Indirect fitting procedure to separate the effects of mobility degradation and source-and-drain resistance in MOSFET parameter extraction

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\textbf{Abstract}

A new procedure is presented to separate the effects of source-and-drain series resistance and mobility degradation factor in the extraction of MOSFET model parameters. It requires only a single test device and it is based on fitting the \(I_D(V_{GS}, V_{DS})\) equation to the measured characteristics. Two types of bidimensional fitting are explored: direct fitting to the drain current and indirect fitting to the measured source-to-drain resistance. The indirect fitting is shown to be advantageous in terms of fewer number of iterations needed and wider extent of initial guess values range.

\begin{equation}
I_D = \frac{K}{1 + \theta(V_{GS} - V_T)} \left( V_{GS} - V_T - \frac{V_{DS}}{2} \right) V_{DS},
\end{equation}

where
\begin{align}
V_{GS} &= V_{GS} - I_D \frac{R}{2} \\
V_{DS} &= V_{DS} - I_D R \\
K &= \frac{W}{L} C_{ox} \mu_v
\end{align}

\(V_{GS}\) is the externally applied gate voltage, \(V_{DS}\) is the externally applied drain voltage, \(R\) is the total source-and-drain series resistance, \(\mu_v\) is the low-field mobility, \(\theta\) is the mobility degradation factor due to the gate field [9], \(x\) is the bulk-charge factor which accounts for threshold voltage dependence on channel potential due to depletion thickness nonuniformity along the channel [10], and the rest of the parameters have their usual meaning.

1. Introduction

Accurate model parameter extraction is crucial for modeling modern MOSFET devices. Extensive work abounds in the literature dedicated to this subject. Free-carrier mobility degradation and source-and-drain series resistance are two parameters of special importance for MOSFET characterization that are particularly cumbersome to extract independently from each other. Both of these parameters produce similar effects on the device's transfer characteristics, \(I-V\) characteristics, and output characteristics, \(I-V\) characteristics, which complicates their accurate extraction.

Several ingenious procedures have been proposed to circumvent this difficulty [1–5]. Another method was proposed to extract these parameters from the drain current versus gate voltage characteristics in the saturation region using several devices of different mask channel lengths [6]. An alternative procedure was recently proposed to extract the source-and-drain series resistance independently of mobility degradation by using bias conditions under which the channel carrier mobility is kept constant [7].

In what follows we present a new procedure based on the above-threshold and below-saturation \(I-V\) characteristics of a single transistor. It is based on bidimensional fitting of the \(I_D(V_{GS}, V_{DS})\) model equation to measured data. This permits to separate the effects of mobility degradation and source-and-drain series resistance. The procedure is validated using synthetic data, and then applied to experimental data from a single MOSFET. The computational efficiency is finally analyzed in terms of number of iterations and the extent of initial values range tolerance.

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The above-threshold drain current of a MOSFET in the triode region may be modeled by an equation of the form [8]:
traditional methods are usually not computationally efficient because of the implicit nature of (1)–(3). The extraction of \( R \) and \( \theta \) usually turns out to be difficult, considering, as already mentioned, the similar effect that the total source-and-drain series resistance and the mobility degradation have on the shape of the \( I_D(V_{CS}) \) characteristic for small \( V_{DS} \). Alternatively, direct bidimensional \((V_{CS}, V_{DS})\) fitting of (1) to the measured \( I_D(V_{CS}, V_{DS}) \) characteristics is possible to extract model parameters \( R \), \( \theta \), \( \alpha \), and \( K \), using the, previously extracted, value of \( V_T \).

Instead of trying to fit the implicit model represented by (1)–(3) directly to the measured \( I-V \) characteristics, we can transform it into an explicit equation. Substituting (2) and (3) into (1), and solving for the drain current, after some algebraic manipulations, an explicit form of \( I_D(V_{CS}, V_{DS}) \) is obtained:

\[
I_D = \frac{V_{DS}}{R_{ON} + \sqrt{R_{ON}^2 - R_0^2}}
\]

(5)

where

\[
R_{ON} = \frac{R}{2} + \frac{(1 + \theta(V_{CS} - V_T))}{2 K(V_{CS} - V_T - \frac{\alpha V_{DS}}{2})} + \frac{R(1 - \alpha)V_{DS}}{4(V_{CS} - V_T - \frac{\alpha V_{DS}}{2})},
\]

(6)

and

\[
R_0^2 = \frac{V_{DS} R_0}{2 K V_{CS} - V_T - \frac{\alpha V_{DS}}{2}}.
\]

(7)

Here the subscripts “0” and “ON” have been chosen because \( R_0^2 = 0 \) for the particular case of \( V_{DS} = 0 \), and therefore Eq. (3) implies that \( R_{ON} \) represents half of the measured source-to-drain resistance \((R_{m} = V_{DS}/I_D)\).

Fitting of this explicit Eq. (5) already represents a significant computational improvement over trying to extract the parameters using the implicit Eq. (1). However, the computational efficiency of the fitting may be further improved by rearranging the model equation and performing an indirect fitting procedure, as presented in the next section.

3. Indirect fitting

To improve the numerical fitting efficiency, we propose to do an indirect bidimensional \((V_{CS}, V_{DS})\) fit to the measured source-to-drain resistance, instead of directly fitting the current (1), or (5). The measured source-to-drain resistance is:

\[
R_m = \frac{V_{DS}}{I_D}.
\]

(8)

Substituting the variable \( I_D \) by \( V_{DS}/R_m \) in (1)–(3) and after some algebraic manipulations yields:

\[
a_{vd} V_{DS} + a_{VC}(V_{CS} - V_T) - 2R_m = 0,
\]

(9)

where the two coefficients are given by:

\[
a_{vd} = R_m R K (2\alpha - 1) - R_m^2 K \alpha + R^2 K (1 - \alpha) + R \theta
\]

(10)

and

\[
a_{VC} = 2 R_m^2 K - 2 R_m \theta - 2 R_m R K.
\]

(11)

The proposed parameter extraction procedure proceeds as follows: (a) Measure \( I_D(V_{CS}) \) at low \( V_{DS} \) and extract the threshold voltage, \( V_T \), value using any of the known conventional \( V_T \) extraction methods [12]. Here we have used a transition method developed a few years ago [13]. (b) With this value of \( V_T \) and the measured \( I_D(V_{CS}, V_{DS}) \) data, proceed to extract the parameters \( R \), \( \theta \), \( \alpha \), and \( K \), by bidimensional \((V_{CS}, V_{DS})\) fitting of Eq. (9). It is important to observe that Eqs. (9)–(11), required for indirect fitting, are much simpler than Eqs. (5)–(7), utilized for direct fitting. Observe the straightforward functional dependence Eq. (9) has on the independent variables \( V_{CS}, V_{DS} \), as compared to the more complex dependence of Eqs. (5)–(7) on these variables. Consequently, the proposed indirect fitting procedure is expected to be more computationally efficient than direct fitting, as will be confirmed in the following sections. Although, it is reasonable to expect direct fitting to be more accurate than indirect fitting, because it directly uses the measured dependent variable \( I_D \) instead of the derived variable \( R_m \), our results presented in the next section reveal that both methods produce similar accuracies.

4. Results

Fig. 1a presents three below-saturation output characteristics of an experimental 0.15 μm channel length 4 nm oxide thickness test DRAM MOSFET, measured at three above-threshold gate voltages. This is the set of data points used in this demonstration of the extraction procedure. The figure also contains the resultant three model playbacks obtained using the parameters extracted by applying the indirect procedure of bidimensionally fitting \( R_m \), as expressed by (9), to the values of the measured source-to-drain resistance. Fig. 1b shows the corresponding measured and model playback \( R_m \) as functions of \( V_{DS} \). The parameters extracted from indirect fitting, and used for the model playbacks are shown in the figures. On the other hand, if direct fitting were used instead, a very similar set of parameter values would be extracted, resulting in comparable accuracy of the direct and indirect fitting procedures, as will be shown in the next section. The most important difference between both of them lays on the dissimilar computational efficiency and robustness inherent to each procedure, with the indirect fitting being substantially better, as will also be shown in the next section.

For completeness sake, we also present in Fig. 2a experimental above-threshold transfer characteristics of the same MOSFET,
measured at three low drain voltages, together with the three resultant model playbacks. The playbacks are only illustrated for strong inversion condition where the model is valid. Fig. 2b shows the corresponding measured and model playback $R_m$ as functions of $V_{GS}$. We have chosen to present in Fig. 2b curves for these three low values of drain bias (10, 20, and 50 mV) to illustrate the fact that the differences in measured source-to-drain resistance are almost imperceptible at such low drain voltages. This means that it is nearly impossible to separately extract the total source-and-drain series resistance and the mobility degradation factor at such low drain voltages using a single device. Therefore the bidimensional fitting extraction should be performed at larger values of drain bias, as we have done in this example by using drain-to-source voltages up to 0.5 V.

5. Discussion

Fig. 3 portrays the convergence of the parameters as they are being extracted using each of the two bidimensional fitting procedures studied: “Direct fit” to (5)–(7) and “Indirect fit” to (9). The figure presents their evolution from the initial guess values, converging towards their final values, as the number of iterations progresses. For convenience of discussion, we define here a general factor $f$ which represents the quotient between the initial guess and the correct parameter value. The initial guesses used for this illustration are 3.89 times their final values ($f = 3.89$). Initial guess values corresponding to $f = 3.89$ were chosen because the direct procedure starts to fail to converge when the initial guess values used are above a value of $f = 3.9$, as will be seen in Fig. 4. Both fitting procedures were applied using a Levenberg–Marquardt type algorithm [14] with a step size of $10^{-3}$ and a tolerance of $10^{-10}$.

It is evident from Fig. 3 that the use of the indirect fitting procedure provides a clear advantage in computational efficiency, since it requires a considerably smaller number of iterations to converge to the correct values: about 28 for indirect fitting against around 100 for direct fitting, under similar computational conditions in this particular example. This corroborates, as expected, that Eqs. (9)–(11), because of their simpler functional dependence on the independent variables ($V_{GS}, V_{DS}$), impose a lesser computational burden than Eqs. (5)–(7), when used for fitting the measured data.

To further compare the relative computational robustness of both direct and indirect procedures, we measured the number of iterations required for each procedure to converge to the correct parameter values, within the established tolerance, as a function of the extent of the initial guess range. Fig. 4 indicates that the direct and indirect procedures require comparable numbers of
iterations for values of $f$ of up to about 2.25. Beyond this initial guess range, the figure shows that the direct fit requires increasingly larger numbers of iterations than the indirect fit. This number of iterations needed starts to grow quickly upon reaching about 3.5, until the direct procedure is no longer capable of converging to the correct parameter values for $f$ above 3.9. Conversely, the number of iterations needed by the indirect fitting procedure remains around 28 even for larger values of $f$ up to about 6.5, where the indirect procedure also fails to converge to the correct parameter values. All these results are, of course, valid for this particular example and for the computational constraints indicated. However, it seems reasonable to expect in general the same type of trends observed, specially pertaining to the relative computational efficiency and robustness of both bidimensional fitting procedures.

Finally, in order to compare the resulting accuracy of direct and indirect fitting, we show in Fig. 5 the relative errors of drain current versus drain voltage, at two strong inversion values of gate bias, as modeled using the parameters extracted using each of the two fitting procedures. According to this figure, indirect fitting using (9) seems to produce slightly more accurate results than directly fitting to (5)–(7), for $V_{GS} = 2$ V. On the other hand, direct fitting turns out to be slightly better than indirect fitting for $V_{GS} = 3$ V. One may conclude that, since the differences are minor, both fitting procedures are comparable in their accuracy.

The dependence of the series resistance with respect to gate voltage could [15,16] be approximately extracted by the present method. For example, if the bidimensional fitting is performed for three close values of gate bias, the extracted parameters would correspond to the average gate bias. Furthermore, we would like to point out that by generalizing Eq. (9) this indirect bidimensional fitting procedure could be applied to more complex models, including carrier velocity saturation and other short channel effects [17]. The generalization could be done by using a more sophisticated model than the one presented in (1)–(4). It is important to note that such a generalization of (9) does not require an explicit form of $I_D(V_{GS}, V_{DS})$.

6. Conclusions

We have presented a new procedure to separately extract the mobility degradation factor and the source-and-drain series resistance parameters of MOSFET models. Additionally, bulk-charge factor, and $K$ are extracted. The procedure is based on the bidimensional fitting of their current–voltage characteristics model equations to the measured $I_D(V_{GS}, V_{DS})$ data. The proposed procedure is applicable to a single device and it is able to surmount the usual difficulty of separating the effects of source-and-drain series resistance and mobility degradation factor. The important aspect is the use of indirect fitting of the source-to-drain resistance expression to the measured data. Direct fitting of the drain current explicit expression to the measured current data was also used as a comparison, and indirect fitting showed clear advantages over direct fitting regarding computational efficiency and robustness. This suggests that it might be worth exploring this idea of indirect fitting also in other similar parameter extraction situations, as it could result in improved convergence.

References