The introduction of intuitionistic fuzzy sets is due to K. T. Atanassov, who also proposed some problems about this subject. D. Çoker defined the intuitionistic fuzzy topological spaces and, with some coworkers, studied these spaces. In this paper, we define and study the notion of quasicoincidence for intuitionistic fuzzy points and obtain a characterization of continuity for maps between intuitionistic fuzzy topological spaces.

The introduction of “intuitionistic fuzzy sets” is due to Atanassov [1], and this theory has been developed in many papers [2, 3, 4]. This author proposed as an open problem “to investigate the topological and geometric properties of the IFSs” remarking that “some first steps in this direction are made” [4].

In this paper, we define for intuitionistic fuzzy sets the notion of quasicoincidence and the corresponding neighborhood structure (see [9]). These concepts allow us to obtain a characterization of continuity for maps between two intuitionistic fuzzy topological spaces.

(For notions on ordinary fuzzy topology used in this paper, see [7, 8].)

First, we list some previous definitions.

**Definition 1** [1]. Let \( X \) be a nonempty set. An intuitionistic fuzzy set (IFS) \( A \) of \( X \) is an object having the form

\[
A = \{ ⟨x, μ_A(x), γ_A(x)⟩ \mid x ∈ X \},
\]

where the functions \( μ_A : X → I \) and \( γ_A : X → I \) denote the degree of membership and the degree of nonmembership of each element \( x ∈ X \) to an ordinary subset of \( X \), and \( 0 ≤ μ_A(x) + γ_A(x) ≤ 1 \) for each \( x ∈ X \).

**Notation 2.** \( 0_∞ = ⟨x, 0, 1⟩ \) and \( 1_∞ = ⟨x, 1, 0⟩ \).

**Definition 3** [2]. Let \( X \) be a nonempty set, and let \( A \) and \( B \) be two IFSs of \( X \). Then,

(a) \( A ⊆ B \) if \( μ_A(x) ≤ μ_B(x) \) and \( γ_A(x) ≥ γ_B(x) \);

(b) \( A = B \) if \( A ⊆ B \) and \( B ⊆ A \);
Definition 4 [5]. Let \( \{A_j \mid j \in J\} \) be an arbitrary family of IFSs of \( X \). Then,

\[(a) \bigcap A_j = \{(x, \land \mu_{A_j}, \lor \gamma_{A_j}) \mid x \in X\};
(b) \bigcup A_j = \{(x, \lor \mu_{A_j}, \land \gamma_{A_j}) \mid x \in X\}.
\]

For other definitions concerning IFSs used in this paper, see [5, 6].

Definition 5. Let \( A = \{\langle x, \mu_A, \gamma_A \rangle \mid x \in X\} \) and \( B = \{\langle x, \mu_B, \gamma_B \rangle \mid x \in X\} \) be two IFSs. Say that \( A \) quasicoincides with \( B \), denoted by \( AqB \) if \( \mu_A \) quasicoincides with \( \mu_B \) and \( \gamma_A \) quasicoincides with \( \gamma_B \).

Remark 6. If \( AqB \), we have that \( A \cap B \neq \emptyset \). \( (\mu_A \mu_B \) implies that \( \mu_A \land \mu_B \neq 0 \), then \( A \cap B \neq \emptyset \).

Remark 7. If \( A \) and \( B \) verify that \( \gamma_A = \mu_A' \) and \( \gamma_B = \mu_B' \), then \( AqB \) if, and only if, \( \mu \mu_B \).

Proposition 8. Let \( A \) and \( B \) be two IFSs of \( X \), let \( f : X \to Y \) be a map between two nonempty sets \( X \) and \( Y \), then if \( AqB \), \( f(A)qf(B) \).

Proof. \( AqB \) if and only if \( \mu_A \mu_B \) and \( \gamma_A \gamma_B \). Then, we have that \( f(\mu_A)f(\mu_B) \) and \( f(\gamma_A)f(\gamma_B) \), that is, \( (1 - f(\gamma_A))(1 - f(\gamma_B)) \). Thus \( f(A)f(B) \).

Proposition 9. Let \( X \) and \( Y \) be two nonempty sets, let \( f : X \to Y \) be a map, let \( A \) be an IFS of \( X \), and let \( C \) be an IFS of \( Y \), then if \( AqC \), \( Af^{-1}(C) \).

Proof. \( f(A)fC \) if and only if \( f(\mu_A)fC \) and \( f(\gamma_A)fC \). Then, \( \mu_A f^{-1}(\mu_C) \) and \( \gamma_A f^{-1}(\gamma_C) \), that is, \( \gamma_A f^{-1}(\gamma_C) \) (because \( f^{-1}(\gamma_C) = f^{-1}(\gamma_C) \)).

Remark 10. If \( A, B, \) and \( C \) are IFSs of \( X \), such that \( AqB, \) and \( B \subseteq C, \) then \( AqC \).

Definition 11. Let \( (X, \tau) \) be an IFTS, and let \( p \) be an IFP of \( X \). Say that an IFS \( N \) of \( X \) is a \( Q \)-neighborhood of \( p \) if there exists an IFOS \( A \) of \( (X, \tau) \) such that \( pqA \) and \( A \subseteq N \).

Theorem 12. Let \( (X, \tau) \) be an IFTS, let \( p \) be an IFP of \( X \), and let \( \mathcal{U}_Q(p) \) be the family of all the \( Q \)-neighborhoods of \( p \) in \( (X, \tau) \), then,

\[(1) N \in \mathcal{U}_Q(p) \implies pqN,
(2) N_1, N_2 \in \mathcal{U}_Q(p) \implies N_1 \cap N_2 \in \mathcal{U}_Q(p),
(3) if N \in \mathcal{U}_Q(p) and N \subseteq M, then \( M \in \mathcal{U}_Q(p),
(4) if N \in \mathcal{U}_Q(p), \) there exists \( M \in \mathcal{U}_Q(p), \) \( M \subseteq N, \) such that, \( \) for every IFP \( e \) which quasicoincides with \( M, M \in \mathcal{U}_Q(e).\)

Proof. (1) \( N \in \mathcal{U}_Q(p) \) if and only if there exists an IFOS \( A \) such that \( pqA \) and \( A \subseteq N, \) then \( pqN \) (by Remark 10).

(2) \( N_1, N_2 \) are \( Q \)-neighborhoods of \( p \) if and only if there exist two IFOSs \( A_i \) such that \( pqA_i, A_i \subseteq N_i (i = 1, 2) \), then, if \( p = c(\alpha, \beta) \), we have that \( c_\alpha \mu_A, \mu_A, \mu_N, c_\beta \gamma_A, \gamma_N \), \( (i = 1, 2) \), then \( c_\alpha \mu_{A_1 \cap A_2}, \mu_{A_1 \cap A_2} \subseteq \mu_{N_1 \cap N_2}, c_\beta \gamma_{A_1 \cap A_2}, \gamma_{A_1 \cap A_2} \subseteq \gamma_{N_1 \cap N_2} \), and \( pqA_1 \cap A_2, A_1 \cap A_2 \subseteq N_1 \cap N_2 \), with \( A_1 \cap A_2 \) an IFOS.

(3) It is obvious.
(4) $N \in \mathcal{U}_Q(p)$ if and only if there exists an IFOS $A$ such that $pqA$ and $A \subseteq N$, then $A$ is also a $Q$-neighborhood of $p$, and for each IFP $e$ such that $eqA$, $A$ is a $Q$-neighborhood of $e$. \hfill \Box

**Proposition 13.** Let $X$ be a nonempty set, for each IFP $p$ of $X$, let $\mathcal{U}_Q(p)$ be a family of IFPs verifying (1), (2), and (3) of the theorem, then $\tau = \{U \in \mathcal{U}_Q(p) \mid pqU\}$ is an IFT in $X$. If also the family verifies (4), then $\mathcal{U}_Q(p)$ is the system of $Q$-neighborhoods of $p$ in $(X, \tau)$.

**Proof.** $1_\sim \in \tau$ by (3).

$U_i \in \tau$ ($i = 1, 2$), and $pq(U_1 \cap U_2)$, then $U_i \in \mathcal{U}_Q(p)$ and $U_1 \cap U_2 \in \mathcal{U}_Q(p)$ by (2).

$\{U_j\}_{j \in J} \in \tau$, $pq \cup U_j$ with $p = c(\alpha, \beta)$ if and only if $c_\alpha q\mu_{U_j}$ and $c_{1-\beta} q\nu_{U_j}$ and it is equivalent to $c_\alpha q\mu_{U_{j_0}}$ (for some $j_0$ of $J$) and $c_{1-\beta} q\nu_{U_j}$ (for all $j$ of $J$). Then $pqU_{j_0}$ for some $j_0 \in J$, $U_{j_0} \in \mathcal{U}_Q(p)$ for some $j_0 \in J$, and $\cup U_j \in \mathcal{U}_Q(p)$ by (3).

Finally, if $N \in \mathcal{U}_Q(p)$, there exists $M \in \mathcal{U}_Q(p)$, $M \subseteq N$, such that for every IFP $e$ which quasi-coincides with $M$, we have that $M \in \mathcal{U}_Q(e)$, then $M \in \tau$, $pqM$, $M \subseteq N$, and $N$ is a $Q$-neighborhood of $p$ in $(X, \tau)$. Conversely, for every $Q$-neighborhood $N$ of $p$ in $(X, \tau)$, there is an $A \in \tau$ such that $pqA$, $A \subseteq N$, then for every IFP $e$ which quasi-coincides with $A$, we have that $A \in \mathcal{U}_Q(e)$, thus $N \in \mathcal{U}_Q(p)$. \hfill \Box

**Proposition 14.** Let $X$, $Y$ be two nonempty sets, let $f : X \rightarrow Y$ be a map, let $\tau$ be an IFT in $X$, and let $s$ be an IFT in $Y$. Then, $f : (X, \tau) \rightarrow (Y, s)$ is continuous if, and only if, for each IFP $p$ of $X$, and for each $Q$-neighborhood $V$ of $f(p)$, there exists a $Q$-neighborhood $U$ of $p$ such that $f(U) \subseteq V$.

**Proof.** If $V$ is a $Q$-neighborhood of $f(p)$, there exists an IFOS $G$ such that $f(p)qG$ and $G \subseteq V$, then $pqf^{-1}(G)$ (by Proposition 9), and $f^{-1}(G)$ is an IFOS such that $f^{-1}(G) \subseteq f^{-1}(V)$. Thus, $f^{-1}(V)$ is a $Q$-neighborhood of $p$ and $f(f^{-1}(V)) \subseteq V$.

Conversely, for each $G \subseteq s$, we have that, for every IFP $p$ such that $pqf^{-1}(G)$ is $f(p)qf(f^{-1}(G))$ (by Proposition 8), then $f(p)qG$, and $G$ is a $Q$-neighborhood of $f(p)$. By the hypothesis, there exists a $Q$-neighborhood $U$ of $p$ such that $f(U) \subseteq G$, then $U \subseteq f^{-1}(G)$ and $f^{-1}(G) \in \mathcal{U}_Q(p)$. From Proposition 13, it follows that $f^{-1}(G) \in \tau$. \hfill \Box

**References**


1542 Quasicoincidence for intuitionistic fuzzy points


Francisco Gallego Lupiáñez: Departamento de Geometría y Topología, Facultad de Ciencias Matemáticas, Universidad Complutense de Madrid, 28040 Madrid, Spain

E-mail address: fg.lupianez@mat.ucm.es
Submit your manuscripts at http://www.hindawi.com