Impact and compensation of sample clock offset on up-link SC-CDMA systems

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Abstract—Single-carrier code division multiple access (SC-CDMA), also named cyclic-prefix CDMA in the literature, is a promising air interface for the up-link of the 4G cellular wireless communication systems. It enables the high capacity intrinsically offered by CDMA by making the equalization of the multi-path channels and the mitigation of the resulting interference possible at a low complexity. This paper studies the impact of multi-user sample clock offset on the performance of up-link SC-CDMA and proposes a compensation strategy at the receiver. It is shown that clock offset leads to an unacceptable degradation of the performance if it is not compensated. In a simultaneous multi-user up-link system, the sample clock offset is usually pre-compensated at the transmitters. This paper demonstrates that the compensation can also be done at the receiver. In a first step the optimal linear multi-user joint detector is designed according to the MMSE criterion, taking the effect of clock offset into account. In a second step the MMSE joint detector is simplified to a low complexity detector incurring a negligible loss of performance.

I. INTRODUCTION

Cellular systems of the third generation (3G) are based on the direct-sequence code division multiple access (DS-CDMA) technique [1]. DS-CDMA increases the systems intrinsic capacity and offers interesting networking abilities. First, the communicating users do not need to be time synchronized in the up-link. Second, soft hand-over is supported between two cells making use of different codes at the base stations. However, the system suffers from intersymbol interference (ISI) and multiuser interference (MUI) caused by multi-path propagation, leading to a significant loss of performance.

In order to enable the design of low complexity transceivers that can cope with multi-path channels, next generation cellular systems could combine the DS-CDMA accessing scheme with the single-carrier block transmission (SCBT), also known as single-carrier (SC) modulation with cyclic prefix [2], [3]. Similarly to orthogonal frequency division multiplexing (OFDM), SCBT transforms a time dispersive channel into a set of parallel independent flat sub-channels that can be equalized at a low complexity. Since the SCBT technique benefits from a low peak-to-average power ratio (PAPR), it has been recognized as an interesting alternative to OFDM in the up-link, that could significantly reduce the constraints on the analog front-end as well as the processing complexity at the terminal [4]. DS-CDMA is applied on top of the SCBT equalized channel. The DS-CDMA signals are either spread across the SC sub-channels, leading to single-carrier CDMA (SC-CDMA) [5], [6], or across the SCBT blocks, leading to single-carrier block-spread CDMA (SCBS-CDMA) [7], [8]. SC-CDMA and SCBS-CDMA can be seen as the SC counter-parts of multi-carrier CDMA (MC-CDMA) [9], [10] and multi-carrier block-spread CDMA (MCBS-CDMA) [11], [12], [13] respectively. SCBS-CDMA preserves the orthogonality amongst the users, regardless of the underlying multi-path channel, which enables perfect user separation through low complexity code correlation. It entails however a larger symbol latency than SC-CDMA, that makes it impractical in medium-to-high mobility cellular environments. For time-selective channels, SC-CDMA is the only viable air interface.

In an actual communication system the synchronization between transmitter and receiver is not perfect. Synchronization errors can lead to unacceptable performance degradation. If we want to implement an SC-CDMA system, the impact of synchronization errors have to studied and if necessary, compensated. One of these synchronization errors is sample clock offset. The impact of sample clock offset has already been studied for MC-CDMA [14], [15] and for MC-DS-CDMA systems [16].

In this paper, we investigate the impact of sample clock offset on an SC-CDMA system with simultaneous multi-user up-link. A strategy for compensation of the multi-user sample clock offset at the receiver is also proposed. The paper is organized as follows: The model of the system is first introduced in section II. In a second step the impact of clock offset is analyzed in section III. In a third step a number of methods to compensate for the clock offset are proposed in section IV. The conclusion is in section V.

II. SYSTEM MODEL

Figure 1. gives an overview of the ideal SC-CDMA up-link system model. Each user terminal groups and modulates data and applies CDMA processing. We make the assumption of periodic signals. In practice this is done by adding a cyclic prefix at the transmitter and removing it at the receiver. All user terminals transmit simultaneously. Each transmitted signal experiences a different channel on its way to the base-station. At the base-station receiver, the signals from all user terminals are summed and noise is added. A multi-user joint detector is applied to recover the data symbols from each user.
A. Transmit model

The transmit processing consists of the following steps:
- Starting form a stream of bits, the bits are grouped together and mapped on a modulated symbol.
- In a second step, a set of symbols is grouped together in a symbol block; the number of symbols in the block is \( B \).
- In a third step, the symbol block is spread by the CDMA code into a transmit block. In the frequency domain a transmit block uses \( Q \) discrete sub-channels. We operate in an environment where multiple users transmit simultaneously to the base station. The number of users is \( M \). CDMA codes are used to separate the signals from different users. The spreading factor of the CDMA system is \( N \). Note that \( N \geq M \) and \( Q = NB \).
- In a fourth step, a signal frame is build from a series of consecutive transmit blocks. We number the transmit blocks with an index \( b \) starting from 1 till the length of the frame.

The \( b \)th transmit block in the signal frame from a particular user \( m \) generated at the user’s transmitter can be represented as:

\[
\mathbf{T}_m^b = \mathbf{Q}^m \mathbf{z}_m^b
\]

(1)

with
- \( \mathbf{z}_m^b \) is a modulated symbol vector in transmit block \( b \) from user \( m \). The size of \( \mathbf{z}_m^b \) is \( B \times 1 \).
- \( \mathbf{Q}^m \) is used to implement the spreading operation with the CDMA code for user \( m \). We have \( \mathbf{Q}^m = \mathbf{I}_B \otimes \mathbf{c}^m \), this represents the Kronecker product of an identity matrix of size \( B \) with the code vector \( \mathbf{c}^m \) for user \( m \). The size of \( \mathbf{Q}^m \) is \( Q \times B \), and the size of \( \mathbf{c}^m \) is \( N \times 1 \).

The contribution at the receiver of a single user \( m \) can be described as

\[
\mathbf{R}_m^b = \mathbf{F}_Q^H \mathbf{F}_Q \mathbf{Q}^m \mathbf{T}_m^b.
\]

(2)

\( \mathbf{F}_Q^H \mathbf{F}_Q \mathbf{Q}^m \) is a circular matrix that represents the time domain convolution with the channel in a matricial way. \( \mathbf{\Lambda}^m \) is a diagonal matrix of size \( Q \) representing the channel in the frequency domain for user \( m \). \( \mathbf{F}_Q \) is the square Fourier transform matrix of size \( Q \), \( \mathbf{F}_Q^H \) is the Hermitian transpose of \( \mathbf{F}_Q \), and corresponds to the inverse Fourier transform.

B. Clock offset model

Remark that eq.2 can be rewritten as:

\[
\mathbf{R}_m^b = \mathbf{F}_Q^H \mathbf{v}_m^b \quad \text{with} \quad \mathbf{v}_m^b = \mathbf{\Lambda}^m \mathbf{F}_Q \mathbf{Q}^m \mathbf{z}_m^b.
\]

(3)

The contribution \( \mathbf{R}_m^b \) of a particular user \( m \) is an Inverse Fourier transform of a vector in the frequency domain. So like in OFDM the time domain signal is a weighted sum of \( e^{j2\pi f_o l} \) functions with \( f_o l \) the sample frequency at the transmitter and \( l = -Q \ldots Q - 1 \). If there was no clock offset, this functions would be sampled at times \( \hat{t}(z) = (z-1)T_s^m + z = 1 \ldots Q \) and \( T_s^m = 1/f_s^m \). Note that \( T_s^m \) is user dependent, each user has a different sample clock.

The clock offset will cause a re-sampling at different time moments \( \hat{t}(z) \). The relative position of the \( \hat{t}(z) \) samples will evolve from block to block. In order to describe this evolution, we start with the first block \( b = 1 \) of the data frame. Let’s suppose in a first step that the first sample of \( \hat{t}(z) \) starts exactly at \( t = 0 \). We have \( \hat{t}(z) = (z-1)T_s \), with \( T_s \) the period of the base station receiver clock. In a second step, we introduce a shift \( t_0 \) of the \( \hat{t}(z) \) sampling moments away from \( t = 0 \). \( t_0 \) can be anywhere in the range \( 0 \leq t_0 < T_s \). We get:

\[
\hat{t}(z) = (z-1)T_s + t_0.
\]

(4)

For subsequent blocks \( (b = 2 \ldots) \), the shift from the block center will be \( t_0 - (b-1)(C_p + Q)(T_s^m - T_s) \) with \( C_p \) the number of samples in the cyclic prefix. By defining \( (T_s^m - T_s) = \Delta_s^m \) we get a sample series \( \hat{t}_b(z) \) for block \( b \) equal to:

\[
\hat{t}_b(z) = (z-1)T_s + t_0 - (b-1)(C_p + Q)\Delta_s^m.
\]

(5)

Since we want to highlight the difference with the original sample set \( t(z) = (z-1)T_s^m \), we rewrite eq.5:

\[
\hat{t}_b(z) = (z-1)(T_s^m - \Delta_s^m) + t_0 - (b-1)(C_p + Q)\Delta_s^m.
\]

(6)

Substituting this in the Inverse Fourier transform formula yields:

\[
r_b^m(z) = \sum_{t=-Q}^{Q-1} v_m^b(t) e^{j2\pi f_o t[z-(1-\Delta_s^m)] + \Delta_s^m} e^{-j2\pi ((b-1)(C_p+Q)\Delta_s^m)}. \quad (7)
\]

If we look at the time components between the square brackets, we see first a time dependent term \( (z-1)(1-\Delta_s^m) \). This term is slightly different from \( (z-1) \) because \( \Delta_s^m \) is small (order of 40ppm = parts per million). This term will destroy the orthogonality after the Fourier transform at the receiver and cause a small amount of inter sub-channel interference.
The second term \( \frac{1}{T_{ch}} \) represents a constant time offset and finds its origin in the phase offset between the sampling clock at the transmitter and receiver. Note that it is independent from the transmit block. A time delay corresponds to a per sub-channel phase rotation in the frequency domain proportional with the sub-channel frequency.

The third term \(-(b-1)(C_p+Q)\Delta_{m}^{T_{ch}}\) is also a time delay constant in a transmit block but increasing from transmit block to transmit block. The effect is the same as term 2 but here the impact grows from transmit block to transmit block.

Eq.7 can be expressed in a matricial way as follows:

\[
\mathbf{L}^m = \mathbf{G}^m \mathbf{D}^m \mathbf{v}^m
\]

with

- \( \mathbf{G}^m = (\mathbf{F}^H)^{(1-\frac{\Delta_{m}^{T_{ch}}}{\pi})} \) where raising \( \mathbf{F}^H \) to the power \( (1-\frac{\Delta_{m}^{T_{ch}}}{\pi}) \) must be seen as raising each element of \( \mathbf{F}^H \) to that power, and

- \( \mathbf{D}^m = \begin{pmatrix} d_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & d_{Q-1} \end{pmatrix} \)
a diagonal matrix with \( d_l = e^{j2\pi\alpha^m_{l}} \) with \( l = -\frac{Q}{2}, \ldots, \frac{Q}{2} - 1 \)
and \( \alpha^m_{l} = \frac{l}{T_{ch}} - (b-1)(C_p+Q)\Delta_{m}^{T_{ch}} \).

### C. Matricial model at the receiver

At the receiver the contribution of all users is summed and noise is added. Combining eq.3 and 8, summing over all the users, and adding noise yields a total signal \( \mathbf{R}_b \):

\[
\mathbf{R}_b = \sum_{m=1}^{M} \mathbf{G}^m \mathbf{D}^m \mathbf{\Lambda}^m \mathbf{F}^m \mathbf{s}_b^m + \mathbf{n}_b
\]

with \( \mathbf{n}_b \) a vector of independent white Gaussian noise samples of size \( Q \times 1 \) for block \( b \).

Finally we express also the sum over the different users in a matricial way. We get:

\[
\mathbf{R}_b = \mathbf{SGD} \mathbf{\Lambda} \mathbf{F} \mathbf{s}_b + \mathbf{n}_b
\]

with

\[
\mathbf{S} = \begin{pmatrix} \mathbf{I}^1_Q & \cdots & \mathbf{I}^M_Q \end{pmatrix}, \quad \mathbf{G} = \begin{pmatrix} \mathbf{G}^1 & \cdots & \mathbf{O}_Q \\ \vdots & \ddots & \vdots \\ \mathbf{O}_Q & \cdots & \mathbf{G}^M \end{pmatrix},
\]

\[
\mathbf{D}_b = \begin{pmatrix} \mathbf{D}^1_Q & \cdots & \mathbf{O}_Q \\ \vdots & \ddots & \vdots \\ \mathbf{O}_Q & \cdots & \mathbf{D}^M_Q \end{pmatrix}, \quad \mathbf{\Lambda} = \begin{pmatrix} \mathbf{I}^1_Q & \cdots & \mathbf{O}_Q \\ \vdots & \ddots & \vdots \\ \mathbf{O}_Q & \cdots & \mathbf{I}^M_Q \end{pmatrix},
\]

\[
\mathbf{F} = \begin{pmatrix} \mathbf{F}^1_Q \cdots \mathbf{O}_Q \\ \vdots \cdots \cdots \vdots \\ \mathbf{O}_Q \cdots \mathbf{F}^Q_{B \times B} \end{pmatrix}, \quad \mathbf{\Theta} = \begin{pmatrix} \mathbf{\Theta}^1 \cdots \mathbf{O}_{Q \times B} \\ \vdots \cdots \cdots \vdots \\ \mathbf{O}_{Q \times B} \cdots \mathbf{\Theta}^M_{Q \times B} \end{pmatrix},
\]

\[
\mathbf{s}_b = \begin{pmatrix} s^1_b \\ \vdots \\ s^M_b \end{pmatrix}, \quad \mathbf{n}_b = \begin{pmatrix} n^1_b \\ \vdots \\ n^M_b \end{pmatrix}.
\]

Where \( \mathbf{O}_{Q \times B} \) is a matrix of zeros with the size \( Q \times B \), \( \mathbf{I}^m_Q \) is a square matrix of zeros of size \( Q \) and \( \mathbf{I}^m_Q \) an identity matrix of size \( Q \), the superscript is only there to show that \( \mathbf{s}_b \) is repeated \( M \) times.

### III. Analyzing the impact of clock offset

We start with a classical MMSE joint detector at the receiver to estimate the modulated symbols. The MMSE joint detector is:

\[
\mathbf{M} = \sigma^2 \mathbf{I}_{BM} + \mathbf{T}^H \mathbf{T}^{-1} \mathbf{T}^H \mathbf{T} \quad \text{with} \quad \mathbf{T} = \mathbf{F}^H \mathbf{S} \mathbf{D} \mathbf{\Lambda} \mathbf{F} \mathbf{\Theta} \mathbf{s}_b
\]

It has been shown in [5] that \( \mathbf{T}^H \mathbf{T} \) can be transformed to a block diagonal matrix, simplifying the implementation of the matrix inversion. Applying the MMSE detector to the received signal eq.10 and expanding \( \mathbf{T}^H \) yields:

\[
\mathbf{\hat{s}}_b = [\sigma^2 \mathbf{I}_{BM} + \mathbf{T}^H \mathbf{T}^{-1} \mathbf{\Theta}^H \mathbf{F}^H \mathbf{\Lambda} \mathbf{F} \mathbf{\Theta} \mathbf{s}_b + M \mathbf{n}_b]
\]

Let’s examine first the effect of \( \mathbf{G} \). The inner product is

\[
\mathbf{S}^H \mathbf{F}^m \mathbf{G} \mathbf{S} = \begin{pmatrix} \mathbf{F}^m & \mathbf{G}^M \\ \vdots & \vdots \\ \mathbf{F}^1 & \mathbf{G}^1 \end{pmatrix} \begin{pmatrix} \mathbf{F}^1 & \mathbf{G}^M \\ \vdots & \vdots \\ \mathbf{F}^1 & \mathbf{G}^1 \end{pmatrix}
\]

![Fig. 2. degradation of bit error rate for successive blocks](image-url)
We see the general term \( F_s G^m \). In the absence of clock offset, we have \( G^m = F_s^H \) and \( F^m G^m = F_s^H F_s^H = I_Q \). So the clock offset destroys the orthogonality of the Fourier transform. \( F_s G^m \) is no longer the identity matrix and non-diagonal terms will appear. When \( F_s G^m \) block is on the diagonal, the non-diagonal terms in \( F_s G^m \) represent inter sub-channel interference. When \( F_s G^m \) block is not on the diagonal it represents multi-user interference and the non-diagonal elements in \( F_s G^m \) now represent extra multi-user interference terms, due to clock offset.

To get a quantitative idea, we calculate the diagonal and non-diagonal terms of \( F_s G^m \). I can be shown that the term on column \( k \) and row \( l \) is equal to:

\[
1 \sin(\pi(k - l - k \frac{\Delta}{2})) Q \sin(\pi(k - l - k \frac{\Delta}{2})) e^{-i\pi l \Delta}.
\]

For the diagonal elements \( k = l \), the amplitude is approximately equal to 1 and the effect is essentially a small rotation. If we assume for example a 128 carrier system with a clock offset of 40ppm, then we get a maximum difference in rotation between the sub-channels of about 0.9 degrees. 

The energy of the amplitude distortion is \(-90\,db\). For the non-diagonal elements the angle will vary over the entire \( 2\pi \) range and the maximum energy of the amplitude is about 52db below the amplitude energy of the diagonal elements.

So we expect the impact to be small.

Secondly we examine the effect of \( D_s \), while neglecting the effect of \( G_s \) so we put \( G^m = I_Q \). We get from eq.12:

\[
\tilde{s}_0 = [\sigma_n^2 I_{BM} + T^H T]^{-1} H^H S^H D_s \Lambda F \tilde{\Theta} \tilde{s}_b + M \tilde{\omega}_b.
\]

\( D_s \) is a diagonal matrix with, on its diagonal, terms equal to \( e^{i2\pi \alpha^m_i} \) where \( \alpha^m_i \) is increasing for subsequent blocks (see eq.8) and dependent on the user in the up-link case (different clock offset). The effect of \( e^{i2\pi \alpha^m_i} \) in the product \( D_s \Lambda \) is a sub-channel dependent rotation. It is like the transmitted signal passes through a transformed channel \( \tilde{\Lambda} = D_s^{-1} \Lambda \). The MMSE detector is constructed for \( \tilde{\Lambda} \), so applying the MMSE detector will result in incorrect channel compensation and in incorrect mitigation of the multi-user interference. In order to quantize the effect of \( D_s \) only, we assume no noise \((\sigma_n = 0)\) and an ideal channel \((\tilde{\Lambda} = I)\). In this case \( T^H T = I_{BM} \) and eq.12 becomes:

\[
\tilde{s}_b = \tilde{\Theta}^H S^H D_s F \tilde{\Theta} \tilde{s}_b.
\]

If the term \( F^H D_s F \) is not present, the matricial product in eq.15 would be equal to \( \Theta^H S^H \tilde{\Theta} \), which is equal to the identity matrix due to the orthogonality of the codes. The presence of \( F^H D_s F \) destroys the orthogonality of the codes and causes multi-user interference. \( F^H D_s F \) is a block-diagonal matrix with blocks \( F^H D_s F \) on the diagonal. It can be shown that for such a block the term on column \( k \) and row \( l \) is equal to:

\[
\frac{1}{Q} \sin(\pi(k - l - \alpha^m_i)) Q \sin(\pi(k - l - \alpha^m_i)) e^{-i\pi l \Delta}.
\]

If we look at the amplitude part of eq.16, we see it is a fraction of two sinusoids. The sinusoid in the denominator varies \( Q \) times slower than the one in the numerator. The amplitude will reach its maximum when the denominator tends to zero. This is the case for \( \alpha^m_i \approx (n - m) - Qi \) with \( i \) an integer. Using the definition of \( \alpha^m_i \) in eq.8, we obtain equivalently that the denominator tends to zero for \( b - 1 = \frac{Qi - (k - l)}{(C + Q) \Delta} + \epsilon \) with \(-1 < \epsilon < 1 \), \( \epsilon \) is added because \( b \) has to be an integer value. For these values of \( b \) the absolute value of the amplitude becomes:

\[
\frac{1}{Q} \sin(\pi \epsilon (C + Q) \Delta \frac{\alpha^m_i}{\Delta}) \left( C + Q \Delta \frac{\alpha^m_i}{\Delta} \right).
\]

Since the angles are very small \( \sin x \) can be approximated by \( x \) and the maximum of the absolute amplitude tends to 1. So the multi-user interference term can become as big as the useful signal itself. To explore the degradation caused by clock offset, a simulation for a 128 carrier system at a sample frequency of 5MHz and a sample clock offset of 40ppm has been made. We used a spreading factor of 8 and simulated a QAM64 modulation with a set of 500 multi-path channels for respectively 1,4 and 8 users. In Figure 2 we plotted the BER as the number of blocks progresses. We assume that the clock offset is zero on the first block. We observe a rapid degradation to a BER of 0.5. Note that the degradation is faster as the number of users increases, this is because the individual multi-user interference contributions add up.

IV. MODIFIED MMSE DETECTOR FOR THE COMPENSATION OF CLOCK OFFSET

We can change the detection with a new MMSE detector optimized for clock offset. The MMSE detector becomes:

\[
M_2 = [\sigma_n^2 I_{BM} + T^H T]^{-1} T^H F^H D_s \Lambda F \Theta \tilde{s}_b.
\]

Note that \( T^H T \) can no longer be transformed to a block-diagonal matrix due to the presence of \( G_s \). This means the inversion of a matrix of size \( BM \). A less complex detector can be constructed if we ignore the effect of \( G_s \) (the effect is small anyway). Then we get:

\[
M_3 = [\sigma_n^2 I_{BM} + T^H T]^{-1} T^H F^H D_s \Lambda F \Theta \tilde{s}_b.
\]

Because \( D_s \Lambda \) is diagonal, \( T^H T \) can be transformed to a block-diagonal matrix, and there is no increase in complexity for the inversion when compared to the MMSE detector without clock offset compensation. In Figure 3, the results of bit-error rate simulations are shown for a system at full user load. We see that the impact of clock offset is compensated by the ideal detector M2. Note that there is a
small performance degradation when compared to the system without clock offset using detector M. We also see that at 40ppm there is no noticeable performance difference between M2 and M3. The performance difference becomes only noticeable for clock offsets higher than 1000ppm.

Finally, if the clock offset is known at the transmitter, we can precompensate the clock offset. We apply a derotator $D_H$ in the transmitter before the inverse Fourier transform. The signal at the receiver will become:

$$ R_o = S G D A D_H^T F \Theta s_b + n_b. \quad (20) $$

$D$, and $A$ are both diagonal matrices, so there order can be switched, we get the product $D A D_H^T F$ that cancels out. We get:

$$ R_o = S G A F \Theta s_b + n_b. \quad (21) $$

Only the small effect of $G$ remains.

V. Conclusion

A matricial model for up-link SC-CDMA in the presence of clock offset has been established. We have shown that the impact of clock offset can be divided in 2 effects. The first effect hampers the orthogonality of the Fourier transform and is independent of the block sequence number. The second effect corresponds to a per sub-channel rotation that increases from block to block. If not compensated for, the multi-user interference caused by the second effect will destroy the detection. We have derived the MMSE joint detector at the receiver that is optimized for clock offset. We have simplified the optimal MMSE joint detector to an implementable solution and have shown that for normal clock sample offsets the degradation when compared with the optimal MMSE detector is negligible.

References


