A SIMPLE ADAPTIVE INTERPOLATION APPROACH BASED ON VARYING IMAGE LOCAL ACTIVITY LEVELS

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In this paper, a simple adaptive interpolation approach is proposed for image interpolation. It depends on modifying the warped distance technique for image interpolation considering the local activity levels in each region of the image. This is performed by weighting the pixels used in the interpolation process with different adaptive weights. The adaptation can be extended to different traditional interpolation techniques such as bilinear, bicubic and cubic spline techniques as well as the warped distance technique. Our study shows that the adaptive weighting of pixels in interpolation yields better results than that obtained using only traditional interpolation methods or by using the warped distance technique. The computation cost of the suggested interpolation approach is moderate and acceptable.

Keywords: Image processing; image interpolation; bilinear interpolation; bicubic interpolation and cubic spline interpolation.
1. Introduction

Image interpolation has a wide range of applications in image processing systems. It allows the user to change the size of images interactively, to concentrate on some details or to get a better overview. The importance of image interpolation ranges from medical imaging to military applications or to consumer electronics [Thevenaz, Blu & Unser, 2000; Carey, Chuang & Hemami, 1999; Ramponi, 1999].

In an idealized world of linear and stationary systems, an optimal approach for interpolation exists. This approach uses the popular sinc kernel as an interpolating function. However, the sinc kernel decays too slowly at infinity and it is difficult to realize this function physically. Hence different approximations such as the bilinear, bicubic and cubic spline operators have been proposed [Ramponi, 1999; Han & Kim, 2001; Hou & Andrews, 1978] to solve this problem. These techniques are counteracted by the fact that real data cannot be accurately modeled by a linear and stationary space invariant system [Thevenaz, Blu & Unser, 2000; Carey, Chuang & Hemami, 1999; Ramponi, 1999].

Recently, a linear space variant approach was proposed for image interpolation [Ramponi, 1999]. This approach is based on the evaluation of a “warped distance” between the pixel to be interpolated and each of its neighbors. The warping process is performed by moving the estimate of the pixel towards the more homogeneous neighboring side.

Recently [El-Khamy et al., 2005] presented a different technique for the adaptive acquisition of high resolution images from available low resolution images based on a least squares block by block approach. This algorithm is based on the segmentation of the image to overlapping blocks and the interpolation of each block separately. The obtained results using this algorithm were compared to the results obtained through using both the cubic O-MOMS and its adaptive warped distance variant [Ramponi, 1999], and it showed better performance.

In this paper, we propose a new adaptive approach for image interpolation. This approach depends on weighting the pixels to be included in the interpolation process in a space variant manner. This weighting approach is used both with traditional approximations as well as with the warped distance technique.

The paper is organized as follows. Section 2 describes the traditional techniques used in linear space invariant image interpolation. Section 3 discusses the warped distance image interpolation technique and how to implement it. In Sec. 4, the suggested adaptive image interpolation model is introduced. Section 5 discusses the error analysis for edge interpolation with the suggested adaptive scheme. The results of the traditional image interpolation and the suggested image interpolation schemes are introduced in Sec. 6. Section 7 concludes the paper.

2. Linear Space Invariant Image Interpolation

The process of image interpolation aims at estimating intermediate pixels between the known pixel values as shown in Fig. 1. To estimate the intermediate pixel at position \( x \), the neighboring pixels and the distance \( s \) are incorporated into the estimation process.

For equally spaced 1D sampled data, \( f(x_k) \), many interpolation functions can be used. The value to be interpolated, \( \hat{f}(x) \), can, in general, be written in the form [Hou & Andrews, 1978; Unser, 1999]:

\[
\hat{f}(x) = \sum_{k=-\infty}^{\infty} c_k \beta(x - x_k),
\]

where \( \hat{f}(x) \) is the corresponding interpolated function, \( \beta(x) \) is the interpolation kernel, and \( x \) and \( x_k \) represent continuous and discrete spatial distance, respectively.

From the classical Sampling theory, if \( f(x) \) is band limited to \((-\pi, \pi)\), then [Hou & Andrews, 1978; Unser, 1999]:

\[
\hat{f}(x) = \sum_{k} f(x_k) \text{sinc}(x - x_k).
\]

This is known as the ideal interpolation. From numerical computations point of view, the ideal interpolation formula is not practical due to the slow rate of decay of the interpolation kernel.
sinc(\(x\)). So, approximations such as the bilinear, bicubic and cubic spline interpolation techniques are used as alternatives [Hou & Andrews, 1978; Unser, 1999].

As shown in Fig. 1, we define the distance between, \(x_k\) and \(x_{k+1}\) as [Ramponi, 1999; Han & Kim, 2001]:

\[
s = x - x_k, \quad 1 - s = x_{k+1} - x.
\]

For the bilinear, Bicubic and Cubic spline image interpolation algorithms we have [Ramponi, 1999; Han & Kim, 2001]:

(i) **Bilinear**

\[
\hat{f}(x) = (1 - s)f(x_k) + sf(x_{k+1}).
\]

(ii) **Bicubic**

\[
\hat{f}(x) = \hat{f}(x_{k-1})(-s^3 + 2s^2 - s)/2 \\
+ f(x_k)(3s^3 - 5s^2 + 2)/2 \\
+ f(x_{k+1})(-s^3 + 4s^2 + s)/2 \\
+ f(x_{k+2})(s^3 - s^2)/2.
\]

(iii) **Cubic Spline**

\[
\hat{f}(x) = f(x_{k-1}) \\
\times [(3 + s)^3 - 4(2 + s)^3 + 6(1 + s)^3 - 4s^3]/6 \\
+ f(x_k)[(2 + s)^3 - 4(1 + s)^3 + 6s^3]/6 \\
+ f(x_{k+1})[(1 + s)^3 - 4s^3]/6 + f(x_{k+2})s^3/6.
\]

A fast implementation of the cubic spline image interpolation process can be performed using a digital filtering approach to speed up the computation but at the price of a large amount of blurring in the resulting image [Hou & Andrews, 1978]. For 2D image interpolation, these techniques are applied along rows and then along columns [Ramponi, 1999].

### 3. Warped Distance Image Interpolation

The idea of warped distance can be used in any of the three techniques which are mentioned previously in Sec. 2 to improve its performance. This idea is based on modifying the distance \(s\) and using a new distance \(s'\) based on the homogeneity or inhomogeneity in the neighborhood of each estimated pixel. The warped distance \(s'\) can be estimated using the following relation [Ramponi, 1999]:

\[
s' = s - \tau A_k s(s - 1),
\]

where \(A_k\) refers to the asymmetry of the data in the neighborhood of \(x\) and it is defined as:

\[
A_k = \frac{|f(x_{k+1}) - f(x_{k-1})| - |f(x_{k+2}) - f(x_k)|}{L - 1},
\]

where \(L = 256\) for 8 bit pixels. The scaling factor \(L\) is to keep \(A_k\) in the range of \([-1, 1]\).

The parameter \(\tau\) controls the intensity of warping. It has a positive value and may be equal to 1 or 2. The desired effect of this warping is to avoid blurring of the edges in the interpolation process.

### 4. Adaptive Image Interpolation

Instead of using the traditional image interpolation techniques mentioned in Sec. 2, a new approach is suggested in this section. This approach depends on weighting the values of the pixels incorporated into the image interpolation by space variant adaptive weights. The distance \(s\) is kept fixed.

For adaptive bilinear image interpolation, Eq. (3) may be developed as follows:

\[
\hat{f}(x) = (1 - s)a_0 f(x_k) + sa_1 f(x_{k+1}),
\]

where

\[
a_0 = 1 - \lambda A_k, \quad a_1 = 1 + \lambda A_k.
\]

The adaptive bicubic image interpolation formula can also be modified to:

\[
\hat{f}(x) = a_{-1} f(x_{k-1})(-s^3 + 2s^2 - s)/2 \\
+ a_0 f(x_k)(3s^3 - 5s^2 + 2)/2 \\
+ a_1 f(x_{k+1})(-s^3 + 4s^2 + s)/2 \\
+ a_2 f(x_{k+2})(s^3 - s^2)/2.
\]
Also, the adaptive cubic spline image interpolation formula can be written in the form:

\[
\hat{f}(x) = a_{-1}f(x_{k-1}) + a_0f(x_k) + a_1f(x_{k+1}) + a_2f(x_{k+2})
\]

where in (10) and (11),

\[
a_{-1} = a_0 = 1 - \lambda A_k, \quad a_1 = a_2 = 1 + \lambda A_k,
\]

and \(\lambda\) is a constant in the range of 1 to 5. The constant \(\lambda\) controls the intensity of weighting used for neighboring pixels. Thus the weighting coefficients are updated at each pixel depending on the asymmetry \(A_k\) at this pixel. We note the following special cases:

(i) It is noted that for homogeneous regions, the value of \(A_k\) tends to zero which leads to \(a_{-1} = a_0 = a_1 = a_2 = 1\). This is equivalent to the traditional image interpolation process.

(ii) For positive values of \(A_k\), which means that there is an edge that is more homogeneous on the right side, the weights of the pixels on the right side \((a_1\) and \(a_2)\) are increased and the weights of the pixels on the left hand side \((a_{-1}\) and \(a_0)\) are decreased. This is expected to yield images with better visual quality.

(iii) For negative values of \(A_k\), \(a_{-1}\) and \(a_0\) are increased and \(a_1\) and \(a_2\) are decreased and the same effect is obtained.

The proposed adaptive weighting technique can also be applied to warped distance image interpolation in all the methods mentioned in Sec. 2 and better results are expected.

5. Error Analysis for Edge Interpolation

One of the best models for edges is the sigmoidal model. In this model the image data is assumed to satisfy the following equation [Ramponi, 1999]:

\[
f(x) = \frac{1}{(1 + e^{-C(x-x_0)})}.
\]

The edge location is at \(x = 0\). The steepness of the edge is controlled by the constant \(C\) and its amplitude is normalized to 1. The constant \(C\) ranges from 1 (for an edge of large width) to 5 for an edge of small width. We define the distances between the position \(x\) and its nearest neighbors \(x_0\) and \(x_1\) as:

\[
s = x - x_0 \quad \text{and} \quad 1 - s = x_1 - x.
\]

This formula is expected to yield less error at the interpolated edges.

If we estimate the function \(f(x)\) at position \(x\) using the bilinear interpolator, the result, \(\hat{f}_L(x)\), will be given by (3) with \(k = 0\). The corresponding estimation error will thus be given by the following equation [Ramponi, 1999]:

\[
\varepsilon_L(x, s, C) = f(x) - \hat{f}_L(x)
\]

\[
= \frac{1}{1 + e^{-C(x-x_0)}} - \frac{1 - s}{1 + e^{-C(x-s)}} - \frac{a_0(1 - s)}{1 + e^{-C(x-s+1)}}.
\]

Now, if the idea of adaptive weighting of the pixels is incorporated into the interpolation process, the error formula will be modified to:

\[
\varepsilon_L(x, s, C) = f(x) - \hat{f}_L(x)
\]

\[
= \frac{1}{1 + e^{-C(x-x_0)}} - \frac{a_0(1 - s)}{1 + e^{-C(x-s)}} - \frac{a_1 s}{1 + e^{-C(x-s+1)}}.
\]

This formula is expected to yield less error at the interpolated edges.

The mean square error between the original image and the interpolated image can be calculated using the following formula:

\[
\text{MSE} = \frac{1}{MN} \sum_{m=1}^{M} \sum_{n=1}^{N} [f(m, n) - \hat{f}(m, n)]^2,
\]

where \(f(m, n)\) is the original image and \(\hat{f}(m, n)\) is the interpolated image. \(M\) and \(N\) are the dimensions of the interpolated image.
6. Simulation Results

In this section, the traditional image interpolation methods (bilinear, bicubic and cubic spline interpolation methods) are tested on a down sampled Mandrill image. The Mandrill image is chosen because of the large number of edges existing in this image. The low-resolution image is obtained by down sampling the Mandrill image by 2 in horizontal and vertical directions to yield a $240 \times 240$ low-resolution image. This image is then interpolated to its original resolution of $480 \times 480$. The mean square error (MSE) between the original image and the interpolated images is then estimated. The proposed adaptive image interpolation method is tested on the same image for both bilinear, bicubic and cubic spline methods. The suggested adaptive technique is also tested with the warped distance technique in all the methods mentioned. The MSE and computation time results are tabulated in Table 1. The results show that the proposed adaptive weighted image interpolation yields better results than the traditional interpolation techniques or warped distance interpolation techniques. Also, the adaptive weighted image interpolation with warping gives better results than those of adaptive weighted interpolation only. The computation cost of the suggested interpolation approach is moderate and acceptable.

Figure 2 illustrates the original Mandrill image and Fig. 3 illustrates the down sampled low-resolution image. The details of the image are blurred in Fig. 3. Figures 4 to 7 illustrate the results of image interpolation using the bicubic method with standard interpolation, warped distance interpolation, adaptive weighted interpolation and adaptive weighted warped distance interpolation respectively. It is clear that the adaptive weighted technique yields the best results from the MSE point of view.

Figures 8 and 9 illustrate the squared error at each pixel for the traditional bicubic interpolation technique and the weighted warped distance bicubic interpolation technique. It is clear that in Fig. 8, some pixels have larger squared error values than in Fig. 9.

![Fig. 2. Original Mandrill image.](image1)

![Fig. 3. Low resolution Mandrill image.](image2)

<table>
<thead>
<tr>
<th>Interpolation</th>
<th>Warped distance interpolation</th>
<th>Adaptive weighted interpolation</th>
<th>Adaptive weighted interpolation with warping</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bilinear</td>
<td>MSE = 845.09</td>
<td>MSE = 830.72</td>
<td>MSE = 808.26</td>
</tr>
<tr>
<td></td>
<td>CPU = 3.87 s</td>
<td>CPU = 5.02 s</td>
<td>CPU = 6.15 s</td>
</tr>
<tr>
<td>Bicubic</td>
<td>MSE = 826.7</td>
<td>MSE = 807.4</td>
<td>MSE = 791</td>
</tr>
<tr>
<td></td>
<td>CPU = 5.27 s</td>
<td>CPU = 6.05 s</td>
<td>CPU = 7.1 s</td>
</tr>
<tr>
<td>B-spline</td>
<td>MSE = 855.5</td>
<td>MSE = 846.23</td>
<td>MSE = 819.96</td>
</tr>
<tr>
<td></td>
<td>CPU = 11.64 s</td>
<td>CPU = 12.27 s</td>
<td>CPU = 13.01 s</td>
</tr>
</tbody>
</table>

Table 1. Interpolation results of the low resolution Mandrill image.
Fig. 4. Bicubic Interpolation (no warping). MSE = 826.7.

Fig. 5. Bicubic interpolation with warping. MSE = 807.4.

Fig. 6. Adaptive Image Interpolation (no warping) MSE = 791.

Fig. 7. Adaptive image interpolation with warping. MSE = 786.4.

Fig. 8. Squared error for bicubic image interpolation.

Fig. 9. Squared error for adaptive bicubic image interpolation.
Table 2. Interpolation results of the low-resolution Cameraman image.

<table>
<thead>
<tr>
<th>Interpolation with no warping</th>
<th>Warped distance interpolation</th>
<th>Adaptive weighted interpolation</th>
<th>Adaptive weighted interpolation with warping</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bilinear</strong></td>
<td><strong>MSE = 249.1</strong></td>
<td><strong>MSE = 244.34</strong></td>
<td><strong>MSE = 235.44</strong></td>
</tr>
<tr>
<td><strong>CPU = 1.1 s</strong></td>
<td><strong>CPU = 1.48 s</strong></td>
<td><strong>CPU = 1.75 s</strong></td>
<td><strong>CPU = 1.97 s</strong></td>
</tr>
<tr>
<td><strong>Bicubic</strong></td>
<td><strong>MSE = 240.4</strong></td>
<td><strong>MSE = 234.08</strong></td>
<td><strong>MSE = 225.7</strong></td>
</tr>
<tr>
<td><strong>CPU = 1.5 s</strong></td>
<td><strong>CPU = 1.72 s</strong></td>
<td><strong>CPU = 2.02 s</strong></td>
<td><strong>CPU = 2.27 s</strong></td>
</tr>
<tr>
<td><strong>B-spline</strong></td>
<td><strong>MSE = 254.09</strong></td>
<td><strong>MSE = 250.7</strong></td>
<td><strong>MSE = 240.76</strong></td>
</tr>
<tr>
<td><strong>CPU = 3.31 s</strong></td>
<td><strong>CPU = 3.49 s</strong></td>
<td><strong>CPU = 3.7 s</strong></td>
<td><strong>CPU = 3.78 s</strong></td>
</tr>
</tbody>
</table>

The same interpolation experiments have been made on the Cameraman image as it contains lower number of edges and large flat areas and the results are tabulated in Table 2.

7. Conclusion

In this paper an adaptive technique is suggested for image interpolation using the bilinear, bicubic and the cubic spline methods by adaptively weighting the pixels incorporated into the interpolation process. This adaptive technique is compared to the traditional techniques of interpolation and the warped distance technique of interpolation. Results show that the adaptive technique is superior to both the traditional technique and the warped distance technique for both bilinear, bicubic and cubic spline methods. The suggested adaptive technique is also tested with the warped distance technique for image interpolation and better results are obtained. These obtained better results are due to the ability of the proposed technique to preserve the details of the image. The computation cost of the suggested interpolation approach is moderate and acceptable.

References


Biography

**Said E. El-Khamy** received the PhD degree from the University of Massachusetts, USA in 1971. He is a professor and past chairman of the Department of Electrical Engineering, Alexandria University, Egypt. His current research areas of interest include modern signal processing and their applications in image processing and communication systems. He has published about two hundreds scientific papers and has earned many national and international research awards among which are the IEEE, R.W.P. King best paper award in 1980, and the Egypt’s State Appreciation award (highest) in Engineering Sciences for 2004. He is a Fellow of the IEEE since 1999.

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