We present a novel algorithm performing projective rectification which does not require explicit computation of the epipolar geometry, and specifically of the fundamental matrix. Instead of finding the epipoles and computing two homographies mapping the epipoles to infinity, as done in recent work on projective rectification, we exploit the fact that the fundamental matrix of a pair of rectified images has a particular, known form. This allows us to set up a minimization that yields the rectifying homographies directly from image correspondences. Experimental results show that our method works quite robustly even in the presence of noise, and with inaccurate point correspondences. The code of our implementation will be made available at the author’s web site.

1. Introduction

The cornerstone of stereo and motion 2-frame analysis is the solution of the correspondence problem, that is, determining which parts of two images, say $I_1$ and $I_2$, are projections of the same scene elements. The solution of this problem is represented as a correspondence or disparity map, which relates pixels in $I_1$ to pixels in $I_2$. In order to determine the pixel in $I_2$ corresponding to a pixel in $I_1$, a search maximizing a similarity criterion must be performed over some region of $I_2$ [1]. Such regions reduce to lines (the epipolar lines) if the epipolar geometry, codified in the fundamental matrix, is known [3, 18]. Therefore, if the fundamental matrix is known, the correspondence problem is reduced from a 2D search to a 1D search problem.

 Needless to say, the search for corresponding points can be simplified if the two images can be warped [17] in such a way that any two corresponding points lie on the same scanline in the two images (in other words, the epipolar lines are parallel to the horizontal image axes and are the same in the two images). This process is called rectification [3, 7, 4]. The rectified images can be regarded as acquired by cameras rotated with respect to the original ones. Note that most of the stereo algorithms presented in the computer vision literature assume rectified images.

The concept of rectification has been known for long to photogrammetrists [15]. Photogrammetric approaches, like most of the computer vision ones [1, 2, 8, 4], assume known projection matrices, that is, calibrated cameras. Only recently algorithms not assuming full calibration of the stereo rig, but only knowledge of the epipolar geometry, have been presented for generic images [7, 10, 12, 13]. In particular, [7] gives a theoretical presentation of projective rectification. All these algorithms either rely on explicit estimates of the fundamental matrix, which can be determined in several ways [9, 18], or make assumptions on the stereo geometry; e.g., [10] assumes that the cameras rotate only around a particular axis.

This paper presents a novel algorithm for projective rectification which does not require an explicit estimate of the epipolar geometry, and specifically of the fundamental matrix. Instead of first finding the fundamental matrix and then computing two homographies mapping the epipoles to infinity, as done by recent work on projective rectification [7], we exploit the fact that the fundamental matrix of a pair of rectified images has a particular, known form to set up a minimization yielding the rectifying homographies directly from image correspondences. The two transformations computed by our algorithm can then be used to estimate the epipolar geometry between the two original images. Our method builds on top of results presented by Hartley in [7].

In Section 2 we summarize the relevant properties of the epipolar geometry of rectified images. In Section 3 the class of rectifying homographies is characterized. Section 4 presents our method for rectification. Experimental results are shown in Section 5. The last section is dedicated to final remarks and a brief discussion.
1.1. Notation

It is convenient to cast our presentation from the point of view projective geometry [14], whereby the image planes are considered as projective planes, and image points are represented as 3D column vectors. A rectifying transformation is a linear one to one transformation of the projective plane, called homography, represented by 3×3 non-singular matrix.

We indicate column vectors by bold lower-case letters, such as $\mathbf{a}$. Row vectors are denoted by transposed column vectors, e.g., $\mathbf{a}^t$. Matrices are denoted by bold upper-case letters, e.g., $\mathbf{M}$. Scalars are denoted by italic letters.

Given a vector $\mathbf{a} = [a, b, c]^t$ we denote by $[\mathbf{a}]$ the rank-2 skew-symmetric matrix used in place of the vector product by $\mathbf{a}$.

$$
\begin{bmatrix}
0 & -c & b \\
 c & 0 & -a \\
-b & a & 0
\end{bmatrix}
$$

2. The fundamental matrix of two rectified images

It is well known that any two corresponding image points $(\mathbf{p}_1, \mathbf{p}_2)$ are related via the fundamental matrix $\mathbf{F}$ by the algebraic relation

$$\mathbf{p}_1^t \mathbf{F} \mathbf{p}_2 = 0. \tag{1}$$

$\mathbf{F}$ is defined up to a scale factor and usually computed from 8 or more point correspondences using linear [6] or more accurate nonlinear methods [18]. If we interpret $\mathbf{F} \mathbf{p}_2$ as a line in the projective plane, equation (1) tells us that $\mathbf{p}_1^t$ is constrained to lie on $\mathbf{F} \mathbf{p}_2$, the epipolar line of $\mathbf{p}_2$ in $\mathbf{I}_1$ (the epipolar line of $\mathbf{p}_1$ in $\mathbf{I}_2$ is given by $\mathbf{F}^t \mathbf{p}_1$). The null spaces of $\mathbf{F}$ and $\mathbf{F}^t$ define the epipoles in the projective plane, $\mathbf{e}_1$ and $\mathbf{e}_2$, geometrically the projections on the two image planes of the centers of projection of the cameras.

It can be proven [7] that $\mathbf{F}$ can be factorized as

$$\mathbf{F} = [\mathbf{e}_1]^t \mathbf{M}, \tag{2}$$

where $\mathbf{M}$ is a three-parameter family of non-singular matrices. Moreover, $\mathbf{M}$ is such that $\mathbf{M} \mathbf{e}_2 = \mathbf{e}_1$ and for each point $\mathbf{p}_2$ the point $\mathbf{M} \mathbf{p}_2$ must lie on $\mathbf{F} \mathbf{p}_2$.

If the images are rectified, the epipoles are

$$\mathbf{e}_1 = \mathbf{e}_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}. \tag{3}$$

If $\mathbf{p}_2 = [X, Y, Z]^t$ we have that $\mathbf{F} \mathbf{p}_2 = [0, 1, -Y]^t$, and therefore $\mathbf{M} \mathbf{p}_2 = [X, Y, 1]^t$. It is evident that in such case $\mathbf{M}$ can be chosen as the identity. Consequently, for a pair of rectified images we have $\mathbf{F} = [\mathbf{e}_1]^t$, that is,

$$\mathbf{F} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}. \tag{4}$$

3. The class of rectifying homographies

Let us call $\mathbf{H}_1$ and $\mathbf{H}_2$ the two homographies rectifying $\mathbf{I}_1$ and $\mathbf{I}_2$, respectively. The pair of rectifying homographies is not unique; in general, some rectifying homographies are unacceptable, for instance as they cause too large projective distortions (e.g., some points can be mapped to infinity). It is therefore necessary to constrain any algorithm computing $\mathbf{H}_1$ and $\mathbf{H}_2$ in such a way that the rectified images do not look too different from the original ones.

Hartley [7] uses the conditions that one of the two homographies, say $\mathbf{H}_2$, should be close to a rigid transformation in the neighborhood of a selected point $\mathbf{p}_0$. He shows that, in this case, $\mathbf{H}_2$ can be conveniently written as

$$\mathbf{H}_2 = \mathbf{KRT}, \tag{5}$$

where $\mathbf{T}$ is a translation taking $\mathbf{p}_0$ to the origin, $\mathbf{R}$ is a rotation mapping the epipoles (which is assumed known) to a point $[f, 0, 1]^t$ on the $x$ axis, and $\mathbf{K}$ is a transformation mapping $[f, 0, 1]^t$ to $[1, 0, 0]^t$ and acting as the identity map close to the origin. In particular, $\mathbf{K}$ is given by the matrix

$$\mathbf{K} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -f & 0 & 1 \end{bmatrix}. \tag{6}$$

In this way, the homography $\mathbf{H}_2$ depends only on two parameters: $f$ and the angle of rotation, $\theta$.

It is proven in [7] that if $\mathbf{H}_2$ is given and the fundamental matrix is factorized as $[\mathbf{e}_1]^t \mathbf{M}$ then $\mathbf{H}_1$ is one of the form $(\mathbf{1} + \mathbf{H}_2 \mathbf{e}_1 \mathbf{a}^t) \mathbf{H}_2 \mathbf{M}$ for some vector $\mathbf{a}$, and where $\mathbf{M}$ is one of the matrices obtained by the factorisation (2).

4. Description of the method

Assume that the fundamental matrix of the rectified pair, $\mathbf{F}$, is given by equation (4). Then, for each pair of corresponding points $(\mathbf{p}_1, \mathbf{p}_2)$ the following relation holds

$$(\mathbf{H}_2 \mathbf{p}_2)^t \mathbf{F} \mathbf{H}_1 \mathbf{p}_1 = 0. \tag{7}$$

If $\mathbf{H}_2$ is factorized as per equation (5), the number of unknowns is 10: 2 for $\mathbf{H}_2$ and 8 for $\mathbf{H}_1$. The idea of our method is simply to identify two homographies $\mathbf{H}_1$ and $\mathbf{H}_2$,
satisfying equation (7). To do that, given $N$ point correspondences $(p_{i1}, p_{2i})$, $i = 1, \ldots, N$, between the two images, we seek the two homographies $H_1$ and $H_2$ minimizing the cost function

$$J(H_1, H_2) = \sum_{i=1}^{N} [(H_2 p_{2i}^{tr})^t F H_1 p_{1i}]^2. \quad (8)$$

Since equation (7) involves only the second and third rows of the two homographies (because the first row of $F$ is the null vector). We could actually choose any three real numbers such that $H_1$ is full rank matrix. However rectification is performed in order to simplify a stereo matching procedure, and a not careful choice for the first row of $H_1$ can be cause the two resampled images to look too different and therefore cause the failure of the matching. Then it is necessary to introduce some constraint in order to determine the first row of $H_1$. We decided to determine the missing parameters of $H_1$, by minimizing the sum of squared distances

$$\sum_{i=1}^{N} |(H_1 p_{1i})_x - (H_2 p_{2i})_x|^2 \quad (9)$$

where $(\cdot)_x$ indicates $x$ coordinates. In other words we want that the $x$ coordinates of correspondent points are not too different. This choice proved to give good results for standard stereo-rig setups. The minimization of (9) is a simple, linear least-squares problem, which can be solved using known linear techniques [5].

The knowledge of the two rectifying homographies $H_1$ and $H_2$ gives the epipoles $e_1$ and $e_2$ of the two original images, since it is $H_2^{-1}[1, 0, 0]^t = e_1$, $i = 1, 2$. Moreover, $H_1$ and $H_2$ are such that $H_1^t F H_2$ is the fundamental matrix between the two original images. Therefore the method returns also a complete estimation of the epipolar geometry between the two original images, which is implied by the knowledge of 8 or more correspondences.

The minimization of (8) is a non-linear least squares minimization problem. To perform this minimization we use the Levenberg-Marquardt algorithm [11]. A good initial estimate is important to guarantee the convergence to a right solution. To overcome this problem we perform a hierarchical rectification, which first rectifies subsampled images, where the difference between coordinates is smaller. The estimated homographies for the subsampled level are then used to initialize the estimation for the finer levels, avoiding in this way local minima. A similar approach has been used for the case of image registration, see for example [16].

In the appendix a summary of the algorithm in a MATLAB-like language is given. The C implementation will be made available from the author’s web page.

5. Experimental results

In order to estimate the accuracy of the rectification process we measure the mean of the error index $r_i = |(H_1 p_{1i})_y - (H_2 p_{2i})_y|$, i.e. the distance along the vertical axis of corresponding points after rectification.

First of all the algorithm has been tried on a set of synthetic data. The synthetic data are a set of 100 3D points, randomly drawn from a cube of size $20 \times 20 \times 20$, placed 10 units in front of the left camera, imaged by two syntethic cameras of unit focal length, with the matrix of intrinsic parameters given in Figure 1. The point correspondences $(p_{1i}, p_{2i})$ are perturbed by adding increasing amounts of Gaussian noise to both the integer coordinates of $p_{2i}$. The fixed point $p_0$ (see Section 3) has been chosen as the barycentrum of the point set $\{p_{2i}\}$.

For each run of the experiment only 50 points have been used to compute the homographies, while the error has been computed using all the 100 points. In order to give an accurate estimation of the effect of the rectification on the images, the error has been computed using the true image points, that is the integer coordinates not corrupted by the additive noise.

The results in Figure 2 are relative to two different displacements of the cameras: the first one is only a translational displacements, the second one is rototranslational. The error measure shown in the graphs are averaged over several trials. They show the algorithm perform quite well (an average of 0.5 pixels error) even in presence of significant noise.

The experiments on real data shown in this paper are relative to 8 stereo pairs available from the INRIA-Synthim WWW site (see acknowledgements section). For all the stereo pairs a few pont correspondences have been manually selected, so that thei accuracy was limited. No errors have been computed for the real data, since no ground truth was available. The results on the real images show that the algorithm performs quite well, even with not accurate point correspondences.

The resampled images look quite similar; in two cases, Figure 5 and Figure 6, the resampled images look different, but this is due to the large difference between the viewpoints.
6. Conclusions

We have presented novel method to perform uncalibrated rectification of a pair of stereo images. The novelty of the algorithm lies in the fact that no previous estimation of the epipolar geometry between the original images is necessary.

The method relies on a non-linear minimization plus a linear least square estimation. The algorithm is practically as computationally expensive as methods requiring knowledge of the epipolar geometry and nonlinear estimates of the fundamental matrix. Our implementation will be publicly available from the author’s web site.

The algorithm has been tested on both synthetic and real data and yielded accurate results, even if the point correspondences were not very accurate. The algorithm returns also an estimate of the fundamental matrix between the two original images; current work is devoted to assess the accuracy of such estimates and to compare them with the results obtained by other algorithms.

Acknowledgements

The real stereo pairs shown in this paper are available from INRIA-Syntim under Copyright (http://www-syntim.inria.fr/syntim/analyse/paires-eng.html).

A. Summary of the algorithm

function [H1,H2] = rectify(P1,P2, scalemax)
% rectify computes the two rectifying % homographies H1 and H2
% P1 and P2 are two matrices storing the % correspondent points
% scalemax is the starting scale factor for % the hierachical minimization
% parameters initialization
% H1 is set to the identity matrix
H1 = I;
% set the parameter f
f = 1;
%set the rotation angle
\theta = 0;

% set the scaling factor
scale = scalemax;
scalestep = 2;

while scale >= 1

% points coordinates scaling
p1 = P1/scale;
p2 = P2/scale;
% Levenberg-Marquardt performing minimization of (7)
%returning H1 (the last two rows), f and \theta
(H1, f, \theta) = LM(P1,P2,H1,f,\theta);
% scaling the transformations (hierarchical rectification)
if scale > 1 then

f = f*scalestep;
H1(1,3) = H1(1,3)*scalestep;

end
H1(2,3) = H1(2,3) * scalestep;
H1(3,1) = H1(3,1) / scalestep;
H1(3,2) = H1(3,2) / scalestep;
scale = scale / scalestep;

endif

endwhile

% compute the matrix H2 from f and \theta
% see equation (5)
H2 = transformation(f, \theta);

% estimate the first row of H1, see equation (9)
H1 = estimateH1(H1, H2, p1, p2);

References

Fig. 5. Top row: the original Aout stereo pair. 11 point correspondences have been manually selected for this experiment. The two images in the bottom row are the rectified ones.

Fig. 6. Top row: the original Rubik stereo pair. 13 point correspondences have been manually selected for this experiment. The two images in the bottom row are the rectified ones.

Fig. 7. Top row: the original Tot stereo pair. 14 point correspondences have been manually selected for this experiment. The two images in the bottom row are the rectified ones.

Fig. 8. Top row: the original Angle stereo pair. 14 point correspondences have been manually selected for this experiment. The two images in the bottom row are the rectified ones.

Fig. 9. Top row: the original BalMouss stereo pair. 16 point correspondences have been manually selected for this experiment. The two images in the bottom row are the rectified ones.

Fig. 10. Top row: the original BatInria stereo pair. 19 point correspondences have been manually selected for this experiment. The two images in the bottom row are the rectified ones.