them. Among this information, we would find the predictable difficulties of the students, bound to the resolution of the exercise (for example, think of using the theorem...); the typical errors (for example, the answer 5 for \(\sqrt{10} + \sqrt{15}\)); the importance of the exercise with regard to the objectives of the program: is it essential for this competency? Let us note that it is not an exercise which is essential for a competency but a type of exercise, hence the importance of exercise patterns.

The characteristic most often quoted is the difficulty, which the teachers wish to link to a competency to be reached. But as Gérard Tisseau points out: "an exercise has no intrinsic difficulty, but a statistical difficulty for a certain type of student at a certain moment of their learning path" [11]. The same exercise will be more or less difficult according to the school level where it is tackled, if it is about the discovery of a concept or about its assessment.

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\(^2\) The term capacité seems to have replaced in the programs 2007 the term compétence used in the programs 2004-2005.

### 3. Use of competencies to classify and to search for exercises

From the requests of the teachers, we propose a system of classification and search for the exercises allowing the teachers and the students to obtain exercises corresponding to their criteria.

#### 3.1 Classification of the exercises

The first set of criteria is school level and program. However the school programs often change (the BO of April 19th, 2007 modifies the program of mathematics defined in 2004-2005). We, thus, looked for a classification of the mathematics connected to the mathematical concepts and independent of programs. We chose to use the taxonomy created by the "mathematics taxonomy committee" (Math NSDL1) [10], who grouped together and merged several existing taxonomies to propose a common classification for the groups working on digital resources. This classification contains four hierarchical levels, it represents a common base for the classification; the authors indicate that we can add supplementary levels of detail. Indeed, such a structure is not precise enough to be able to index the resources, because in such a structure, exercises of very different levels such as:

- Ex1: Simplify \(\sqrt{a}\) where \(a\) is a square
- Ex2: Expand and simplify \((a\sqrt{b} + c\sqrt{d})\) where \(b\) and \(d\) are squares
- Ex3: Expand and simplify \((a\sqrt{b} + c\sqrt{d})\)^2 where \(bd\) is not a square
- Ex4: Fill the blanks in \(\sqrt{a} (...) + c\sqrt{b} = \ldots + d\sqrt{a}\) where \(ab\) is a square
- Ex5: Fill the blanks in \((\sqrt{a} - \ldots \sqrt{b})^2 = \ldots - c\sqrt{ab}\) occur all in Numbers and Computation \(\rightarrow\) Arithmetic \(\rightarrow\) Operations \(\rightarrow\) Roots. The first three exercises consist in developing and/or in simplifying an expression. The student has to start from the left hand side to arrive at the result. The last two exercises are more complex, because it is necessary to work simultaneously on both sides of equality. To take into account these differences we introduce into the hierarchy a supplementary level, the competencies. These exercises are about different competencies: know how to "simplify an expression with square roots" for the three first ones, know how to "complete an expression with square roots" for the two last ones.

For a given competency, to distinguish the different exercises, one has to determine precisely the sub-competencies that every exercise aims at implementing as well as the necessary pre-requirements. Every competency breaks down into a set of sub-
competencies. Every sub-competency can rely on the other sub-competencies of the same competency and on the other competencies, which constitute the prerequisites. Every sub-competency must be clarified by the specific properties of the parameters, which we call the context. A supplementary level is not necessary because subcompetency with its context and exercise pattern description (statement + constraints between parameters) are similar. For the competency "simplify an expression with square roots" involved in the first three exercises, we distinguished thirteen sub-competencies, some examples of which we present in the following table:

<table>
<thead>
<tr>
<th>Sub-competency and its context</th>
<th>Necessary sub-competency</th>
<th>Another necessary competency</th>
<th>Exercises</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sc1: Simplify √a where a=b²</td>
<td></td>
<td>Break down an integer into prime factors</td>
<td>Ex 1</td>
</tr>
<tr>
<td>Sc2 : Simplify √a√b where ab is a square but a and b are not</td>
<td>Sc1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sc10 : Simplify(m√a ± n√b)/√c where ac=d² and bc=e²</td>
<td>Sc2</td>
<td>Expand a literal expression</td>
<td>Ex 2</td>
</tr>
<tr>
<td>Sc13 : Simplify (m√a ± n√b)² where ab is not a square</td>
<td>Sc10</td>
<td>Expand an expression using remarkable identities</td>
<td>Ex 3</td>
</tr>
</tbody>
</table>

We shall notice that the sub-competency sc13 has as a prerequisite the competency "to break down an integer into prime factors" by transitivity on the necessary sub-competencies.

The characterization of the exercises is refined by the list of competencies and sub-competencies to be implemented. We thus propose a four level hierarchy description constituting a tree, the elements of the fourth level being connected with the graph of the competencies and the corresponding sub-competencies.

3.2 Search for exercises

Concerning the search for exercises, two important criteria desired by the teachers were not taken into account in our description: the school level and the level of difficulty. As already seen, the school level often varies. Thus, we wished to give this information in a way independent of the graph above: it is a part of the description of the exercises. The same exercise can be entered several times into the base with different levels. To search for an exercise, we can select the level, then clarify a competency, or search for all the exercises for a given competency, independently of the level. Then, the problem is to order the exercises selected to present them to the teacher. To order the competencies, we introduced the notion of weight for every competency. This weight is not fixed and can be modified. The rank of an exercise for a given competency is then calculated from the weights of the other competencies which it implements.

As regards the difficulty of an exercise, which is not easy to appreciate, we chose to associate its description with statistics on the scores obtained by the students. Those associated to the rank of the exercise for the competency and with the school level, make it possible to estimate the difficulty on a qualitative scale.

In France, (a+b)²= a²+2ab+b², (a-b)²= a²-2ab+b² and (a+b)(a-b)=a²-b² are named remarkable identities and are taught in secondary school.
4. Implementation

We developed software which makes it possible to enter a taxonomy with four levels and competencies and sub-competencies such as we have defined above. In this software, elements of the taxonomy, competencies and sub-competencies can be added by the teachers. This software can be used by teachers to classify and look for their own exercises but its aim is to make it possible to group together exercises created by various teachers and shared by a community. Naturally, the fact that we can enrich the classification makes it a tool of cooperative work and can lead to the corresponding problems.

Two ways to use this are available: the first one to modify and enrich the base (Figure 2), the second one to search for exercises and to create worksheets.

To add exercises to the base, teachers choose a path among the elements of the taxonomy and a competency; then they describe the exercise by defining the title, the language, a school level and the sub-competency corresponding to the exercise. They can also give additional information such as the necessity to use a medium, an indication of duration, the maximum score, etc. This interface of storage also makes it possible to define new competencies and sub-competencies with their contexts and their relations with the other sub-competencies and the competencies.

Figure 2. Interface to add an exercise pattern to the base

Searching for the exercises corresponding to a competency arranges them according to their rank. Teachers can click on one exercise to view details (Figure 3), or select several exercises to create interactive sheets of exercise templates. These worksheets are recorded in a data base to be used by all, teachers and students.

Figure 3. Presentation of an exercise
To choose an exercise pattern, the users can see the associated competencies and sub-competencies, discover the constraints between the parameters and resolve so many instances as they wish to test feedback.

5. Discussion and conclusion

Numerous sites propose educational resources on the Web, we extracted two projects which seem to us representative of libraries of interactive exercises. The European WebALT project (Web Advanced Learning Technologies) [6] aims at the construction of a repository of mathematical resources, more specifically exercises. The storage and the search for the exercises are based on the taxonomy of MathNSDL. WebALT noticed that the elements of this taxonomy are of too high a level to characterize the exercises and thus enriched this taxonomy by adding elements making it possible to come down in the hierarchy. This classification serves for indexing and for searching for the mathematical resources in the WALTER repository. The classification is described by using metadata which, when possible, derive from LOM. In France the site Mathenpoche, of the sesamath association [7], has been created and enriched by math teachers. It contains more than a thousand exercises parameterized for the four classes of secondary school. The search is made first by choosing the level, then by using a classification founded on the various chapters of the school programs. Every chapter is divided into series, each series containing a sequence of exercises. Every exercise has a title and contains 10 questions. The users can either execute the series of questions directly, or look at all the exercises of a series. In that case, for each exercise, they will have the description of the objective and then the description of its various questions.

Using just the existing taxonomies seems insufficient to discriminate between the exercises. Using competencies to tag educational resources is a proposition of the community as it is claimed by Sampson & Fytros [8]. We agree with this proposition by associating a graph of competencies and sub-competencies with the exercises. These competencies are similar to those used in Lingot to characterize the exercises [5], as well as to those used in Aplusix to classify their exercise patterns [4]. Furthermore, our process is characterized by the will to put the teachers at the centre of the system. They can visualize this graph and develop it by the addition of sub-competencies and competencies or by the modification of the weights associated with the competencies. The graph is used as a base for a fast search for exercises corresponding to what they are looking for. The developed tools are put at the teachers’ disposal; they allow them to use our exercises and our graph but also to create their own taxonomy and graph.

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10. References