Abstract—Energy efficiency and sensing accuracy have both been attractive research fields in sensor networks. Achieving both objectives is possible in a compromise model. In this paper we formulate one such problem and use a game theoretic approach for its solution. The interaction between sensor nodes is modeled as a cooperative bargaining game, where individual sensors cooperate for achieving the application sensing requirements while minimizing and balancing the energy consumption. We use Kalai-Smorodinsky Bargaining Solution to find a distribution rule that optimizes the trade-off in the compromise problem. Based on the distribution rule, we propose a lightweight distributed algorithm in order to schedule nodes for performing the sensing task. Simulation shows a superiority in terms of scalability over a similar algorithm.

Index Terms—Coverage, energy-efficiency, cooperative game, bargaining solution, bankruptcy problem

I. INTRODUCTION

Energy conservation in wireless sensor networks to extend overall network lifetime has been of paramount importance in sensor network design [2]. On the other hand, the main goal of a sensor network deployment is to fulfill the specific application requirements, e.g. to satisfy a level of sensing quality at all times for the monitoring environment. In this context, the efficient operation of a sensor network can be defined as satisfying the application requirements during a maximally prolonged lifetime. We identify the compromise problem of energy-efficient coverage in WSNs, in which the two objectives of maintaining qualitative sensing and energy conservation are contradictory.

The common approach to address this problem in literature has been to put as many sensors as possible into sleep mode while still achieving a certain degree of coverage. The works in [4, 1] try to split sensors of the network into maximum possible sets each of which is capable to cover the sensing area, and schedule the sets to be activated one after the other to extend network lifetime. [11] minimizes the overlapping area between active sensors to maximize the number of sleeping ones, while [9] employs an eligibility rule to achieve the same goal. Achievements of these works are however limited by constraints of sensor devices, and the inherent binary nature of the well-known sensing disk model, where successful sensing of an object by a sensor is quantified simply as 0 or 1 depending on whether the object lies within the given sensing disk radius.

Game Theory concepts have been applied to address coverage problem in WSNs. In [10, 5], authors model the operation of sensor network as non-cooperative games of sensor nodes, where nodes behave selfishly to fulfill their local interests. Our cooperative approach is driven by the observation that sensors are identically deployed in order to perform a certain task, i.e. the sensing task, which is a key task of the sensor network itself. Thus giving rise to cooperative behaviour amongst individual sensors.

Our main contributions in this paper are: (i) a novel approach using game theoretic bargaining solution for sensor networks; (ii) a sensing model that is more realistic than the conventional sensing disk model, and allows higher flexibility in optimizing networks’ sensing activity; and (iii) a lightweight, localized & distributed algorithm that schedules network sensing activity to meet the energy-efficiency/sensing accuracy trade-off.

II. MODEL AND ASSUMPTION

Consider a deployed-with-coverage-redundancy network of \( N \) stationary sensors monitoring \( M \) stationary objects in a geographic area. Let \( g^{QS} \) be the application specified threshold value. Our objectives are i) qualitative sensing, i.e. keeping the sensing gain for all objects at a value of at least \( g^{QS} \) at all times; and ii) energy efficiency, i.e. minimizing and fairly distributing the energy consumption throughout all nodes in the network.

We assume there is a fixed, small time period \( \zeta \) (round), during which the network is considered static (i.e., no sensor state transition). Let \( a_i(t) \) be the schedule mask function of sensor \( i \) at round \( t \), which equals to 1 if \( i \) is scheduled active during \( t \), or 0 otherwise. Function \( a_i(t) \) is drawn from the schedule profile \( a(t) = \{a_i(t)\} \), which belongs to the schedule space \( A = \{a(0), a(1), \ldots, a(T)\} \), where \( T \times \zeta \) is the expected operational life time of the network (detail in II.C).

A. Sensing Model

We consider the case where objects of interest emit some form of signal, such as sound or light or any wave, whose intensity attenuates with distance. The sensors can detect the objects or their status to the extent that they receive their signal. We model the sensing function of an individual sensor \( i \) for an object \( o \), at round \( t \) as

\[
g_i(o,t) = \begin{cases} 
    a_i(t) \times \frac{\gamma(o)}{\pi^{d(o)}} & \text{if } \frac{\gamma(o)}{\pi^{d(o)}} \geq g^E \\
    0 & \text{otherwise}
\end{cases}
\]

(1)

where \( \gamma(o) \) is the original intensity of the source object, \( \alpha \) is the path-loss factor, \( d(i, o) \) is the Euclidean distance between \( o \) and \( i \), and \( g^E \) is an application-specified effective threshold. \( g_i(o, t) \) represents the gain that sensor \( i \) obtains from object \( o \), at round \( t \). In case \( g_i(o, t) \geq g^E \), \( i \) is said to obtain effective gain from \( o \). We call sensor \( i \) an effective sensor of the object \( o \), and \( o \) is an effective object to \( i \). The effective threshold helps sensors to omit weak signals (emitted from far/lowed-intensity objects), and process only signals whose intensity is greater than \( g^E \). Since sensor hardware varies with applications, it is realistic to let applications determine the value of \( g^E \).
Our sensing function \( g_i(o) \) receives values from the continuous domain \([0, \gamma_i o / d_i(i, o)]\) (equation (1)). The conventional binary sensing disk model (i.e., \( g_i(o) \in \{0, 1\} \)) is therefore a special case of our sensing model.

We define the \textit{aggregated} effective gain, that a sensor \( i \) obtains from the set of \( M \) target objects at round \( t \) as \( g_i(t) = \sum_{j=1}^{M} g_i(o_j, t) \).

### B. Energy Consumption Abstraction

Sensor devices vary with different applications and hardware [2]. To cope with this variation at a high level, we assume that it costs \( e_i \) amount of energy for a sensor \( i \) to perform its sensing task and its required communication activity during a round \( t \). \( e_i \) is an abstract hardware-dependent function that can be easily substituted by a detailed description in any particular sensing application.

The energy consumption of sensor \( i \) during \( t \) therefore is \( a_i(t) \times e_i \). Let \( l_i(t-1) \) denotes sensor \( i \)'s residual energy after round \( t-1 \). We have \( l_i(t) = l_i(t-1) - a_i(t) \times e_i \). The network residual energy after round \( t \) therefore is \( L(t) = \sum_{i=1}^{N} l_i(t) \), and its mean is \( \mu(t) = \frac{L(t)}{N} \). The deviation from the mean is \( \delta(t) = \frac{1}{N} \sum_{i=1}^{N} \| l_i(t) - \mu(t) \| \).

### C. Objective Elaboration

Our first objective involves ensuring that every object within the sensing area is monitored by the network with sensing gain no less than \( g_i^{QS} \). We assume an additive function for combining the sensing gain of different sensors over the same object. More sophisticated functions can also be well applied (which is one of our future research directions), however we stay simple here to focus on the problem at high level.

Consider a round \( t \) and a target object \( o \) within the sensing area. The gain that the entire network gets from \( o \) is \( G(o, t) = \sum_{i=1}^{N} g_i(o, t) = \sum_{k=1}^{K} g_k(o, t) \), where \( K \leq N \) is number of effective sensors of object \( o \). In order to formally consider the qualitative sensing objective, we associate with every object \( o \) a \textit{satisfaction function} \( \phi(o, t) \) that gets the value of 1 if \( o \) is monitored by the network with sensing gain no less than \( g_i^{QS} \), or gets to 0 otherwise. Our qualitative sensing objective can now be expressed as

\[
\prod_{j=1}^{M} \phi(o, t) = 1 \quad (2)
\]

Equation (2) can also be seen as the constraint to our second objective of energy-efficiency. Here the aim is to extend the time duration \( T \times \zeta \) during which (2) holds true. We call such time duration the \textit{operational lifetime} of the network. To maximize \( T \times \zeta \), we maximize the possible total number of rounds \( T \), while subjecting to condition (2). This equals to minimizing the network energy consumption \( E(t) = L(t) - L(t-1) \) during each round \( t \), while subjecting to (2). From definition of \( L(t) \) we have \( E(t) = \sum_{i=1}^{N} a_i(t) + e_i \), i.e., a minimized schedule profile \( a(t) \) is needed to minimize network energy consumption at each round \( t \), therefore maximize total number of rounds \( T \).

On the other hand, scheduling the sensors to \textit{wake up} and \textit{sleep} in a way that they equally share the sensing load will further increase the overall network lifetime by keeping each individual node alive (preventing full battery depletions, which negatively affects (2)) as long as possible. Such balanced distribution of the sensing load across the network requires minimizing the value of \( \delta(t) \).

### III. COOPERATIVE BEHAVIOR OF SENSORS IN WSNs

In proposed game, sensors are assumed to cooperate on sensing task in order to fulfill the application requirement, and their utility function is characterized by their effective gain and energy condition. Our assumption is justified by the fact that all the sensors are deployed to carry out sensing activity i.e., to obtain sensing gain from objects. The gain of sensor is monotonically increasing in sensing activity. This nature of sensors fits to the cooperative game concept (coalition formation), we assume that coalition is \textit{TU} (transferable utility).

Consider an arbitrary object within the geographic area. We know that a sensor alone is only able to cover the object if its gain is more than an application-specified threshold. The sensors’ cooperation is not bounded by any condition, as joining coalition always increase sensors’ gain. Thus we always have a coalition of the object’s effective sensors in its vicinity to do the sensing job. This coalition is sensing redundant by default due to the deployment assumption of the sensor network.

The second objective of energy efficiency can also be obtained within the design of our cooperative game. Take a coalition of sensors obtaining redundant gain above the application threshold from an object. To save energy, this sensing redundancy should be minimized by putting some sensors in sleep mode, and letting only those whose gains are together summed up satisfying the qualitative sensing condition in (2) stay awake. If we consider the application threshold as a kind of sensing good, then we have the problem of fair assignment of sensing good among sensors, to decide who stay awake (and thus get more gain). To make the “sensing good” notation clear, consider a coalition \( S \subseteq N \) of sensors obtaining redundant gain, let the smallest coalition that satisfies the qualitative sensing objective be represented by \( \tilde{S} \subseteq S \), then sensing good is defined as gain obtained by \( \tilde{S} \).

Now that the problem is reduced to fair distribution of sensing good, well-known game theoretic approach “bargaining solution” fits best to find such distribution among members of coalition.

### IV. THE EFFICIENT OPERATION PROBLEM AS A BARGAINING GAME

Background on bankruptcy problem and bargaining solutions can be referred in our earlier work [7]. Here, we denote Bankruptcy problem as a pair \((B, Q)\), where \( B \) represents the amount of estate, and \( Q \) is the set of investments of \( n \) creditors \( Q = \{q_1, ..., q_n\} \), such that \( 0 \leq B \leq \sum_{i=1}^{n} q_i \).

Consider an allocation \( X_j(n) = (x_{1j}, ..., x_{nj}) \subset X(n) \) of the estate among \( n \) creditors, where \( X(n) \) is the set of all possible allocations. Then \( X_j(n) \) should satisfy

\[
0 \leq x_{ij} \leq q_i \quad \text{and} \quad \sum_{j=1}^{n} x_{ij} = B. \quad (3)
\]

A question here is how do creditors come to a fair agreement of such allocation. In cooperative game theory [8], this process of players negotiating towards an agreement of mutual benefits is modeled as a bargaining game.

In proposed game, sensors cooperatively play to achieve their goal of increasing their sensing activity to satisfy the application requirement. Recall that for an object that is being monitored, there is a coalition of effective sensors, yielding redundant gain beyond the application threshold. To conserve network energy, such redundancy should be minimized, which corresponds to putting some of the sensors of the coalition into sleep. However none of the sensors want to go to sleep as doing so means to reduce their sensing activity (i.e., sensing gain). Instead, they try
to prove themselves suitable for the sensing job by competing for the greater portion out of the sensing good. This translates the problem of efficient operation of WSNs into the problem of allocation of sensing good among sensors of a coalition.

We model this problem as a Bankruptcy problem, in which sensors are analogous to creditors and sensing good is analogous to the estate to be distributed. We use the well-suited game theoretic Bargaining approach to describe the behavior of sensors as bargaining over the amount of sensing good to seek for a suitable allocation that corresponds to their investments. An allocation to a sensor \( i \) is regarded as the utility function of the sensor. This utility function is derived from the real capacity of the sensor in terms of its aggregated effective gain \( q_i \) and its residual energy \( l_i \). Based on the real capacity, the greater the value (as a function of capacity) a sensor can offer, the bigger utility it will get. This is analogous to the creditors’ situation, where their demand is derived by their investment. We use this analogy to regard a sensor’s function of capacity as its investment in the game.

To formally detail the problem with necessary notations, we regard the application threshold \( g^{QS} \) as the sensing good to be distributed among \( u \) effective sensors of an object (i.e., a coalition). The generic investment of a sensor \( i \) is: \( q_i = f_{inv}(g_i, l_i) \). The Bargaining problem is thus given by:

\[
(S(g^{QS}, X(u)), d) = \{ x_i \mid d \leq x_i \leq q_i, \sum_{i=1}^{u} x_i = g^{QS} \} \tag{4}
\]

where \( S \subset \mathbb{R}^T \) is the convex set of all possible allocations to \( u \) sensors. The disagreement point is set to \( d = 0 \) due to our assumption of full cooperation among effective sensors of each object.

The Bargaining game of (4) is driven by the investment of sensors, as investment is the key to differentiate them. Owing to our localized and distributed approach, the cooperative game is an incomplete information game, where sensors make decisions based only on their aggregated effective gain and residual energy. The private information of sensors is not common knowledge but the game rules are. So the dominating policy is based on the sensing activity coupon to other sensors with higher residual energy, even though their aggregated effective gain may be lesser than its. In other word, the sensor is self-care and cooperative to its playmates.

V. SOLUTION TO THE BARGAINING GAME

A. Kalai-Smorodinsky Bargaining Solution - KSBS

According to Kalai & Smorodinsky [6], the feasibility set can be extended such that a player may get more benefit without compromising the benefit of others. Given the KSBS axioms, in our case a sensor’s utility is increased in the direction of its investment. The outcome of the game is therefore supposed to lie in all players’ satisfaction.

Now, we have our bargaining game as given in equation (4). Let the ideal point (desired solution) be represented by \( Z(S(g^{QS}, X(u)), 0) \), whereas the ideal point is consequence of the selection of sensors’ investments. Since sensors are role-flat and independent, we carry the computation of allocation for only the scenario of 2 sensors for the sake of simplicity. The final solution should be scalable to any n-sensor scenario (\( n \geq 2 \)).

The ideal point in the 2-sensor scenario is where both the sensors have made their maximum possible investment, that is \( Z(S(g^{QS}, X(u)), 0) = \psi(f_{max}^{inv}(i), f_{max}^{inv}(j)) \). The ideal solution therefore lies at this ideal point. Now let \( X_{i,j}^* \) be the allocation to the two sensor such that \( \sum_{i,j \in N} X_{i,j}^* = g^{QS} \). For sensor \( i \), rather than \( X_i^* = \psi(f_{max}^{inv}(i), f_{max}^{inv}(j)) \) as it would violate the Bankruptcy condition (3), we have \( X_i^* < \psi(f_{max}^{inv}(i), f_{max}^{inv}(j)) \) owing to the convex feasibility set given in (4). To find \( X_{i,j}^* \), we join the 0-disagreement point to the ideal point. The intersecting point on this line segment is the desired optimal solution, which is proportional to the investment made by each sensor.

Thus, the optimal distribution rule for the formulated bargaining problem in (4) is the proportional distribution with respect to the investment that a sensor has made in (5). We can then calculate the allocated portion of sensing good for a sensor \( i \) as \( a_i = \frac{g^{QS} \times \sum_{j \in N} \frac{f_{max}^{inv}(j)}{\sum_{j \in N} f_{max}^{inv}(j)}}{\sum_{i \in N} \frac{f_{max}^{inv}(i)}{\sum_{i \in N} f_{max}^{inv}(i)}} \), which also is the distribution rule of the bargaining game.

B. Proposed Distributed Algorithm

In our algorithm, each sensor maintains a two-entry gain table consisting of its effective objects and the corresponding effective gains. Before the first game takes place, the table must be locally advertised once for coalition formation. Each object \( o \) will then be associated with a coalition of \( K \) effective sensors monitoring it.

At the beginning of a round \( t \), every sensor of a coalition locally calculates its investment according to (5). Investments are then exchanged among sensors so that each sensor \( i \) can play the game by calculating, following the distribution rule, its own allocated portion of sensing good and other coalition members’. Based on this values, the sensor sorts the coalition’s members into an ascending list, and use the list for making state transition decision. It first initiates a variable \( K \) and top-down visits the list’s entries, while summing up the effective gains of coalition members, until the entry where \( \phi(o, t) = 1 \). The sensor marks itself active if its position in the list falls anywhere between \([1, K]\), or inactive otherwise. Repeating this process for all its \( m \) ascending lists (\( m \leq M \) is number of effective objects of the sensor), sensor \( i \) decides to stay awake if it is marked active in every repetition i.e., its schedule mask function \( a_i(t) = 1 \). Otherwise it decides to sleep for the rest of the round, i.e., \( a_i(t) = 0 \). That way, the schedule profile \( a(t) \) for round \( t \) is built distributively by all sensors of the network. Since the distribution rule is optimal, \( a(t) \) is the optimal schedule profile.

If sensor \( i \) depletes/fails at a round \( t \), its neighbors will notice no investment is advertised in next round. They will internally
remove the entry corresponding to sensor \( i \) in their gain table to update coalition membership information.

Note that the communication overhead is caused only by the exchange of investments within neighborhood. To even further reduce that, we propose a triggering mechanism to lower the frequency of making decision, rather than playing the game for each sensing activity (round). Each sensor maintains an energy-critical threshold \( t_{th_e} \), which is updated right after a game has been played. Active sensors check their energy condition for every round if it falls below \( t_{th_e} \), and send out a trigger message to request for playing a new game. Otherwise, they simply do the sensing job and no game is played (i.e., no communication overhead during the round). We experimentally choose \( t_{th_e} = \frac{\text{ln}(m)}{\text{ln}(L)} \) to drive active sensors to trigger the game more frequently as their energy condition get lower, leaving chances for other sensors with better energy status to take over the sensing job. This way the sensing load is actually spread over the network more fairly, which equals to minimizing the value of \( \delta \) (in subsection II.B).

VI. NUMERICAL ANALYSIS

In this section we evaluate and compare proposed algorithm’s performance against the results in [3]. The work in [3] addresses a similar target coverage problem to ours. A set of known-location objects is monitored by a sensor network with a certain degree of coverage (1-coverage in this case). To extend the network lifetime, the authors proposed a scheduling method that organizes sensors into maximal number of set covers that are activated successively. The scheduling activity is modeled as the maximum set covers problem. The problem is proved to be NP-complete and is solved heuristically using centralized, linear programming-based approach, which seems to require excessive computation that may not fit to sensor networks. Proof of concept of our approach is verified through simulation which demonstrates superior performance in terms of scalability. The negligible gap between GTA curves of 15 and 25 objects confirms this promising result.

VII. CONCLUSION

This paper models the compromise between energy conservation and sensing coverage in a novel way - the cooperative game theoretic approach. We model the problem as a bankruptcy problem and associate it in a bargaining game context. The game is solved using the well-known KSBS to find an appropriate distribution rule, based on which we propose a distributed scheduling algorithm to guide sensors in doing their sensing task. The proof of concept is presented by simulating our algorithm and evaluating the performance in different aspects: load balancing and network lifetime improvement. We also compare our approach with another non-game-theoretic approach, demonstrating superior performance in terms of scalability and performance.

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