Delegation vs. Control of Component Procurement*

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A manufacturer must choose to delegate component procurement to its tier-1 supplier, or to control component procurement by contracting with both the tier-1 supplier and the tier-2 component supplier. Both suppliers have private cost information and the manufacturer has an alternative source of supply with cost known to all parties. This paper proves that if the firms may use arbitrarily complex contracts, then the manufacturer has the same expected profit with delegation as with control of component procurement. If, however, the firms use simple price-only contracts, then the manufacturer may achieve strictly greater expected profit with either delegation or control, depending upon the price of the alternative source, the selling price of the end product and, most importantly, what the manufacturer knows about her suppliers’ costs. A numerical study shows that over a wide range of conditions, if the manufacturer chooses delegation versus control correctly, then she achieves nearly as much expected profit with price-only contracts as with the complex optimal contracts. However, the study identifies conditions under which price-only contracts perform poorly, and also shows that the loss may be extremely high when the manufacturer chooses delegation versus control incorrectly.

Key words: contract design, asymmetric information, component procurement, multi-tier supply chain, delegation, control, decentralization

1. Introduction

Should a manufacturer contract with component (tier-2) suppliers, or delegate that responsibility to its direct (tier-1) supplier? Manufacturers, retailers, and service providers have wrestled repeatedly with that question and, even within the same industry or over time, arrived at different answers. In the electronics industry, Sun Microsystems controls component procurement and even purchases many subcomponents itself (Chin 2003). In contrast, Cisco tends to delegate component procurement to its contract manufacturers (Nagarajan and Bassok 2004). Between these

* This research was funded in part by NSF grants #NSF/CAREER-0547021 and #NSF/PECASE-0239840.
extremes, HP delegates procurement of commodity components to its contract manufacturers but controls the purchase of strategic components (Carbone 2004). Historically, GM and other American automobile manufacturers relied on large purchasing departments to contract with thousands of component suppliers (Carr and Truesdale 1992) but, over the past two decades, they have increasingly delegated responsibility to tier-1 suppliers for component procurement and assembly (Marsh 1996, McIvor, Humphreys, and McAleer 1998). Historically and now, in comparison with American automobile manufacturers, Japanese automobile manufacturers delegate greater responsibility for component procurement to select tier-1 suppliers (McIvor, Humphreys, and McAleer 1998). In the past, retailer Wal-Mart simply purchased consumer goods from tier-1 suppliers. However, to obtain innovative, environmentally-friendly products, Wal-Mart is beginning to work with lower-tier suppliers (e.g., organic cotton farmers) (Denend and Plambeck 2007). Similarly, to acquire innovative, environmentally-friendly trucks, overnight-delivery-service provider FedEx has started to work with a hybrid transmission supplier in addition to its truck assembler (Hoyt and Plambeck 2005). Increasingly, airline carriers like Delta, Air China, and Emirates contract directly with aircraft engine manufacturers like GE and United Technologies in addition to aircraft assemblers like Boeing and Airbus (CFM 2007a, 2007b, CNN World Business 2003). For digital storage of medical images, healthcare service providers are shifting from contracting only with imaging hardware vendors, which in turn purchase the required storage hardware, to contracting directly with both imaging and storage hardware vendors (Daher 2006).

Many factors influence whether a firm should control or delegate component procurement. Delegation reduces managerial effort in procurement, which might be better directed toward other activities, and reduces the number of employees needed in the purchasing department. However, delegation might result in a degradation of quality and delivery performance, or higher cost of components (Amaral, Billington, and Tsay 2005). For example, a large manufacturer that employs multiple contract manufacturers may negotiate a low price with a component supplier by purchasing a large quantity, aggregated on behalf of all its contract manufacturers (Ellram and Billington
In this paper, we examine how asymmetric cost information and the use of a simple versus complex contract structure influences the optimal decision to delegate or control component procurement.

Multi-tier contracting under asymmetric information is addressed in the economics literature; Mookherjee (2006) provides a thorough review. The related economics literature on delegation takes as its starting point that, when the Revelation Principle holds, a manufacturer achieves weakly lower expected profit when she chooses to delegate responsibility for contracting with the tier-2 supplier to her tier-1 supplier, rather than contract with both suppliers. Researchers have identified the following conditions for the manufacturer to achieve the same expected profit with delegation: (1) Either the subcontracting cost incurred or the quantity produced by the tier-1 supplier (especially for substitute components) is observable by the manufacturer (Baron and Basenko 1992, 1994 and Melumad, Mookherjee, and Reichelstein (MMR) 1992, 1995). (2) The manufacturer contracts with the tier-1 supplier before the latter contracts with the tier-2 supplier (MMR 1995). (3) The tier-1 supplier is risk neutral (McAfee and McMillan 1995). Other researchers have proposed models in which the Revelation Principle fails and delegation may result in strictly greater expected profit for the manufacturer. In particular, these researchers have incorporated information processing costs (Mount and Reiter 1995, Radner 1992, and van Zandt 1997a, 1997b), cost of complex contracts (MMR 1992, 1997), and collusion among suppliers (Mookherjee and Tsumagari 2004). This paper is differentiated by its newsvendor formulation, incorporation of an alternative supply option, and consideration of simple price-only contracts. Moreover, as opposed to MMR (1995) who conclude that delegation is always preferable to control when complex contracts are costly, we find that either delegation or control may be preferable depending on what the manufacturer knows about the cost structure of her suppliers.

In the operations management literature, several papers address contracting with a tier-1 supplier under asymmetric cost information. Ha (2001), Corbett (2001), Corbett and de Groote (2000), Corbett, Zhou, and Tang (2004), and Lutze and Özer (2004) design optimal contracts which may be arbitrarily complex. Unfortunately, such contracts almost always include non-linear terms, which
are difficult to implement and enforce. Laffont and Martimort (2002) and Wilson (1993) explain how a menu of relatively simple (e.g., piece-wise linear) contracts may be near optimal. Similarly, Cachon and Zhang (2006) find that in a production-inventory system wherein the supplier’s capacity cost is unknown to the manufacturer, simple linear contracts are near optimal. In a numerical study of a 1-tier version of the model in this paper, Feng and Zhang (2006) find that price-only contracts are near optimal. In contrast, for the 2-tier supply chain, we find that price-only contracts may perform poorly, especially when the manufacturer makes an error in her choice to control or delegate component procurement.

Operations management researchers have only recently started to model asymmetric information in multi-tier supply chains. For example, in Chen, Deng, and Huang (2006) a manufacturer must decide how to allocate his capacity among multiple retailers, each of whom has private information about her end customers’ demand, or delegate responsibility for that allocation to a distributor, who also has some private demand information. In the work most similar to our own, Guo, Song, and Wang (2006) consider delegation versus control of component procurement for a price-setting manufacturer with a linear, deterministic demand function, and no information whatsoever about the component supplier’s cost. In a single-period model, the authors find that delegation reduces the manufacturer’s profit, as the tier-1 supplier obtains extra profit by lying about the component supplier’s cost. However, delegation may be profitable in a two-period model. In the first period, each supplier may reject a wholesale price to signal that his cost is high and obtain a higher wholesale price in the second period. Delegation increases the probability that both suppliers participate in the first period. In a single-period model, we show how the manufacturer’s optimal component procurement strategy (delegation versus control) depends upon the complexity of contracts used, the manufacturer’s knowledge of each supplier’s cost, the selling price for the end product, and the alternative source of supply.

The paper is organized as follows. We present our model in Section 2. We identify business-environment conditions where delegation (versus control) of component procurement is optimal, first under the idealistic assumption that the firms can use complex optimal contracts in Section 3
and then in Section 4 under the realistic assumption that the firms use simple price-only contracts. In Section 5, we explain when and why price-only contracts perform well, relative to the complex optimal contracts. We conclude the paper with managerial insights in Section 6. Proofs of all results are in the appendix. Throughout the paper, subscripts under expectation operators denote the random variable over which expectation is taken and we use the convention $(x)^+$ for $\max(x, 0)$. Comparison terms such as greater, smaller, increasing, and decreasing should be interpreted in the weak sense (allowing for equality).

2. Model Formulation

A manufacturer (M) has a linear, 2-tier supply chain. The tier-2 supplier (S2) incurs cost $c_2$ per unit to produce a component. The tier-1 supplier (S1) incurs cost $c_1$ per unit to transform the component into a final product and deliver it to M. Each supplier knows his own cost $c_i$, whereas the other two firms know only that $c_i$ has probability density function $f_i(c_i)$ with associated cumulative distribution function $F_i(c_i)$ and support $\Omega_i := [c_i, \bar{c}_i]$ where $0 \leq c_i < \bar{c}_i < \infty$, and that $c_1$ and $c_2$ are independent. This prior belief is common knowledge. We assume that $F_i(c_i)$ is log-concave, which is satisfied by many common distributions, including the uniform, normal, exponential, and triangular (Bagnoli and Bergstrom 2005). The log-concavity of $F_i(c_i)$ implies that $h_i(c_i) := \frac{F_i(c_i)}{f_i(c_i)}$ is increasing in $c_i$. The total cost, defined as

$$k_i(c_i) := c_i + h_i(c_i)$$

will play an important role in subsequent analysis.

M must take delivery before realizing demand for the product $D \geq 0$, which has probability density function $g(D) > 0$ and cumulative distribution function $G(D)^1$. The selling price is $p$ per unit and salvage value is zero. We assume $p > c_1 + c_2$ (otherwise production is not profitable).

We assume that M, S1, and S2 are risk neutral; each seeks to maximize his own expected profit. We take M’s perspective, and focus on the fundamental question: should M delegate or control

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1 Our model can be used for a price setting M with a deterministic demand. The only assumption we need is that the price of the product is decreasing in total quantity M sells. One example is the conjugate pair of “deterministic and linearly decreasing price function” and “stochastic uniform demand function.”
component procurement? If M chooses control, then she contracts with both S1 and S2 herself. If M chooses delegation, she contracts only with S1, and S1 contracts with S2. Throughout the paper, the superscript $c$ is mnemonic for control, and the superscript $d$ is mnemonic for delegation.

M has an alternative source from which she can purchase additional units of the product for $\overline{w}$ per unit before demand is realized. Let $Q$ denote the number of units procured from S1 and S2 and $Q_A$ the number of units procured from this alternative source. Given $Q$, M chooses $Q_A$ to maximize $pE\left[\min(D, Q + Q_A)\right] - \overline{w}Q_A$. By standard newsvendor analysis, the optimal $Q_A$ is equal to $(Q_n(\overline{w}) - Q)^+$, where $Q_n(w) := G^{-1}(1 - w/p)$. We assume that $\overline{w} \in (c_1 + c_2, p]$, meaning that S1 and S2 are more efficient than the alternative source with strictly positive probability.

3. Delegation vs. Control with Complex Optimal Contracts

In this section, we adopt the idealistic assumption that the firms can implement arbitrarily complex contracts, contingent on the costs reported by the supplier(s). The results in this section will serve as a benchmark for subsequent analysis. The sequence of events under control and delegation, which is similar to that in the related economics literature (see, e.g., MMR 1995), is given in Table 1.

With arbitrarily complex contracts, the Revelation Principle implies that by exerting control M can replicate any outcome that occurs under delegation, and therefore M has weakly greater expected profit with control than with delegation (Mookherjee 2006). The remaining question is: can M achieve the same expected profit with control and delegation?

**Control:** By the Revelation Principle, M can restrict attention to contracts under which both S1 and S2 report their costs truthfully (Laffont and Martimort 2002). Specifically, M can design a menu of contracts $\{Q(c_1, c_2), t_1(c_1, c_2), t_2(c_1, c_2)\}_{c_1, c_2 \in \Omega_1 \times \Omega_2}$, where $Q(c_1, c_2)$ is the quantity to be produced by both S1 and S2, $t_1(c_1, c_2)$ is the payment from M to S1 and $t_2(c_1, c_2)$ is the payment from M to S2, contingent on the costs $c_1$ and $c_2$ reported by S1 and S2, respectively. (An optimal contract has both S1 and S2 produce the same quantity because S1 requires one component from S2 to produce each unit of the final product). The optimal menu of contracts for M to offer to S1 and S2 is the solution to:
Table 1 The sequence of events under control and delegation.

<table>
<thead>
<tr>
<th>Control</th>
<th>Delegation</th>
</tr>
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<tbody>
<tr>
<td>1. M offers a menu of contracts to S1 and S2: ( {(Q(c_1,c_2), t_1(c_1,c_2), t_2(c_1,c_2)) } ) ( \times \Omega_1 \times \Omega_2 ).</td>
<td>(1.) M offers a menu of contracts to S1: ( {t_1(Q,c_1)} ).</td>
</tr>
<tr>
<td>2. Each supplier decides whether or not to accept a contract. If both suppliers accept:</td>
<td>(2.) S1 decides whether or not to accept a contract. If he accepts:</td>
</tr>
<tr>
<td>(i.) S1 and S2 simultaneously report their costs, ( \tilde{c}_1 ) and ( \tilde{c}_2 ), and effectively choose the combination ( {(\tilde{c}_1, \tilde{c}_2), t_1(\tilde{c}_1, \tilde{c}_2), t_2(\tilde{c}_1, \tilde{c}_2)} ).</td>
<td>(i.) S1 with cost ( c_1 ) sends his cost report ( \tilde{c}_1 ) to M. Note that, S1 does not commit to any quantity ( Q ), as he does not know the cost of S2 yet. Having reported his cost as ( \tilde{c}_1 ), S1 offers a menu of contracts to S2: ( {(Q_2(c_2</td>
</tr>
<tr>
<td>(ii.) S2 produces and delivers ( Q(\tilde{c}_1, \tilde{c}_2) ) units to S1 who assembles them and delivers the final product to M.</td>
<td>(ii.) S2 decides whether or not to accept a contract. If he accepts, he sends his cost report ( \tilde{c}_2 ) to S1 and produces and delivers ( Q_2(\tilde{c}_2) ) units. S1 then chooses his production quantity ( Q_1 ), and assembles ( Q = \min(Q_1, Q_2(\tilde{c}_2)) ) units and delivers the final product to M.</td>
</tr>
<tr>
<td>3. If both suppliers accept a contract, M pays ( t_1(\tilde{c}_1, \tilde{c}_2) ) to S1 and ( t_2(\tilde{c}_1, \tilde{c}_2) ) to S2, and she procures an additional ( (Q_n(\pi) - Q(\tilde{c}_1, \tilde{c}_2))^+ ) units from the alternative source. Otherwise, M procures ( Q_n(\pi) ) units from the alternative source.</td>
<td>(3.) If both S1 and S2 accept a contract, M pays ( t_1(Q, \tilde{c}_1) ) to S1 and S1 pays ( t_2(\tilde{c}_2) ) to S2. M procures an additional ( (Q_n(\pi) - Q)^+ ) units from the alternative source. Otherwise, M procures ( Q_n(\pi) ) units from the alternative source.</td>
</tr>
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\[ P1 : \max_{Q, \Omega_1 \times \Omega_2 \rightarrow \mathbb{R}^+} \min_{t_1, t_2 \in \Omega_1 \times \Omega_2} \{E_{c_1,c_2}[pE_D[\min(D, Q(c_1,c_2) + Q_A(c_1,c_2))] - \bar{w}Q_A(c_1,c_2) - t_1(c_1,c_2) - t_2(c_1,c_2)]\} \]

s.t. \( E_{c_j}[t_i(c_i, c_j) - c_iQ(c_i, c_j)] \geq 0 \quad \forall c_i \in \Omega_i, i,j = 1,2, \ j \neq i \) (2)

\( E_{c_j}[t_i(c_i, c_j) - c_iQ(c_i, c_j)] \geq E_{c_j}[t_i(\tilde{c}_i, c_j) - c_iQ(\tilde{c}_i, c_j)] \quad \forall c_i, \tilde{c}_i \in \Omega_i, i,j = 1,2, \ j \neq i \) (3)

where, \( Q_A(c_1,c_2) = (Q_n(\pi) - Q(c_1,c_2))^+ \). M’s problem is to maximize her expected profit subject to two sets of constraints. The first set reflects the fact that both suppliers are guaranteed non-negative expected profits (otherwise they would refuse to participate). The second set ensures that both suppliers report their costs truthfully. Because S1 and S2 do not know each other’s costs, we use the expected profit of each supplier with respect to the other supplier’s cost in these constraints.
The solution to $\textbf{P1}$ is characterized by the following proposition. Note that, in the optimal solution, M procures from either the alternative source, or S1 and S2, but not both.

**Proposition 1.** M’s optimal menu of contracts $\{Q^c(c_1,c_2), t_1^c(c_1,c_2), t_2^c(c_1,c_2)\}$ satisfies:

\[
Q^c(c_1,c_2) = \begin{cases} 
G^{-1}\left(p-k_1(c_1)-k_2(c_2)\right) & \text{if } k_1(c_1) + k_2(c_2) \leq \bar{w}, \\
0 & \text{otherwise}
\end{cases}
\]

\[
t_i^c(c_1,c_2) = c_iQ^c(c_1,c_2) + \int_{c_i}^{c_i} Q^c(c_j,\tau)d\tau, \ i, j = 1, 2, \ j \neq i.
\]

Proposition 1 implies that, due to asymmetric cost information, the supply chain is not coordinated. Comparing $Q^c(c_1,c_2)$ to the supply chain optimal quantity $Q_n(c_1 + c_2)$, we see that M orders less from S1 and S2 than would maximize total expected profit for S1, S2, and M (as $k_i(c_i) \geq c_i$), and the distortion increases with $c_i$ (as $k_i(c_i)$ is increasing in $c_i$). This *under-production* is the first of two types of inefficiency arising from asymmetric cost information. The second type is *over-elimination*: when $c_1 + c_2 < \bar{w} < k_1(c_1) + k_2(c_2)$, M fails to contract with S1 and S2 even though they are more efficient than the alternative source. The underlying reason for both inefficiencies is that due to the asymmetric cost information, M has to pay an “information rent” $t_i(c_1,c_2) - c_iQ^c(c_1,c_2) \geq 0$ to each supplier, that is, M pays each supplier more than his cost.

Proposition 1 defines the optimal complex contracts as a function of both suppliers’ cost reports $(c_1,c_2)$. In practice, a manufacturer may prefer to offer a quantity-contingent payment scheme rather than asking for the cost reports that determine payments and quantities. However, a cost-contingent menu of contracts $\{Q(c_1,c_2), t_1(c_1,c_2), t_2(c_1,c_2)\}$ can easily be transformed to an equivalent quantity-contingent menu of contracts $\{t_1^*(Q_1,Q_2), t_2^*(Q_1,Q_2)\}$ as follows (for $i = 1, 2$):

\[
t_i^*(Q_1,Q_2) = \begin{cases} 
t_i(c_1,c_2) & \text{if } Q_m = E_{c_n}[Q(c_m,c_n)] \text{ for some } c_m \in \Omega_m, m, n = 1, 2, \ m \neq n, \\
0 & \text{otherwise}
\end{cases}
\]

Therefore, we restrict our attention to cost-contingent contracts while analyzing the optimal complex contracts.

**Delegation:** M contracts only with S1, and S1 in turn contracts with S2. We allow for the menu
of contracts that M offers to S1, \( \{ t_1(Q, c_1) \}_{c_1 \in \Omega_1} \), to be contingent on both the cost \( c_1 \) reported by S1 and the quantity \( Q \) delivered by S1. First, we solve for S1’s optimal menu of contracts:

\[
P_2 : \max_{Q_1 \in \mathbb{R}^+} \quad E_{c_2}[t_1(\min(Q_1, Q_2(c_2)), \tilde{c}_1) - c_1 Q_1 - t_2(c_2)]
\]

\[
s.t. \quad t_2(c_2) - c_2 Q_2(c_2) \geq 0 \quad \forall c_2 \in \Omega_2
\]

S1 faces two sets of constraints, which guarantee non-negative expected profit to S2 and that S2 reports his cost truthfully, analogous to the two sets of constraints for M in \( P_1 \). Let \( \Pi_{S1}^d(c_1, \tilde{c}_1) \) be S1’s optimal value of problem \( P_2 \) (given his cost \( c_1 \) and his cost report \( \tilde{c}_1 \)) and \( Q_1^d(c_1, c_2) \) be the variables corresponding to this optimal solution.

We can now formulate M’s problem of choosing the payment function \( t_1(Q, c_1) \) as:

\[
P_3 : \max_{t_1: \mathbb{R}^+ \times \Omega_1 \rightarrow \mathbb{R}} \quad E_{c_1, c_2} \left[ pE_D[\min(D, Q^d(c_1, c_1, c_2) + Q_A(c_1, c_1, c_2))] - \omega Q_A(c_1, c_1, c_2) - t_1(Q^d(c_1, c_1, c_2), c_1) \right]
\]

\[
s.t. \quad Q^d(c_1, \tilde{c}_1, c_2) = \min(Q_1^d(c_1, \tilde{c}_1, c_2), Q_2^d(c_2 | c_1, \tilde{c}_1)) \quad \forall c_1, \tilde{c}_1 \in \Omega_1, \forall c_2 \in \Omega_2
\]

\[
\Pi_{S1}^d(c_1, \tilde{c}_1) \geq 0 \quad \forall c_1 \in \Omega_1
\]

\[
\Pi_{S1}^d(c_1, \tilde{c}_1) \geq \Pi_{S1}^d(c_1, \tilde{c}_1) \quad \forall c_1, \tilde{c}_1 \in \Omega_1.
\]

where, \( Q_A(c_1, \tilde{c}_1, c_2) = (Q_A(\tilde{\pi}) - Q^d(c_1, \tilde{c}_1, c_2))^+ \). As we have already stated that delegation cannot dominate control when complex contracts are employed, our main objective in solving \( P_3 \) is to design a smart payment function \( t_1(Q, c_1) \) that results in the same quantities being produced by S1 and S2 with minimal payments.

**Delegation vs. Control**: We extend the results in MMR (1995) to the case in which the components of the suppliers are perfectly complementary and M has an alternative source, and find that M can achieve the same expected profit by delegating, rather than controlling, component procurement.\(^2\)

\(^2\)In order to prove Proposition 2, we have assumed that S1 cannot communicate with S2 (to learn about \( c_2 \)) before reporting his own cost \( c_1 \) to M. One can relax this assumption at the expense of introducing an alternative assumption that M can observe (and contract on) any cost information communicated from S2 to S1, as in Baron and Baseneko (1994).
Proposition 2. When the firms use complex contracts, M’s expected profit is equal under delegation and control.

Under the optimal menu of contracts, we observe that: (i) S1 reports his cost truthfully under delegation, (ii) the quantity produced with delegation is the same as the quantity produced with control (i.e., $Q_1^d(c_1, c_1, c_2) = Q_2^d(c_2|c_1, c_1) = Q^c(c_1, c_2)$), and (iii) S1 and S2 have the same profit with control and delegation in expectation. In other words, M ensures that S1 does not lie about his cost, aligns S1’s incentives with her own, and thus replicates the outcome of control with delegation. Hence if the firms are able to implement arbitrarily complex optimal contracts, M may choose a procurement strategy of either delegation or control and achieve the same expected profit.

With either delegation or control, the optimal cost-contingent contracts are very complex, as is evident from Proposition 1. Unfortunately, the equivalent optimal quantity-contingent payment scheme, obtained by transformation (4), also is complex. In the simplest imaginable case, where $c_1$ is known and $c_2$ and $D$ are both uniformly distributed, the payment to S2, $t_2^*(Q_1, Q_2)$, is quadratic in $Q_2$. When the manufacturer has uncertainty about both $c_1$ and $c_2$, or the distributions of $c_2$ and $D$ fail to be uniform, we are unable to characterize such structural properties. As the complex optimal contracts are unlikely to be implementable in practice, we will consider simple price-only contracts, which are commonly used in practice.

4. Delegation vs. Control with Price-Only Contracts

In this section, we assume that both M and S1 employ simple price-only contracts.

**Control:** M offers a constant wholesale price per unit $w_1$ to S1 and $w_2$ to S2, such that $w_1 + w_2 \leq \overline{w}$. Supplier $i$ accepts $w_i$ if and only if $c_i \leq w_i$. If both suppliers accept, then M orders her newsvendor quantity from both suppliers, which is $Q_n(w_1 + w_2) = G^{-1}(1 - (w_1 + w_2)/p)$, and in return pays $t_i = w_i Q_n(w_1 + w_2)$ to supplier $i$. Since the alternative source is costlier, M does not procure any additional capacity from the alternative source in case the suppliers accept her offer. If either one of the suppliers rejects the wholesale price offered by M, M purchases only from her alternative source. Her procurement quantity is $Q_A = Q_n(\overline{w})$ and she receives a net profit of
\( \pi := pE[\min(D, Q_n(\pi))] - wQ_n(\pi) \). Thus, M purchases from either S1 and S2 or the alternative source, but not from both. We can formulate M’s problem of selecting \( w_1 \) and \( w_2 \) as follows:

\[
P_4: \max_{w_1, w_2} F_1(w_1)F_2(w_2)\left[pE_D[\min(D, Q_n(w_1 + w_2))] - (w_1 + w_2)Q_n(w_1 + w_2) - \pi + \pi \right]
\]

s.t. \( \underline{c}_i \leq w_i \leq \bar{c}_i \quad i = 1, 2 \)

\( w_1 + w_2 \leq \bar{w} \)

M obtains a profit of \( \lambda(w_1, w_2) := pE_D[\min(D, Q_n(w_1 + w_2))] - (w_1 + w_2)Q_n(w_1 + w_2) \) in the event that both S1 and S2 accept the contract (with probability \( F_1(w_1)F_2(w_2) \)), and otherwise she obtains her alternative source profit \( \pi \). In the event of acceptance, M’s profit \( \lambda(w_1, w_2) \) is a decreasing function of \( w_1 \) and \( w_2 \). Therefore, M faces a trade-off: increasing \( w_1 \) or \( w_2 \) increases the probability that the contract is accepted, but decreases M’s profits in the event of acceptance. An optimal wholesale price \( w_i \) must be in the interval \( [\underline{c}_i, \bar{c}_i] \) because supplier \( i \) will always accept \( \bar{c}_i \) and will never accept less than \( \underline{c}_i \). Clearly, the under-production and over-elimination inefficiencies caused by asymmetric information persist with price-only contracts.

**Delegation**: The sequence of events is as follows: (1) M offers a wholesale price \( w_1 \leq \bar{w} \) to S1. (2) S1 in turn offers a wholesale price \( w_2 \) to S2. (3) S2 either accepts or rejects S1’s offer. (4) If S2 accepts \( w_2 \), then S1 accepts the wholesale price \( w_1 \) offered by M. M purchases \( Q_n(w_1) = G^{-1}(1 - w_1/p) \) units from S1 at price \( w_1 \) per unit, and S1 purchases \( Q_n(w_1) \) units from S2 at price \( w_2 \) per unit. Since the alternative source is costlier, M does not procure any additional capacity from the alternative source in case the suppliers accept the contracts. (5) If S2 rejects S1’s offer, then S1 is forced to reject M’s offer, and M buys only from the alternative source at price \( \bar{w} \) per unit.

To derive M’s optimal wholesale price, we must first consider S1’s response to any given wholesale price. S1, contingent on his cost \( c_1 \) and the wholesale price \( w_1 \) offered by M, selects the wholesale price \( w_2 \) to maximize his expected profit:

\[
P_5: \max_{\underline{c}_2 \leq w_2 \leq \min(p_2, w_1 - c_1)} F_2(w_2)(w_1 - w_2 - c_1)Q_n(w_1).
\]
An optimal wholesale price \( w_2 \) must be in the interval \( [c_2, \min(\bar{c}_2, w_1 - c_1)] \) because S1 cannot profitably pay more than \( w_1 - c_1 \) and S1 will always accept \( \bar{c}_2 \) but never accept less than \( c_2 \). As \( w_1 - c_1 \) increases, so does S1’s incentive to offer a higher wholesale price \( w_2 \) to S2. S2 accepts if and only if \( w_2 \geq c_2 \). The following proposition provides the solution to \( P5 \).

**Proposition 3.** Given a \( w_1 \), the optimal wholesale price \( w_d^2 \) that S1 offers to S2 satisfies:

\[
w_d^2(c_1, w_1) = \begin{cases} 
  c_2 & \text{if } w_1 - c_1 < c_2 \\
  k_2^{-1}(w_1 - c_1) & \text{if } c_2 \leq w_1 - c_1 \leq k_2(\bar{c}_2) \\
  \bar{c}_2 & \text{otherwise}
\end{cases}
\]

Furthermore, \( w_d^2(c_1, w_1) \) is decreasing in \( c_1 \) and increasing in \( w_1 \).

Proposition 3 tells us that M should offer \( w_1 \leq \bar{c}_1 + k_2(\bar{c}_2) \) because offering a higher price would not increase the probability of having both suppliers accept the contract, and therefore could only decrease her expected profit. Using this fact and S1’s best response function \( w_d^2(c_1, w_1) \), we can formulate M’s problem as follows:

\[
P6: \max_{\bar{c}_1 + c_2 \leq w_1 \leq \min(\bar{c}_1 + k_2(\bar{c}_2), \bar{w})} \mathbb{E}_{c_1} \left[ F_2(w_d^2(c_1, w_1)) \right] (pE_D[\min(D, Q_n(w_1))] - w_1Q_n(w_1) - \bar{\pi}) + \bar{\pi},
\]

where \( Q_n(w_1) = G^{-1}(1 - w_1/p) \). The term \( \mathbb{E}_{c_1} [F_2(w_d^2(c_1, w_1))] \) is the expected probability that S2 agrees to participate given the optimal wholesale price of S1, \( w_d^2(c_1, w_1) \).

From Proposition 3, we see that under delegation, S1 keeps a margin for himself by quoting to S2 a wholesale price \( w_d^2 \) smaller than \( w_1 - c_1 \). Hence, delegation causes a double marginalization problem that does not occur when M controls the wholesale price for both suppliers. The double marginalization effect of delegation exacerbates both over-elimination and under-production. From M’s perspective, \( w_d^2 \) is set too low, meaning that the probability that S2 accepts S1’s offer is too low. By increasing \( w_1 \), M motivates S1 to increase \( w_d^2 \) and thus (indirectly) increases the probability that S2 accepts, which partially addresses the over-elimination problem. Unfortunately, this causes M to pay a higher total price per unit under delegation than under control, and order fewer units, worsening the under-production problem.

However, delegation allows S1 to adjust the price offered to S2 based on his own private cost information, which is a benefit that does not occur when M controls the wholesale price for both
suppliers. Specifically, when S1 has a low cost (information that is not available to M), S1 offers a higher wholesale price to S2 than M would offer in the control scenario, and can thus engage S2 when S2 has a high cost and would rejected M’s offer under control. This adjustment effect reduces the over-elimination inefficiency. That is, the probability that both S1 and S2 accept a contract may be higher under delegation. Delegation results in a greater expected profit for M than control when the positive impact of the adjustment effect outweighs the negative impact of the double marginalization effect.

4.1. Delegation vs. Control When \( c_1 \) is Known

We first compare delegation and control in the case that M knows S1’s cost \( c_1 \), but has uncertainty regarding S2’s cost \( c_2 \). Because \( c_1 \) is known by M, the adjustment effect is null and only the double marginalization effect is relevant. Therefore, unlike in the setting with arbitrarily complex contracts, delegation results in strictly lower expected profit for M than control.

**Proposition 4.** Assume the firms use price-only contracts and M knows S1’s cost \( c_1 \). When \( w \leq c_1 + c_2 \), M procures only from her alternative source. Otherwise, M’s expected profit is strictly lower with delegation than with control.

M might consider delegation for convenience or other reasons not represented in our model. Therefore, to assess the magnitude of loss of expected profit with delegation, we undertake a numerical study with the following parameters. Without loss of generality, the selling price \( p \) is normalized to 1 and S1’s cost \( c_1 \) to 0. Demand \( D \) is the maximum of zero and a normally distributed random variable with mean \( \mu = 100 \) and standard deviation \( \sigma = \{10, 20, 50, 100\} \). We consider both uniform and triangular distribution for S2’s cost \( c_2 \), and specify the minimum cost \( c_2 = \mu_2(1 - \Delta_2) \), maximum cost \( \bar{c}_2 = \mu_2(1 + \Delta_2) \), and, for the triangular distribution, mode \( m_2 = \{c_2, \mu_2, \bar{c}_2\} \), with mean \( \mu_2 = \{0.2, 0.4, 0.8, 1\} \) and cost dispersion \( \Delta_2 = \{0.05, 0.1, 0.3, 0.5, 0.7\} \). Note that cost dispersion \( \Delta_2 = (\bar{c}_2 - c_2)/(2\mu_2) \). The unit cost of the alternative source is \( \min(1, c_1 + c_2 + \delta) \), where \( \delta \) varies from 0.05 to 1 in increments of 0.2.
The results, summarized in Table 2, establish that delegation is often very costly for M (when she knows S1’s cost structure). In our experiments with the triangular cost distribution, the loss of expected profit from delegating rather than controlling component procurement is 18.1% on average, but can be as large as 55.6%. Note that S1 cannot set \( w_2 \) higher than \( \bar{w} \) or lower than \( c_2 \), and therefore the loss of expected profit from double marginalization increases with the difference \( \bar{w} - c_2 \). Interestingly, skewing the triangular distribution to the right decreases the detrimental double marginalization effect under delegation and increases M’s expected profit relative to control. As the mode of the distribution increases, which increases \( k_2^{-1}(w_2) \), S1 is motivated to offer a higher \( w_2 \), and, ultimately, M offers a lower \( w_1 \). We observe qualitatively similar results in experiments with the uniform cost distribution.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Min</th>
<th>Median</th>
<th>Max</th>
<th># of Cases</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular</td>
<td>18.143</td>
<td>0.001</td>
<td>12.477</td>
<td>55.556</td>
<td>996</td>
</tr>
<tr>
<td>( \bar{w} - c_2 \geq 0.45 )</td>
<td>25.953</td>
<td>1.417</td>
<td>24.023</td>
<td>55.556</td>
<td>504</td>
</tr>
<tr>
<td>( \bar{w} - c_2 &lt; 0.45 )</td>
<td>10.143</td>
<td>0.001</td>
<td>2.096</td>
<td>55.556</td>
<td>492</td>
</tr>
<tr>
<td>( m_2 = c_2 )</td>
<td>20.258</td>
<td>0.075</td>
<td>16.926</td>
<td>48.855</td>
<td>332</td>
</tr>
<tr>
<td>( m_2 = \mu_2 )</td>
<td>18.376</td>
<td>0.002</td>
<td>13.425</td>
<td>55.556</td>
<td>332</td>
</tr>
<tr>
<td>( m_2 = \bar{c}_2 )</td>
<td>15.794</td>
<td>0.001</td>
<td>6.596</td>
<td>55.556</td>
<td>332</td>
</tr>
<tr>
<td>Uniform</td>
<td>19.341</td>
<td>0.038</td>
<td>12.300</td>
<td>50.000</td>
<td>332</td>
</tr>
</tbody>
</table>

4.2. Delegation vs. Control When \( c_1 \) is Unknown

We now allow M to have uncertainty about both \( c_1 \) and \( c_2 \), so the adjustment effect becomes relevant. For analytical tractability, we assume that \( c_1 \) and \( c_2 \) are identically distributed, i.e., \( F_1 = F_2 = F \). The next two propositions show how M’s optimal procurement strategy - delegation versus control - depends on the cost distribution, the cost of the alternative source, and the selling price. Together, Propositions 5 and 6 establish that the manufacturer has strictly greater expected profit with delegation under some conditions and has strictly greater expected profit with control under other conditions. This stands in stark contrast with the result in Proposition 2 that with arbitrarily complex contracts the manufacturer has the same expected profit under delegation and control.
Proposition 5. Suppose that the firms use price-only contracts and \( c_i \sim U[c, \bar{c}] \). If \( \bar{w} > c + \bar{c} \), there exists a constant \( \bar{p} \) such that if \( p > \bar{p} \), then \( M \) has strictly greater expected profit with control than with delegation. If \( \bar{w} \leq c + \bar{c} \), then \( M \) has the same expected profit with control and delegation.

When costs \( c_1 \) and \( c_2 \) are uniformly distributed, \( M \) prefers control to delegation and, unlike in the setting with complex contracts, that preference may be strict. When the alternative source is expensive (\( \bar{w} > c + \bar{c} \)) and the selling price is high (\( p > \bar{p} \)) so that \( M \) wishes to purchase a relatively large quantity, \( M \) has a strong incentive to buy from \( S_1 \) and \( S_2 \). This exacerbates the double marginalization effect of delegation, as \( M \) offers a high wholesale price \( w_1 \) to increase the probability that \( S_1 \) accepts her offer. Therefore, \( M \) has strictly lower expected profit with delegation than with control. Conversely, when the alternative source is cheap (so the double marginalization effect of delegation is small) and we change the cost distribution from uniform to triangular (to magnify the adjustment effect) we find that \( M \) has strictly greater expected profit with delegation than with control.

Proposition 6. Suppose that the firms use price-only contracts, \( c_i \sim TR[c, m, \bar{c}] \) and \( \bar{w} \leq m + \bar{c} \). The expected profit of \( M \) is strictly greater with delegation than control.

To understand how the cost distribution influences the adjustment effect of delegation, observe from Proposition 3 and equation (1) that \( S_1 \)'s interior optimal price to offer \( S_2 \) satisfies \( w_2^d + h_2(w_2^d) = w_1 - c_1 \). If the alternative source is cheap, as assumed in Proposition 6, then \( h_2 \) for the triangular distribution is smaller than for the uniform distribution, which enables \( S_1 \) to offer a higher price \( w_2^d \), especially when his own cost \( c_1 \) is small.

In an extensive numerical study, we compute the percentage loss in \( M \)'s expected profit from delegating component procurement. The parameters are as described in Subsection 4.1 except we no longer assume that \( c_1 = 0 \) but instead assume that \( c_1 \) has the same triangular distribution as \( c_2 \). The resulting average and maximum loss in Table 3 are much smaller than their counterparts in Table 2 (4.9\% vs. 18.1\% and 24.5\% vs. 55.6\%, respectively), indicating that delegation becomes much more desirable when \( M \) is uncertain about \( S_1 \)'s cost. Negative values in Table 3 indicate
that delegation is strictly more profitable than control. Delegation is most profitable when the cost distribution is skewed to the right \((m = \bar{c})\) and has a large dispersion \(\Delta\). \(h_2\) decreases with the mode \(m\), which enables S1 to offer a higher price \(w'_2\) under delegation, especially when his cost \(c_1\) is low, corresponding to the large dispersion \(\Delta\).) Delegation is also most profitable when the alternative source is cheap, which reinforces our insights from Propositions 5 and 6. This supports HP’s mixed strategy of delegating procurement of “commodity” components, for which HP has a low-cost alternative source, but controlling procurement of “strategic” components for which no alternative source exists.

<table>
<thead>
<tr>
<th>Table 3</th>
<th>The percentage loss in M’s expected profit from using delegation rather than control.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
</tr>
<tr>
<td>All Cases</td>
<td>4.865</td>
</tr>
<tr>
<td>(\Delta \geq 0.5)</td>
<td>3.568</td>
</tr>
<tr>
<td>(\Delta &lt; 0.5)</td>
<td>6.302</td>
</tr>
<tr>
<td>(m = c)</td>
<td>6.720</td>
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<tr>
<td>(m = \mu_c)</td>
<td>4.977</td>
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<tr>
<td>(m = \bar{c})</td>
<td>2.896</td>
</tr>
<tr>
<td>(w \geq 0.76)</td>
<td>7.053</td>
</tr>
<tr>
<td>(w &lt; 0.76)</td>
<td>2.676</td>
</tr>
</tbody>
</table>

5. Effectiveness of Price-Only Contracts

This section characterizes a broad range of conditions under which price-only contracts yield nearly as much expected profit for a manufacturer as complex optimal contracts. Under these conditions, a manufacturer should use price-only contracts because they are so attractively simple and easy to implement. However, this section also characterizes conditions under which price-only contracts perform very poorly. We start our analysis with the case that M knows S1’s cost \(c_1\). For this case, Propositions 2 and 4 imply that an optimal strategy for M is to simply pay \(c_1\) per unit to S1 and contract directly with S2. In effect, M and S2 constitute a 1-tier supply chain, so the results in Subsection 5.1 are immediately relevant for a manufacturer contracting with a single supplier that has private cost information. In Subsection 5.2, we allow for uncertainty about both S1’s and S2’s cost.
5.1. When \( c_1 \) is known

In characterizing M’s expected profit with optimal complex contracts we can, based on Proposition 2, assume that M contracts directly with both S1 and S2. Because \( c_1 \) is known, M reimburses S1 exactly \( c_1 \) per unit. Therefore, M can focus on designing a menu of contracts for S2, \( \{Q(c_2), t_2(c_2)\}_{c_2 \in \Omega_2} \), where \( Q(c_2) \) denotes production quantity and \( t_2(c_2) \) the payment from M to S2. Contingent on S2’s cost report \( c_2 \) and associated production quantity \( Q(c_2) \), M also purchases \( Q(c_2) \) units from S1 and pays \( t_1 = c_1 Q(c_2) \) to S1. M’s optimal menu of contracts is a special case of the optimal solution derived in Proposition 1.

**Corollary 1.** M’s optimal menu of contracts \( \{Q^*(c_2), t_2^*(c_2)\} \) satisfies:

\[
\{Q^*(c_2), t_2^*(c_2)\} = \begin{cases} 
G^{-1}\left(\frac{p-k_2(c_2)-c_1}{p}\right), c_2 Q^*(c_2) + \int_{c_2}^{c_1} Q^*(\tau)d\tau \quad & \text{for } c_2 \leq c_2 \leq c_t \\
0, 0 \quad & \text{for } c_t < c_2 \leq \overline{c}_2 
\end{cases}
\]

where the cut-off level

\[
c_t = \begin{cases} 
\overline{c}_2 \quad & \text{if } k_2(\overline{c}_2) + c_1 \leq \overline{w} \\
k_2^{-1}(\overline{w} - c_1) \quad & \text{if } k_2(\overline{c}_2) + c_1 > \overline{w} 
\end{cases}
\]

increases with \( \overline{w} \).

M employs a cut-off policy: for a S2 with cost greater than \( c_t \), M offers a null contract and exercises only her alternative source. From a S2 with cost less than \( c_t \), M orders her newsvendor quantity. M’s total unit cost from S2 is \( k_2(c_2) \) as defined in equation (1).

In characterizing M’s expected profit with price-only contracts we can, by Proposition 4, assume that M contracts directly with both S1 and S2. Because \( c_1 \) is known, M optimally offers a constant wholesale price per unit \( w_1 = c_1 \) to S1. M’s problem is to find the optimal wholesale price per unit \( w_2 \) to offer S2 to maximize her expected profit:

\[
P4': \max_{c_2 \leq w_2 \leq \min(\overline{c}_2, \overline{w} - c_1)} F_2(w_2) [pE_D[\min(D, Q_n(w_2 + c_1))] - (w_2 + c_1)Q_n(w_2 + c_1) - \overline{\pi}] + \overline{\pi}.
\]

The optimal \( w_2 \) must satisfy the following equation (see the proof of Proposition 7):

\[
\lambda(c_1, w_2) - \overline{\pi} = Q_n(w_2 + c_1)h_2(w_2). \tag{5}
\]

In comparing the optimal complex and price-only contracts, we find that using a price-only contract always worsens over-elimination but may ameliorate under-production.
Proposition 7. The probability that $M$ procures from $S1$ and $S2$ is lower with the price-only contract than the complex contract:

$$w^c_2 \leq c_t,$$

and is strictly so if $k_2(\bar{c}_2) + c_1 > \bar{w}$. However, the optimal production quantity is strictly higher with the price-only contract than the complex contract:

$$Q_n(w^c_2 + c_1) > Q^c(c_2)$$

if and only if $c_2 > k_2^{-1}(w^c_2)$.

Figure 1 illustrates the result in Proposition 7 that for high realizations of $S2$’s cost $c_2$, the price-only contract results in a strictly larger production quantity than the complex optimal contract.

**Figure 1** Production quantities under the optimal complex contract, $Q^c(c_2)$, the optimal price-only contract, $Q_n(w^c_2)$, and the supply chain optimum, $Q_n(c_2)$, as a function of $S2$’s cost $c_2$. The parameters are $p = 1$, $\bar{w} = 0.84$, $c_1 = 0$, and $c_2 \sim U[0.24, 1.36]$.

Proposition 8. The difference between $M$’s expected profit under the optimal complex contract and the optimal price-only contract is increasing in $\bar{w}$.

More generally, to understand when a price-only contract will be effective (or not), one must think of the price-only contract as an approximation for the complex optimal contract, and consider
the following two effects. First, when the cost range \([c_u, c_t]\) over which the price-only contract must approximate the complex optimal contract is large, the approximation will be relatively poor. We call this the \textit{cost range effect} and it increases with the cost dispersion \(\triangle_2\) and alternative-source price \(\bar{w}\) (since \(c_t\) increases with \(\bar{w}\)). Second, because the production quantity is constant under the price-only contract, the quality of the approximation decreases with the quantity variation of \(Q^c(c_2)\) under the complex contract. We call this the \textit{quantity variation effect} and it depends on the demand and cost distributions. Figure 1 shows that as the coefficient of variation of demand distribution increases from the left panel (a) to the right panel (b), the quantity under the complex contract \(Q^c(c_2)\) declines more rapidly with \(c_2\) and the quantity variation effect increases. Since the demand distribution does not affect the cut-off level \(c_t\), the cost range effect does not change. Hence the performance of the price-only contract deteriorates due to the quantity variation effect.

These two effects provide intuition for the results of our numerical study. The parameters are as specified in Subsection 4.1, except we only consider three levels for the standard deviation of the demand distribution \(\sigma = \{20, 50, 100\}\). Table 4 reports the loss of expected profit from using a price-only contract rather than the complex optimal contract. When the distribution of S2’s cost is uniform, the price-only contract performs well, as the loss of expected profit from using a price-only contract is less than 0.5% for half of all parameter settings, and has a mean value of 2.0%. The loss tends to increase with the alternative source price \(\bar{w}\), the cost dispersion \(\triangle_2\), and the coefficient of variation (c.v.) of demand, and the maximum loss of 11.0% occurs when all three of these parameters are at their maximum levels. In that specific instance, with a high cut-off level of 0.58 and highly variable demand, both the cost range and quantity variation effects are in full effect against price-only contracts. These qualitative insights are replicated with the triangular distribution for S2’s cost. However, we observe an even lower mean loss with the triangular distribution than the uniform because of the reduction in uncertainty (M can infer that S2’s cost is more likely to be near the mode \(m_2\)). In contrast, the maximum loss is higher, due to the quantity variation effect. M’s optimal quantity \(Q^c(c_1, c_2)\) decreases rapidly with \(c_2\) as \(k_2(c_2) + c_1\) approaches \(p\), which the price-only contract cannot mimic. With a right-skewed triangular cost
distribution, \( k_2(c_2) + c_1 \) is more likely to be close to \( p \) than with the uniform distribution. Therefore, we observe the maximum loss of 15.2% with the right-skewed triangular cost distribution, and the maximal values of the alternative source price \( w \), cost dispersion \( \Delta_2 \), and the coefficient of variation of demand.

Based on numerical experiments with \( w = \infty \) (no alternative source), Feng and Zhang (2006) conclude that price-only contracts perform “remarkably well” and report a maximum expected profit loss of 8.1% as opposed to our 15.2%. That Feng and Zhang find more positive results for price-only contracts than we do seems surprising in light of Proposition 8, which indicates that the worst performance for price-only contracts corresponds to \( w = \infty \) (no alternative source). Feng and Zhang find more positive results for price-only contracts than we do primarily because they restrict the cost dispersion to be smaller than 30%, whereas we allow for the cost dispersion to be as large as 70%. From Feng and Zhang’s numerical study and our own, one may conclude that the effectiveness of the price-only contract deteriorates as a manufacturer becomes highly uncertain about her supplier’s cost structure.
5.2. When \( c_1 \) is uncertain

In this subsection, we incorporate uncertainty about S1’s cost. We assume that the cost distributions are uniform, so that by Propositions 2 (with complex optimal contracts) and 5 (with simple price-only contracts) M achieves her maximal expected profit with control. That is, M contracts directly with both S1 and S2. We focus on the case \( \bar{w} = \infty \) (no alternative source).

Table 5 presents M’s expected profit loss from using the price-only contract when both \( c_1 \) and \( c_2 \) are uncertain and \( c_i \sim U(c; \bar{c}) \). (The parameters are as described in Subsection 4.1.) As a comparison, we use the same parameters to find the expected loss assuming \( c_1 \) is known and set \( c_1 = \frac{c_1 + c_2}{2} \) in order to have the same total expected cost in both settings.

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
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<th>10(^{th}) Percentile</th>
<th>Median</th>
<th>90(^{th}) Percentile</th>
<th>Max</th>
<th># of Cases</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_2 ) uncertain</td>
<td>1.841</td>
<td>0.001</td>
<td>0.007</td>
<td>0.395</td>
<td>7.334</td>
<td>9.943</td>
<td>60</td>
</tr>
<tr>
<td>( c_1, c_2 ) uncertain</td>
<td>5.773</td>
<td>0.005</td>
<td>0.02</td>
<td>1.285</td>
<td>20.281</td>
<td>23.727</td>
<td>60</td>
</tr>
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</table>

We find that the maximum expected loss more than doubles from 9.9% to 23.7% as the dimension of asymmetric information increases. More dramatically, as we keep all the parameters fixed and study the effect of additional uncertainty in \( c_1 \), we find that the expected loss at least doubles, and can be as large as 9 times. These observations suggest that M would incur significantly higher loss by using price-only contracts to procure from multiple tiers of suppliers, whose costs are uncertain.

6. Conclusions

We have analyzed the newsvendor-type procurement problem of a manufacturer with a 2-tier supply chain and uncertainty about each supplier’s cost structure. The manufacturer contracts with the tier-1 supplier and must decide whether to control component procurement by contracting with the tier-2 supplier, or delegate that responsibility to the tier-1 supplier.

A well-known result from the economics literature is that if the manufacturer can use arbitrarily complex contracts, she should control component procurement; we have proven that the manufacturer achieves exactly the same expected profit with delegation of component procurement. In
both cases, the manufacturer pays an “information rent” to each supplier, purchases fewer units than would maximize the firms’ total expected profits and also, with positive probability, fails to contract with the two suppliers, even though they are more efficient than the alternative source of supply. We call the first type of inefficiency under-production and the second over-elimination.

We have demonstrated that the manufacturer’s optimal contracts are extremely complex, and are therefore unlikely to be implemented in practice. Instead, the manufacturer may simply offer a constant price per unit to each supplier, the scenario labeled “price-only contracts with control” in Figure 2. The inefficiencies of under-production and over-elimination caused by asymmetric cost information persist. However, while over-elimination is worse than with the complex optimal contract, under-production may be less. We have observed a broad range of numerical examples where simple price-only contracts are nearly as effective as the complex optimal contracts. Unfortunately, under the adverse circumstances (when the manufacturer has a very high level of uncertainty about both suppliers’ costs, a low profit margin, a high demand variability and no alternative source of supply) the manufacturer may give up more than 20% of her expected profit by using simple price-only contracts.

![Figure 2](image)

**Figure 2** When the firms use price-only contracts, the manufacturer achieves greater expected profit with delegation than control if and only if the benefits from adjustment (making the price for the tier-2 supplier contingent on the tier-1 supplier’s cost) dominate the losses from double marginalization.

Our most important result is that using simple price-only contracts, rather than complex optimal contracts, breaks the result that the manufacturer achieves the same expected profit with delegation and control of component procurement. Control is strictly more profitable when the manufacturer
understands the cost structure of her tier-1 supplier. Delegation is strictly more profitable when the manufacturer is highly uncertain of both suppliers’ costs, her prior distribution for each supplier’s cost is skewed to the right, her selling price for the end item is low, and she has an attractive alternative source. A problem with price-only contracts and delegation is that the tier-1 supplier keeps a margin for himself, which directly reduces the manufacturer’s profit and exacerbates under-production and over-elimination, as shown in Figure 2. A compensating benefit is that the tier-1 supplier, with exact information about his own cost, does a better job of optimizing the price to offer the tier-2 supplier. Specifically, when the tier-1 supplier’s own cost is low, he adjusts the price for the tier-2 supplier upward, which increases the probability that the manufacturer successfully contracts with the two suppliers. Thus, adjustment by the tier-2 supplier reduces the over-elimination problem, which increases the manufacturer’s expected profit, as shown in Figure 2. In our numerical study, the effect of delegation on the manufacturer’s expected profit ranges from a gain of 18.5% to a loss of 55.6%. We conclude that, over a wide range of supply and demand conditions, the manufacturer can achieve nearly optimal expected profits with price-only contracts, but only if she makes the right decision to delegate or control component procurement.

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**Appendix**

**Proof of Proposition 1:** First, we observe that the set of constraints (2) for \( c_i < \tau_i \) are redundant:

\[
E_{cj} [t_i(c_i, c_j) - c_i Q(c_i, c_j)] \geq E_{cj} [\bar{t}_i(c_i, c_j) - c_i Q(\bar{\tau}_i, c_j)] \\
\geq E_{cj} [t_i(\bar{\tau}_i, c_j) - \bar{\tau}_i Q(\bar{\tau}_i, c_j)] \\
\geq \text{by (3) for } \bar{c}_i = \tau_i \geq 0.
\]

Hence the only remaining constraints are (2) for \( c_i = \tau_i \) and (3). If (2) for \( c_i = \tau_i \) were not binding, then one could increase the objective function by decreasing \( t_i(c_i, c_j) \) by \( \varepsilon > 0 \), \( \forall (c_i, c_j) \in \Omega_i \times \Omega_j \) (note that constraints (3) will not change). Thus, the optimal mechanism must satisfy \( E_{cj} [t_i(\bar{\tau}_i, c_j) - \bar{\tau}_i Q(\bar{\tau}_i, c_j)] = 0 \). Next, we can show that constraints (3) are satisfied if and only if the following conditions hold (see Laffont and Martimort 2002 for details):

\[
E_{cj} [Q(c_i, c_j)] \text{ is decreasing in } c_i, i, j = 1, 2, \ j \neq i
\]

\[
E_{cj} [t_i(c_i, c_j)] = E_{cj} \left[ t_i(\bar{\tau}_i, c_j) - \bar{\tau}_i Q(\bar{\tau}_i, c_j) + c_i Q(c_i, c_j) + \int_{\bar{c}_i}^{\tau_i} Q(\tau, c_j) d\tau \right].
\]

It follows immediately that \( E_{cj} [t_i(c_i, c_j)] = E_{cj} \left[ c_i Q(c_i, c_j) + \int_{c_i}^{\tau_i} Q(\tau, c_j) d\tau \right] \). Using this equation and the fact that \( Q_A(c_1, c_2) = (Q_n(\bar{w}) - Q(c_1, c_2))^+ \), problem P1 reduces to

\[
\max_{Q: \Omega_i \times \Omega_j \rightarrow \mathbb{R}^+} \int_{\tau_1}^{\tau_2} \int_{\tau_1}^{\tau_2} \left[ pE_D[\min(D, \max(Q(c_1, c_2), Q_n(\bar{w}))) - \bar{w}(Q_n(\bar{w}) - Q(c_1, c_2))^+] - (c_1 + c_2)Q(c_1, c_2) - \int_{c_1}^{\tau_1} Q(\tau_2, c_2) d\tau - \int_{c_1}^{\tau_2} Q(c_1, \tau) d\tau \right] dF_2(c_2) dF_1(c_1)
\]

s.t. \( E_{cj} [Q(c_i, c_j)] \) is decreasing in \( c_i, i, j = 1, 2, \ j \neq i \).
Since the costs $c_1$ and $c_2$ are independent, we can use integration by parts to get:

$$\int_{\xi_j} \int_{\xi_i} \left( \int_{c_i}^{w} Q(\tau, c) d\tau \right) dF_i(c_i) dF_j(c_j) = \int_{\xi_j} \left[ F_i(c_i) \int_{c_i}^{w} Q(\tau, c) d\tau \right]_{c_i=E_i}^{\xi_i} dF_j(c_j) + \int F_i(c_i) Q(c_i, c_j) dc_i dF_j(c_j)$$

We now ignore the monotonicity constraints (6) and solve the relaxed program. We can find the optimal $Q(c_1, c_2)$ via pointwise maximization, i.e.:

$$Q^c(c_1, c_2) \in \arg\max_Q \{ pE_D[\min(D, \max(Q, Q_n(\overline{w})))] - \overline{w} (Q_n(\overline{w}) - Q)^+ - (k_1(c_1) + k_2(c_2))Q \}. \quad (8)$$

We fix $c_1$ and $c_2$, and consider the case where $\overline{w} < k_1(c_1) + k_2(c_2)$. In this case, the profit function of $M$ is linearly decreasing in $Q$ for $Q \leq Q_n(\overline{w})$. For $Q > Q_n(\overline{w})$, it is straightforward to show that the profit function is concave in $Q$ with the maximizer $Q_n(k_1(c_1) + k_2(c_2))$ which is smaller than $Q_n(\overline{w})$. Hence we conclude that the objective function is decreasing for $Q > Q_n(\overline{w})$ and immediately conclude that $Q^c(c_1, c_2) = 0$. Next consider the case $\overline{w} \geq k_1(c_1) + k_2(c_2)$. Following the same steps, we can show that the objective function is linearly increasing in $Q$ for $Q \leq Q_n(\overline{w})$ and concave in $Q$ with the maximizer $Q_n(k_1(c_1) + k_2(c_2))$ which is larger than $Q_n(\overline{w})$. This shows that $Q^c(c_1, c_2) = Q_n(k_1(c_1) + k_2(c_2))$.

To complete the proof, observe that as $k_i(c_i)$ is assumed to be increasing in $c_i$, the monotonicity constraints (6) are trivially satisfied. Hence we conclude that $Q^c(c_1, c_2)$ is a solution to the original problem P1. Finally, observe that $t_i(c_1, c_j)$ as stated in Proposition 1 satisfies the set of constraints (7). ■

**Proof of Proposition 2:** We start by solving S1’s problem P2. Having reported his cost as $\tilde{c}_1$, S1’s design of optimal menu of contracts is complicated by two factors: (1) S1 may want to conceal his own cost from S2 as the menu of contracts that S1 offers signal information about S1’s cost (a signaling problem) and (2) S1 does not know the actual cost of S2 and only learns about this cost through S2’s choice of contract (a screening problem). We observe that in this case the signaling problem does not exist because both suppliers’ profits are linear in the payment $t_2$ (this follows
from Proposition 11 in Maskin and Tirole (1990)). The screening problem can be resolved using the same technique as in the proof of Proposition 1. Therefore, we can rewrite S1’s problem as:

\[ P2: \max_{Q_1 \in \mathbb{R}^+} E_{c_2}\left[ t_1(\min(Q_1, Q_2(c_2)), \tilde{c}_1) - c_1Q_1 - k_2(c_2)Q_2(c_2) \right] \]

\[ \text{s.t. } Q_2(c_2) \text{ is decreasing in } c_2 \]

Setting \( Q_1 \) greater than \( Q_2(c_2) \) would only decrease S1’s profit. If S1 were to set \( Q_1 < Q_2(c_2) \), he could increase his profit by decreasing \( Q_2(c_2) \) such that \( Q_2(c_2) = Q_1 \). We conclude that S1 optimally sets \( Q_1 = Q_2(c_2) \).

We device a payment function \( t_1(Q, c_1) \), where \( Q = \min(Q_1, Q_2) \), as follows:

\[ t_1(Q, c_1) := pE_D[\min(D, \max(Q, Q_n(\bar{w}))) - \bar{w}(Q_n(\bar{w}) - Q)^+ - h_1(c_1)Q + \alpha(c_1)], \] where

\[ \alpha(c_1) := -E_{c_2}\left[ pE_D[\min(D, \max(Q^c(c_1, c_2), Q_n(\bar{w}))) - \bar{w}(Q_n(\bar{w}) - Q^c(c_1, c_2))^+ - (k_1(c_1) + k_2(c_2))Q^c(c_1, c_2)] \right] + \int_{c_1}^{\tilde{c}_1} E_{c_2}\left[ Q^d_1(\tau, \tau, c_2) \right] d\tau \].

The remainder of the proof is in four steps.

Step 1: In this step, we show that S1 reports his cost truthfully at stage 2(i) under the above payment function. In other words, we prove that \( c_1 \in \arg \max_{c_1} \Pi^d_1(c_1, \tilde{c}_1) \). By Lemmas 6.1 and 6.3 of Mirrlees (1986), it suffices to show that \( \frac{\partial \Pi^d_1(c_1, \tilde{c}_1)}{\partial c_1} \) is an increasing function of \( \tilde{c}_1 \). Using the envelope theorem, we find that \( \frac{\partial \Pi^d_1(c_1, \tilde{c}_1)}{\partial c_1} = -Q^d_1(c_1, \tilde{c}_1, c_2) \). Given the definition of \( t_1(Q, c_1) \) and using the fact that \( Q = Q_1 = Q_2(c_2) \), \( Q^d_1(c_1, \tilde{c}_1, c_2) \) is a solution to:

\[ \max_{Q \in \mathbb{R}^+} E_{c_2}\left[ pE_D[\min(D, \max(Q, Q_n(\bar{w}))) - \bar{w}(Q_n(\bar{w}) - Q)^+ - h_1(\tilde{c}_1)Q + \alpha(\tilde{c}_1) - c_1Q - k_2(c_2)Q] \right]. \] (9)

As \( h_1(\tilde{c}_1) \) is increasing in \( \tilde{c}_1 \), we conclude that \( Q^d_1(c_1, \tilde{c}_1, c_2) \) is decreasing in \( \tilde{c}_1 \).

Step 2: Given that S1 reports his cost truthfully, i.e., \( \tilde{c}_1 = c_1 \), we now show that \( Q^d_1(c_1, c_1, c_2) = Q^c(c_1, c_2) \). The objective function for \( Q^c(c_1, c_2) \) is as follows (see the proof of Proposition 1):

\[ Q^c(c_1, c_2) \in \arg \max_{Q} \{ pE_D[\min(D, \max(Q, Q_n(\bar{w}))) - \bar{w}(Q_n(\bar{w}) - Q)^+ - (k_1(c_1) + k_2(c_2))Q] \}. \]
By comparing this function to the objective function in (9) for $Q^d(c_1, c_1, c_2)$, we conclude that $Q^d_1(c_1, c_1, c_2) = Q^c_1(c_1, c_2)$. As we have already concluded that $Q^d_1(c_1, c_1, c_2) = Q^d_2(c_1, c_1, c_2)$, this step proves that the quantities produced under delegation and control are the same for every $(c_1, c_2) \in \Omega_1 \times \Omega_2$.

**Step 3:** Next, we show that S1 receives positive expected profits for every $c_1 \in \Omega_1$, i.e., $\Pi^d_{S1}(c_1, c_1) \geq 0$. Using the results of the first two steps and the definition of $\alpha(c_1)$, we find that $\Pi^d_{S1}(c_1, c_1) = \int_{c_1}^{\tau_1} E_{c_2} [Q^d_1(\tau, c_2, c_2) d\tau] = \int_{c_1}^{\tau_1} Q^c(c_1, c_2) d\tau = \Pi^c_{S1}(c_1, c_2)$. We observe that $\Pi^d_{S1}(c_1, c_1)$ is decreasing in $c_1$ and $\Pi^d_{S1}(\tau_1, \tau_1) = 0$, which completes the proof.

**Step 4:** In this last step, we show that both S1 and S2 receives the same profit under delegation and control in expectation. Using the results of the first two steps, we find that

$$
\Pi^d_{S1}(c_1, c_1) = \int_{c_1}^{\tau_1} E_{c_2} [Q^d_1(\tau, c_2, c_2) d\tau] = \int_{c_1}^{\tau_1} E_{c_2} [Q^c(c_1, c_2) d\tau] = \Pi^c_{S1}(c_1, c_2)
$$

To complete the proof of the proposition, using the results of the four steps, we note that as the production quantity for every $(c_1, c_2) \in \Omega_1 \times \Omega_2$ as well as the expected profits of S1 and S2 are the same under delegation and control, M’s expected profit is equal under delegation and control.

**Proof of Proposition 3:** Consider a S1 with cost $c_1$ who has been offered a wholesale price of $w_1$ by M. Let $\Pi_{S1}(c_1, w_2)$ be S1’s objective function as defined in P5. Then

$$
\frac{d\Pi_{S1}(c_1, w_2)}{dw_2} = (w_1 - w_2 - c_1 - h_2(w_2)) f_2(w_2) Q_n(w_1).
$$

The first term in the derivative, $w_1 - c_1 - w_2 - h_2(w_2)$, is a decreasing function of $w_2$. Since $f_2(w_2) > 0$ by assumption and $Q_n(w_1) > 0$, we immediately conclude that $\Pi_{S1}(c_1, w_2)$ is unimodal in $w_2$. We now consider three cases depending on the value of $w_1 - c_1$ to characterize the optimal wholesale price $w^d_2(c_1, w_1)$: (i.) When $w_1 - c_1 < \zeta_2$, for feasible values of $w_2$ ($w_2 \geq \zeta_2$) we have $w_1 - c_1 - w_2 - h_2(w_2) < 0$ implying $w^d_2(c_1, w_1) = \zeta_2$. (ii.) When $w_1 - c_1 > k_2(\zeta_2)$, for feasible values of $w_2$ ($w_2 \leq \zeta_2$) we get $w_1 - c_1 - w_2 - h_2(w_2) > 0$, implying $w^d_2(c_1, w_1) = \zeta_2$. (iii.) When $\zeta_2 \leq w_1 - c_1 \leq k_2(\zeta_2)$, $w^d_2$
satisfies the first order condition $w_2^d + h_2(w_2^d) = w_1 - c_1$, which is equivalent to $w_2^d = k_2^{-1}(w_1 - c_1)$. This completes the proof of Proposition 3. ■

**Proof of Proposition 4:** When $\overbar{w} \leq c_1 + \overbar{c}_2$, M procures from her alternative source. Therefore, we assume $\overbar{w} > c_1 + \overbar{c}_2$.

**Control:** M offers $w_1 = c_1$ to S1. In order to find the optimal wholesale price $w_2$, M solves P4. To make the comparison easier, we drop the subscript 2 of $w_2$ and rewrite the problem P4 of M as follows:

$$\max_{\overbar{w} \leq w \leq \min(\overbar{c}_2, \overbar{w} - c_1)} \Pi_M^d(w) := F_2(w) [pE_D[\min(D, Q_n(w + c_1))] - (w + c_1)Q_n(w + c_1) - \pi] + \pi.$$

**Delegation:** We solve the problem backwards and find that the optimal wholesale price of S1 is $w_2^d(w_1, c_1) = k_2^{-1}(w_1 - c_1)$ using Proposition 3 and our initial assumption on $\overbar{w}$. (Since M knows both $c_1$ and $F_2(c_2)$, it follows that $\overbar{c}_2 + c_1 \leq w_1 \leq k_2(\overbar{c}_2) + c_1$.) Thus, by letting $w := w_1 - c_1$, one can rewrite the problem P6 as follows:

$$\max_{\overbar{w} \leq w \leq \min(\overbar{c}_2(\overbar{c}_2, \overbar{w} - c_1))} \Pi_M^d(w) := F_2(k_2^{-1}(w)) [pE_D[\min(D, Q_n(w + c_1))] - (w + c_1)Q_n(w + c_1) - \pi] + \pi.$$

**Delegation vs. Control:** Since $k_2^{-1}(w) \leq w$, we have $\Pi_M^c(w) \geq \Pi_M^d(w)$. Moreover it is easy to see that $\arg \max \Pi_M^c(w) \leq \overbar{c}_2$, as $\Pi_M^c(w)$ is strictly decreasing for $w \geq \overbar{c}_2$. This shows that, even though the feasible region of $w$ in delegation is larger, M’s optimal profit is higher with control. Moreover, it is strictly so since $\overbar{w} > c_1 + \overbar{c}_2$. ■

**Proof of Proposition 5:** Since we have identical distributions for $c_i$, we drop the subscript $i$ from $\overbar{c}_i$, $\overbar{c}_i$, $F_i$, and $k_i$.

**Control:** As the supplier’s cost distributions are identical, M sets $w_1 = w_2$. To see this, assume that there exists an equilibrium in which $w_2 > w_1$. Observe that since $F(c_1)$ is assumed to be log-concave (see Section 2), we have $\frac{1}{2} \ln F(w_1) + \frac{1}{2} \ln F(w_1) \leq \ln F\left(\frac{w_1 + w_2}{2}\right)$, which, after rearranging terms, implies $F(w_1)F(w_2) \leq F^2\left(\frac{w_1 + w_2}{2}\right)$. Using this inequality and the problem formulation P4, we observe that M could alternatively set $w_1' = w_2' = \frac{w_1 + w_2}{2}$ and thus increase her profit. Therefore, we can rewrite P4 as (let $w = w_1 + w_2$):

$$\max_{\overbar{w} \leq w \leq \min(2\overbar{w}, \overbar{c}_2)} \Pi_M^c(w) := F^2\left(\frac{w}{2}\right)\left[\lambda\left(\frac{w}{2}, \frac{w}{2}\right) - \pi\right] + \pi.$$
Delegation: We begin solving for the optimal $w_2^d$. For simplicity, we drop the subscript 1 from $w_1$. Observe that for $c \sim U[\xi, \bar{c}]$, we have $k(c) = 2c - \xi$ and $k^{-1}(c) = \frac{c + \xi}{2}$. Using Proposition 3, we find that

$$w_2^d(c_1, w) = \begin{cases} \frac{c}{2} & \text{if } w - c_1 \leq \xi \\ \frac{\xi}{2} + \frac{c}{2} & \text{if } \xi \leq w - c_1 < k(\bar{c}) \\ \text{otherwise} \end{cases}.$$

Hence we can rewrite the objective function of $P6$: $\Pi_M^d(w) := E_{c_1}[F(w_2^d(c_1, w))] \left[ \lambda \left( \frac{w}{2}, \frac{w}{2} \right) - \pi \right] + \pi$, where:

$$E_{c_1}[F(w_2^d(c_1, w))] = \left\{ \begin{array}{ll} \frac{w - c_1 + \xi}{2} F(c_1) & \text{if } w \leq \xi + \bar{c} \\ \frac{w - c_1 + \xi}{2} F(c_1) & \text{if } \xi + \bar{c} < w \leq \xi + k(\bar{c}) = 2\bar{c}. \end{array} \right.$$  

Note that due to the feasibility conditions $w \leq 2\bar{c}$, hence $w_2^d(c_1, w)$ cannot be equal to $\bar{c}$.

Delegation vs. Control: Using the functional forms of the probability density and the cumulative distribution functions of the uniform distribution, we find that

$$F^2 \left( \frac{w}{2} \right) = \frac{(w - 2\bar{c})^2}{4(\bar{c} - \xi)^2} \quad \text{and} \quad E_c[F(w_2^d(c_1, w))] = \begin{cases} \frac{(w - 2\bar{c})^2}{4(\bar{c} - \xi)^2} & \text{if } w \leq \xi + \bar{c} \\ \frac{2w - \bar{c} - 2\bar{c}}{4(\bar{c} - \xi)^2} & \text{if } \xi + \bar{c} < w \leq 2\bar{c}. \end{cases}$$

It is straightforward to verify that the second part of Proposition 5 holds. As $\bar{w} \leq \xi + \bar{c}$ and $w$ must be less than $\bar{w}$, we must have $w \leq \xi + \bar{c}$. Since $E_c[F(w_2^d(c_1, w))] = F^2 \left( \frac{w}{2} \right)$ over this range, we conclude that $M$ receives the same expected profit under delegation and control. To show the first part, we start by observing that for $w > \xi + \bar{c}$, we have $F^2 \left( \frac{w}{2} \right) - E_c[F(w_2^d(c_1, w))] = \frac{(w - \bar{c} - \xi)^2}{4(\bar{c} - \xi)^2} > 0$. Hence to complete the proof we need to show that the optimal wholesale price is strictly greater than $\xi + \bar{c}$ for large enough $p$. In order to do so, we first observe that

$$\frac{d\Pi_M^d(w)}{dw} = F^2 \left( \frac{w}{2} \right) f \left( \frac{w}{2} \right) \left[ \lambda \left( \frac{w}{2}, \frac{w}{2} \right) - \pi - h \left( \frac{w}{2} \right) Q_n(w) \right],$$

and $\frac{d\Pi_M^a(w)}{dp} = E_D[\min(D, Q_n(w))] > 0$. This shows that we can find a large enough $p$ such that $\Pi_M^d(w)$ is a monotonically increasing function, which implies that the optimal $w$ must be equal to $\min(2\bar{c}, \bar{w})$. As $\bar{w} > \xi + \bar{c}$, we conclude that the optimal $w$ is strictly greater than $\xi + \bar{c}$.

Proof of Proposition 6: To prove Proposition 6, we compare the expected profits of $M$ under control and delegation. From the proof of Proposition 5, we know that $M$’s profits under control and delegation are as follows:

$$\Pi_M^c(w) := F^2 \left( \frac{w}{2} \right) \left[ \lambda \left( \frac{w}{2}, \frac{w}{2} \right) - \pi \right] + \pi \quad (10)$$
\[
\Pi^d_M(w) := E_c \left[ F\left(w^d_2(c_1, w)\right) \right] \left[ \lambda \left( \frac{w}{2}, \frac{w}{2} \right) - \pi \right] + \pi
\]  
(11)

Note that, \( w = w_1 + w_2 \) under control and \( w = w_1 \) under delegation.

For the triangular distribution, we have:
\[
F(c) = \begin{cases} 
\frac{(c-m)^2}{(\bar{c}-m)(\bar{c}-m)} & \text{if } c \leq m \\
1 - \frac{(\bar{c}-c)^2}{(\bar{c}-m)(\bar{c}-m)} & \text{if } m < c \leq \bar{c}
\end{cases}
\]
and 
\[
k(c) = c + h(c) = \begin{cases} 
1 & \text{if } c \leq m \\
\frac{(c-m)}{m-c} - \frac{3(m-c)^2}{2} & \text{if } m < c \leq \bar{c}
\end{cases}
\]

Notice that \( k(\bar{c}) \to \infty \). Therefore, using Proposition 3, we find \( w^d_2(c_1, w) \) can either be equal to \( \zeta \) or to \( k^{-1}_2(w-c_1) \) for triangular distribution. Depending on the relative values of \( c \) and \( m \), \( k^{-1}_2(w-c_1) \) is equal to either \( \frac{c+2(w-c_1)}{3} \) or to \( \frac{2\pi+(w-c_1)-\sqrt{4\pi^2-c^2-4m^2+(w-c_1-\pi)^2}}{3} \). Therefore, the optimal wholesale price offered by S1 is as follows:

\[
w^d_2(c_1, w) = \begin{cases} 
\frac{2\pi+(w-c_1)-\sqrt{4\pi^2-c^2-4m^2+(w-c_1-\pi)^2}}{3} & \text{if } c_1 < w - \frac{3m-c}{2} \\
\frac{c+2(w-c_1)}{3} & \text{if } w - \frac{3m-c}{2} \leq c_1 < w - \zeta \\
\zeta & \text{if } c_1 \geq w - \zeta.
\end{cases}
\]

Assume that \( \bar{w} \leq m + \zeta \). Due to feasibility, we have \( w \leq \bar{w} \leq m + \zeta \leq \frac{3m+\zeta}{2} \). Hence
\[
E_c \left[ F\left(w^d_2(c_1, w)\right) \right] = \int_{\zeta}^{\bar{w}} F\left(\frac{c+2(w-c_1)}{3}\right) dF(c_1) + \int_{\zeta}^{\bar{w}} F\left(\zeta\right) dF(c_1).
\]

Note that in our case \( \frac{w}{2} \leq \frac{m+c}{2} \leq m \) and \( \frac{c+2(w-c_1)}{3} \leq m \) for \( w - \frac{3m-c}{2} \leq c_1 \). By algebraic manipulation, using the functional forms for the triangular distribution, \( \frac{\rho^2(0)}{E_c[F\left(w^d_2(c_1, w)\right)]]} = \frac{27}{32} \). Since \( \lambda \left( \frac{\pi}{2}, \frac{\pi}{2} \right) - \pi \geq 0 \) for \( w \leq \bar{w} \), we conclude that \( \Pi^d_M(w) > \Pi^c_M(w) \) for every \( w \leq m + \zeta \). This completes the proof. \( \blacksquare \)

**Proof of Proposition 7:** Before we prove the statement of the proposition, we first observe that the optimal wholesale price \( w^\circ \) must satisfy equation (5). Taking the derivative of the objective function in problem \( \textbf{P}4' \), the first order condition is:
\[
f_2(w) [\lambda(c_1, w_2) - \pi - Q_n(w_2 + c_1)h_2(w_2)] = 0.
\]

As \( f_2(w) \) is assumed to be strictly positive, we conclude that \( w^\circ \) must either satisfy equation (5) or be equal to the boundary points \( \{\zeta, \min(\bar{c}, \bar{w} - c_1)\} \). The value of the objective function is equal to \( \pi \) when \( w_2 \) is equal to either \( \epsilon_2 \) or \( \bar{w} - c_1 \). We can always choose a \( w_2 = \min(\bar{c}, \bar{w} - c_1 - \epsilon) \), where \( \epsilon \) is a very small positive number and the value of the objective function will be at least
as \(\pi(c_1, w_2) - \pi\) is decreasing in \(w_2\) and \(\lambda(c_1, \overline{w} - c_1) = \pi\). Therefore, \(c_2\) and \(\overline{w} - c_1\) are weakly dominated. Observe that M’s profit is decreasing in \(w_2\) for \(w_2 \geq \tau_2\). Hence \(\tau_2\) cannot be the optimal solution either. Therefore, we conclude that \(w^*_2\) must satisfy equation (5).

In order to prove the statement of the proposition, we define two functions \(A(w)\) and \(B(w)\) as:

\[
A(w) := pE_D[\min(D, Q_n(c_1 + k(w)))] - (c_1 + k(w))Q_n(c_1 + k(w))
\]

\[
B(w) := pE_D[\min(D, Q_n(w + c_1)))] - (c_1 + k(w))Q_n(w + c_1).
\]

Notice that \(A(w)\) is decreasing in \(k(w)\) and \(k(w)\) is increasing in \(w\). Henceforth we can conclude that \(A(w)\) is decreasing in \(w\). For any fixed value of \(w\), we have that \(A(w) > B(w)\). To see this, note that \(Q_n(c_1 + w + h(w))\) is the unique optimal quantity \(Q\) for the function \(pE_D[\min(D, Q)] - (c_1 + k(w))]Q\).

The proof is trivial for the first case in which \(c_2 = \tau_2\). Otherwise, \(c_2\) is equal to \(k_2^{-1}(\overline{w} - c_1)\) and it directly follows from the definition of \(\pi\) in Section 4 that \(A(c_2) = \pi\). Since we have already shown that \(w^*_2\) satisfies equation (5), using the definitions of \(\lambda(c_1, w_2)\) and \(B(w)\) and rearranging terms, we get \(B(w^*_2) = \pi\). As \(A(w)\) is decreasing in \(w\) and \(A(w) > B(w)\), we find that \(w^*_2 < c_2\). Hence we conclude that \(w^*_2 < c_2\). Finally, if \(k_2(\tau_2) + c_1 > \overline{w}\), then by Corollary 1, \(c_2\) must be equal to \(k_2^{-1}(\overline{w} - c_1)\), and as in the second case above, we conclude that \(w^*_2 < c_2\).

The second part of the proposition follows directly from comparing the quantities \(Q_n(w^*_2 + c_1) = G^{-1}(1 - (w^*_2 + c_1)/p)\) and \(Q^c(\tau_2) = G^{-1}(1 - (k^c_2 + c_1)/p)\).

**Proof of Proposition 8:** As shown in the proof of Proposition 1, M’s optimal profit under the complex contract, which we denote as \(\Pi^\text{complex}_M(\overline{w})\), is the solution to the following problem:

\[
\max_{Q(c_1, c_2)} E_{c_2} \left[ pE_D[\min(D, \max(Q(c_1, c_2), Q_n(\overline{w})))] - \overline{w}(Q_n(\overline{w}) - Q(c_1, c_2)) + (k_1(c_1) + k_2(c_2))Q(c_1, c_2) \right]
\]

s.t. \(Q(c_1, c_2)\) is decreasing in \(c_2\)

We use the envelope theorem to derive the derivative of \(\Pi^\text{complex}_M(\overline{w})\) with respect to \(\overline{w}\). Notice that the constraint does not depend on \(\overline{w}\). Hence a straightforward application of the theorem and using the optimal variables \(Q^c(c_1, c_2)\), as shown in Corollary 1, results in

\[
\frac{d\Pi^\text{complex}_M(\overline{w})}{d\overline{w}} = \int_{\overline{w}}^{\epsilon_2} 0 + \int_{\epsilon_2}^{\tau_2} - (Q_n(\overline{w}) - Q^c(c_1, c_2)) dF^*_2(c_2) = -(1 - F^*_2(c_1)) Q_n(\overline{w}).
\]
As for the price-only contract, we use the problem formulation $P4'$, and define the optimal profit

$$\Pi_{price-only}^{M}(w).$$

For our purposes, we can ignore the dependence of the constraint $w \leq \min(\tilde{c}_2, \bar{w} - c_1)$ on $\bar{w}$, since by the proof of Proposition 7, $\bar{w} - c_1$ is a weakly dominated solution. Hence, using the envelope theorem again, we find

$$\frac{d\Pi_{price-only}^{M}(w)}{dw} = - (1 - F_2(w^c_2)) Q_n(\bar{w}).$$

To complete the proof, observe that

$$\frac{d\Pi_{M}^{complex}(\bar{w})}{d\bar{w}} - \frac{d\Pi_{M}^{price-only}(\bar{w})}{d\bar{w}} = Q_n(\bar{w}) (F_2(c_1) - F_2(w^c_2)) \geq 0,$$

where the last inequality follows from the fact that $w^c_2 \leq c_1$ by Proposition 7. $\blacksquare$