The Role of Instrumentation in the Process of Modeling Real Capacitors

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Abstract—A laboratory experiment for the introductory electrical circuit course is presented. The experiment consists of the measurement of real commercial capacitors' impedance using a relatively low-cost impedance meter, and further analysis on two levels. First, when results follow well the conventional impedance expression of an ideal capacitor, the capacitance can be determined accurately. Second, when deviations are found, the more advanced framework of equivalent circuit representation has to be used. This representation is discussed for the particular case of capacitors with dc leakage. This experimental approach explicitly emphasizes the role of electrical models in engineering and suggests how to use instrumentation to validate them.

Index Terms—Capacitors, electrical modeling, instrumentation, open-ended problems, problem-solving.

I. INTRODUCTION

It is commonly accepted that modeling an electrical system mainly consists of establishing a set of structural relations between suitable parameters which represent its electrical response. But in many cases, the role of instrumentation in the process of model construction is not explicitly acknowledged. Modeling and measuring are always parallel related activities in every electrical laboratory practice. The strain between modeling and measuring is the practical expression of a more fundamental relation, that existing between relevant system features, on one hand, and the conceptual and instrumental means the student possesses for gaining new information about the system, on the other.

The tension between model validity and experimental limitations appears clearly when results depart from the expected value, in making for example a voltage measurement after calculations based on the proposed model of the system. A question unavoidably arises: is this voltage deviation due exclusively to random or systematic errors or, more likely, is it necessary to consider changes in the system model or instrumentation? These kinds of procedural doubts commonly appear during experimental practices in any laboratory of electromagnetism, electronics or materials electrical properties.

In those situations in which the system model is well established in terms of theoretical and technical bases, the measuring problem mainly comprises the estimation of some of the unknown system parameters. From the point of view of the problem-solving discipline, a particular routine must be chosen among an established battery of methods, that is, a strategy is defined by means of several diagnostical decisions [1]. Therefore, only to a limited extent may these problems be regarded as open-ended situations. In other cases, the problem solution consists of a failure-finding task, because the answer simply lies within a malfunctioning device or circuit part.

But, sometimes, the question is more critical and demands higher conceptual and procedural skills and resources. Particularly when measurement deviations from expected values are due to the adoption of a wrong or incomplete model and not to measurement errors. Students who have only experimented with routine and more standardized procedures often become "blocked" because well-defined, step practices are generally unsuited to solve open-ended problems.

Measurements are always a part of the model-creating process, as it is apparent to professional engineers [2]. Explicit awareness of this relationship can improve students' understanding of real-world engineering problems. In this sense, the present contribution is considered an introduction to a particular case of open-ended electrical problems (model construction) that commonly appear in engineers' everyday tasks. It shows the way to combine conceptual and instrumental resources in order to improve the knowledge the students possess of the tested electrical system. The role of instrumentation is explicitly detailed as well as the introduction of significant, new basic knowledge when it is required.

It is obvious that relatively simple systems must be used when introducing students to the relation between modeling and measuring. To illustrate the relationship being investigated, a system has been deliberately chosen among the devices treated in most basic electrical engineering courses: the capacitor. The paper aims to illustrate in a practical case the process of constructing and validating a circuit model for commercial capacitors. Sophisticated instrumentation has to be employed when more elaborate models are investigated. This laboratory experiment teaches the students the importance of using the proper instrumentation to measure and model real-world devices.

II. MODELING REAL CAPACITORS

First, consider a capacitor with plane parallel plates of area $A$, separated a distance $L$, which is much smaller than the size of the plates. Students' common knowledge of these devices includes the expression for its impedance

\[ Z(\omega) = -\frac{j}{\omega C} \]  \hspace{1cm} (1)

with a capacitance $C = \frac{A \varepsilon_0}{L}$.
Next, directly measure the capacitance value of some of the samples by means of a hand multimeter. It is important to make explicitly clear to the students that this measurement has been performed at a fixed frequency as specified in the instrument operation manual. This choice of instrument is incorrect if the aim is to validate (1). A whole study of the impedance of a capacitor is possible only if a change of instrumentation is adopted.

The impedance as a function of frequency can be measured using a HP 4263A inductance–capacitance–resistance (LCR) meter, which operates at frequencies (in Hz) 100, 120, 1 K, 10 K, and 100 K. These instruments assure very high accuracy in impedance measurements by exploiting an ac bridge-balancing technique [3]. Thirteen commercial capacitors of different kinds: three electrolytic (nominal values of capacitance 10, 1, and 0.1 mF); two tantalum (10 and 1 μF); two polyester (1 and 0.1 μF); two polycarbonate (100 and 10 nF); and four ceramic capacitors (1 nF, and 100, 10 and 1 pF) are used.

Impedance can be written in the following form:

\[ Z = |Z| e^{j\phi}. \] (2)

The LCR meter provides both the modulus \(|Z|\) and the phase angle \(\phi\) of the impedance. The ideal capacitor is defined by \(\phi = -90^\circ\) together with

\[ |Z| = \frac{1}{\omega C}. \] (3)

and, therefore, if a capacitor behaves as an ideal capacitor in the frequency range considered, a log–log plot of \(|Z|\) versus frequency must yield a straight line.

Values of the impedance modulus as a function of frequency are shown in Fig. 1. The plots are straight lines for capacitors of polyester, polycarbonate, and ceramics, while those of electrolytic and tantalum capacitors deviate markedly from a straight line. Measurements of the phase angle, shown in Fig. 2, allow an unambiguous identification of each capacitor’s ideality range, and confirm what is appreciated only approximately in the \(|Z|\) versus \(f\) plots: \(\phi\)-values for polyester, polycarbonate, and ceramic capacitors stay very close to \(-90^\circ\) in the available frequency range. On the other hand, electrolytic and tantalum capacitors do not behave at all as ideal capacitors for frequency values above 100 Hz and 10 Hz, respectively. In all these experiments the deviation of \(\phi\)-value as a criterion of model validation has been adopted.

In the preceding exposition, a wholly insulating condition is attributed to the dielectric via (1). If a dc leakage is included in the capacitor model, its ac response must be represented by a resistance–capacitance (RC) parallel circuit model. This conceptual extension, namely the inclusion of circuit theory of lumped parameters, is needed to rightly account for the capacitor responses previously presented. The full equivalent circuit of a capacitor [4], [5], depicted in Fig. 3, includes a resistance due
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Fig. 3. a) Full equivalent circuit of a real capacitor and b) simplified measurement equivalent circuit model.

Fig. 4. Parallel mode capacitance $C_p$ (open symbols) and series mode capacitance $C_s$ (full symbols) as a function of frequency for 13 capacitors of the following types: electrolytic (○), tantalum (□), polyester (△), polycarbonate (◆), and ceramic (◇). Lines are connected between data points to guide the eyes.

to leads and electrodes, and a certain inductance that consists of the inductance of the electrodes and that of the wire leads. The inductance together with the capacitance $C$ defines the resonant frequency. Above this frequency, the capacitor behaves like an inductor. Further extension of capacitor modeling at high frequencies as an example of distributed circuit analysis has been presented by others [6].

Now it is interesting to observe that these kind of lumped parameter models also constitute some of the apparatus elements. Measurement instruments balance the real capacitor against a reference bridge arm that represents the real capacitor by an equivalent circuit model consisting either on a $R_sC_s$ series circuit, or a $R_pC_p$ parallel circuit. Considering the full bridge circuit, the adjustable internal arm can be regarded as a real capacitor replica. Both of these models, parallel and series, have been merged in the representation of Fig. 3(b). Which of them is best suited to measure a given capacitor, has to be assessed on the basis of the capacitor’s full equivalent circuit previously discussed and shown in Fig. 3(a). In general, it can only be said that if the capacitance is large, then the series resistance ($R_s$) is

likely to have relatively more significance than the parallel resistance ($R_p$), which can be neglected. This can be observed by examining the impedance expression associated with the circuit in Fig. 3(b)

$$Z(\omega) = R_s + \frac{1}{1/R_p + j\omega C_p}. \quad (4)$$

Measurements in both series and parallel capacitance modes yield appreciably different results for the higher capacitance capacitors, those that do not follow an ideal behavior, as shown in Fig. 4. Inspection of Fig. 4 shows that when different results appear for a given capacitor, $C_s$ maintains a constant value while $C_p$ decreases steadily with frequency. Therefore results in Fig. 4 support the previous argument that favors the series model circuit with respect to the parallel one in order to represent real capacitors of large capacitance.

The equivalent circuit model in Fig. 3(a) predicts that a capacitor behaves as a resonator at high frequencies. A clear identification of this behavior requires high-resolution in frequency and a large frequency measurement window. This is outside the capabilities of the LCR meter with only five measurement frequencies, and therefore, in order to show with detail the resonance and the inductive behavior at higher frequencies, an advanced research frequency analyzer HP-4192A was used to measure the impedance of the 1-$\mu$F polyester capacitor, as a function of frequency in the range $10^{-10}^7$ Hz (20 values per decade). Results of measurements are shown in Fig. 5. It was found, however, that no parallel resistance $R_p$ was needed to correctly fit the data. These last considerations have finished the modeling process and, therefore, the problem has mainly become an error treatment. Consequently, this capacitor can be represented with excellent accuracy by an impedance of the type

$$Z(\omega) = R_s + j\left(\omega L - \frac{1}{\omega C}\right). \quad (5)$$
in the whole measurement window. An interesting feature of the impedance response of this polyester capacitor, clearly illustrated in Fig. 5, is that passage from capacitive to inductive behavior takes place in a very narrow interval of frequencies (less than a decade of frequency).

III. CONCLUSION

The experimental process proposed in this paper and the discussion of results allow a main conclusion to be drawn: the familiar impedance expression of an ideal capacitor, given in (1), is a simplified model with limitations that students have to be aware of when dealing with real capacitors. An extended framework for analysis, which takes into account the properties of the material that acts as a dielectric and also the different parts that compose a real capacitor, by means of an equivalent circuit representation has been discussed. The significance of the extended model is that it provides not only minor corrections and in certain frequency ranges it replaces entirely (1). The simple experiment presented in this paper may give rise to an interesting reflection on the role of models in electrical engineering and, further, how to use instrumentation in the model validation.

REFERENCES


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