Energy-Aware Fair Routing in Wireless Sensor Networks with Maximum Data Collection

Ka-Lok Hung, Brahim Bensaou and Junhua Zhu
Department of Computer Science
The Hong Kong University of Science and Technology
{ether, brahim, csjhzhu}@cs.ust.hk

Farid Naït-Abdesselam
IRCiCA/LIFL – INRIA POPS
Universite of Lille 1, France
nait@lifl.fr

Abstract—This paper considers the problem of routing in sensor networks from the point of view of data collection. That is, given the initial amount of battery energy in each node, the aim is to determine how much data can each source transmit until the network is partitioned (i.e., until the nodes cannot find end-to-end routes to their respective sinks). In addition, to respond to some specific applications’ requirements, when determining such nodal data volume distribution, fairness among nodes is taken into account. The problem is formulated as a concave utility maximization and a sub-gradient algorithm is proposed to solve it distributively. Some numerical results are given and the convergence of the algorithm is discussed.

I. INTRODUCTION

Recent cost reductions in micro-sensor technology and advances in wireless networks have spun off a range of new applications where, typically a large number of unattended sensors, deployed in an ad hoc fashion, collaborate through multi-hop wireless communication channels to perform some common tasks such as, object tracking, environmental monitoring, and so on. Since such sensors usually have a low battery capacity and since in many applications battery replacement may not be possible, energy is one of the most scarce resources in most sensor networks applications. Therefore, when designing communication protocols for wireless sensor networks, energy conservation must be the primary concern as wireless communication is known to consume most of the energy in the network.

In this paper, we consider the problem of energy-aware routing in wireless sensor networks. We study the problem of energy-efficient routing in sensor networks with the objective of maximizing the volume of data collected by the sinks which are assumed to be endowed with unlimited power sources. Due to the short transmission range of sensor nodes, multi-hop communication is used when forwarding the data from a source to its corresponding sink. The objective of the routing algorithm is to compute the flow value of each link such that the amount of data collected from different nodes is maximized. Furthermore, instead of simply maximizing the total data flow, we focus on maximizing the data collected by each sensor node fairly. To illustrate the importance of such fairness, consider the network shown in Fig. 1.

In the figure, there are six source-sink pairs, (1,7), (3,7), (5,7), (2,8), (4,8) and (6,8). That is, nodes 1, 3 and 5 are sources associated with sink 7 and nodes 2, 4 and 6 are sources associated with sink 8. For simplicity in this example, we assume that the energy consumed by sending and receiving data are fixed and are the same for all nodes and that all nodes have the same initial battery energy. If the routing algorithm only maximizes the data collected without considering fairness, then the optimal solution would be the one where nodes 5 and 6 use up all their battery energy to send their own traffic to the sinks without routing any other data unit on behalf of other nodes. Other nodes are not able to reach the sinks. This is mainly due to the fact that nodes 5 and 6 are energy-bottleneck nodes on all routes leading to the sinks, and that each unit of data generated by nodes 5 and 6 costs less to carry end-to-end in terms of energy than any other nodes’ data. This simple illustration shows that fairness is of utmost importance and must be taken into account if the routing protocol is to ensure that each node is to send a reasonably large amount of data to the sinks. How much data each node is to send depends mostly on the application. For example, in an environmental monitoring system where the data from different sensors are of equal importance, such fairness is indeed important and the max-min principle is more appropriate as it maximizes the amount of data that can be sent by the most constrained node. On the other hand, in an enemy troops movement monitoring system, it may be preferable to be able to monitor such movements with a continuingly increasing accuracy as they get closer and closer. Enemy troops movements that take place far away, from say a strategic position, can be monitored.
less frequently than those that take place at a closer range. In this situation, nodes which are close to the sinks should be allowed to send more traffic than those that are far away. Proportional fairness can be a more appropriate principle for such application, as it penalizes the nodes based on their global use of resources rather than their use of resources on the bottleneck nodes. To encompass this flexibility, in our work, we associate each link with a utility function that depends on the amount of data which originates at the ingress node of the link, and formulate the maximum data collection problem as a convex optimization problem subject to energy constraint and data flow conservation constraints. We then propose a sub-gradient algorithm to solve the problem iteratively.

The rest of the paper is organized as follows. Section II briefly reviews related work on energy aware routing and contrasts our work with it. Section III describes the system model and the problem formulation. Section IV presents a sub-gradient routing algorithm that solves the optimization problem distributively. Section V shows some numerical results. Section VI concludes the paper.

II. RELATED WORK

A number of energy aware routing protocols for wireless sensor networks have been studied in recent years. In the early ad-hoc routing literature, a routing algorithm for minimizing energy consumption was proposed in [2]. The disadvantage of this algorithm is that the energy of the nodes along the smallest energy consumption path drain out very quickly. Instead of minimizing the energy consumption, maximum lifetime routing is proposed in [3], [4]. The lifetime of the network is defined as the time until the first node runs out of energy. The sensors are assumed to generate traffic periodically and the generation rate is known. A linear programming formulation is presented and a heuristic algorithm is proposed to solve the problem approximately. In [5], an iterative max-flow algorithm is presented to address a similar linear programming problem. In [6], the problem is treated as a maximum concurrent flow problem and a distributed routing algorithm is presented to maximize the network lifetime. In [9], a similar maximum lifetime routing problem is defined and a sub-gradient algorithm is proposed.

In some applications, the traffic generation rate may be variable and unpredictable. Online routing algorithms [7], [8] are proposed to address the variable source rates issue. In [7], a max-min $z P_{min}$ algorithm is presented. A message is routed through a path which maximizes the minimum residual energy fraction with the constraint that this path consumes at most $z$ times more energy than the minimum energy consumption path. In [8], a shortest path routing algorithm is proposed. In this formulation, the link cost is based on the cost of transmission and the residual energy fraction of the sender. A logarithmic competitive ratio is achieved if admission control of messages is allowed. Both of the proposed algorithms do not include the energy cost of receiving data which in real world applications is known to be in the same order of magnitude as the cost of sending data.

In [10], the maximum data extraction problem is presented and a sub-gradient algorithm to solve the problem is proposed. This formulation is the most closely related to our work. However, there are two significant differences between our work and that of [10]. i) A linear programming formulation is used for maximizing the total amount of data extracted and thus the fairness issue among different nodes is not considered; ii) The work in [10] considers only one sink in the network whereas in our work, the system model is extended to include multiple possible sinks.

III. MAXIMIZING DATA COLLECTION

A. System Model

The Wireless Sensor Networks are modelled as a directed graph $G(N, L)$ where $N$ is the set of all nodes in the network and $L$ is the set of all directed links $(i, j)$ and $i, j \in N$. Let $out(i)$ be the set of egress nodes of node $i$, that is all nodes that can be reached by node $i$ and $in(i)$ be the set of ingress nodes of node $i$, that is, all the nodes that can reach node $i$. The network topology can be represented by a node-edge incidence matrix $A \in \mathbb{R}^{N \times |L|}$ where each element $a_{il}$ is defined as

$$ a_{il} = \begin{cases} 1, & \text{if link } l \text{ is an outgoing link of node } i \\ -1, & \text{if link } l \text{ is an incoming link of node } i \\ 0, & \text{otherwise}. \end{cases} $$

Let $e_{ij}^x$ be the energy cost for node $i$ to send one byte (one unit of data) to node $j$ and $e_{ij}^r$ be the energy cost for node $i$ to receive one byte from node $j$. Denote by $E_i$ the initial amount of battery energy contained in node $i$.

Let $T$ be the set of sink nodes $T \subset N$. We assume that the sink nodes are not power limited, they do not generate data. Let $S$ be the set of sensor nodes, $S = N \setminus T$. For each sensor node in the $S$ there is exactly one corresponding sink in $T$. For each sink $t \in T$ denote by $S(t)$ the set of sensors associated with sink $t$. Let $x_{ij}^k$ be the flow generated by source $k$, forwarded from node $i$ to node $j$. The total amount of traffic on link $(i, j)$ is thus $x_{ij} = \sum_{k \in S} x_{ij}^k$. The traffic generated by a source $k$ is called commodity $k$, and the flow vector of commodity $k$ is denoted $x^k = \{x_{ij}^k\}$ and the flow vector of all commodities is denoted $x = \{x_{ij}\}$.

B. Problem Formulation

A multi-commodity flow formulation is invoked to model the maximization of data collection problem with fairness. The problem can be written as a convex programming problem where the objective is to maximize the strictly increasing concave utility function for each commodity among different sources’ outgoing links. There are several problem constraints. The first set of constraint is the energy constraints: The energy consumption for transmitting and receiving data for each sensor node $i$ is limited by its initial energy, i.e.,

$$ \sum_{j \in out(i)} e_{ij}^t x_{ij}^k + \sum_{j \in in(i)} e_{ij}^r \sum_{k \in S} x_{ji}^k \leq E_i. \quad (1) $$

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Define an energy consumption matrix $P \in \mathbb{R}^{\|N\| \times \|L\|}$ whose elements are:

$$p_{il} = \begin{cases} e_{ij}^l, & \text{if link } l = (i,j) \text{ exists} \\ e_{ji}^l, & \text{if link } l = (j,i) \text{ exists} \\ 0, & \text{otherwise}; \end{cases}$$

and let $E = \{E_i\}$ be the initial energy vector, the energy constraints for all the nodes (excluding the sinks) can be written in a compact form as $Px \leq E$

The second set of constraints stems from flow conservation considerations. That is, firstly, no source should accept to route its own traffic on behalf of other sources:

$$- \sum_{j \in \text{in}(i)} x_{ji}^i = 0, \quad \forall i \in N.$$  \hspace{1cm} (3)

Secondly, flow conservation holds at each node, for each commodity $i$:

$$\sum_{j \in \text{out}(k)} x_{kj}^i - \sum_{j \in \text{in}(k)} x_{jk}^i = 0 \quad \forall k \in S.$$  \hspace{1cm} (4)

Finally, sinks accept all of their associated sources traffic and nothing else:

$$- \sum_{j \in \text{out}(k)} x_{jk}^i = 0 \quad \forall k \in T, \quad i \notin S(k),$$

$$- \sum_{j \in \text{in}(k)} x_{ij}^i = - \sum_{j \in \text{out}(i)} x_{ij}^i \quad \forall k \in T, \quad i \in S(k).$$  \hspace{1cm} (5)

By defining a modified node-edge incidence matrix $A^i = \{a_{xl}^i\}$ for each commodity $i$ as:

$$a_{xl}^i = \begin{cases} 0, & \text{if } x = i, \quad a_{xl} = 1 \\ a_{xl}, & \text{otherwise}, \end{cases}$$

and flow vector $s^i = \{s_k^i\}$ as:

$$s_k^i = \begin{cases} - \sum_{j \in \text{out}(i)} x_{ij}^i, & \text{if } k \in T, \quad i \in S(k) \\ 0, & \text{otherwise}, \end{cases}$$  \hspace{1cm} (7)

we can these flow constraints compactly as

$$A^i x^i = s^i.$$  \hspace{1cm} (8)

Finally, the last constraint imposes that commodity flow vectors are non-negative: $x^k \succeq 0$

To ensure a certain level of fairness among flows generated at the sources, as well as among first hop neighbors of the sources, each first hop link is associated with a concave non decreasing utility function $U_{ij}()$. The choice of utility function is mainly directed by the fairness principle to uphold, different utility functions lead to different fairness principles [11]. In summary, the energy-aware maximization of data collection can be be formulated as

$$\max_{x} \sum_{i \in S} \sum_{j \in \text{out}(i)} w_{ij} U_{ij}(x_{ij}^i)$$

$$s.t. \quad \begin{array}{l}
Px \leq E \\
A^i x^i = s^i \quad i \in S \\
x^i \succeq 0 \quad i \in S,
\end{array}$$

where $w_{ij}$ is a positive constant. If a common utility function is used for all the flows, the weights $w_{ij}$ can be treated as sensor $i$’s willingness to pay for its own data flow that passes through link $(i,j)$. The flow $x_{ij}^i$ with a larger value of $w_{ij}$ is favored [16]. This weight could also be used to further differentiate routes according to another criterion than energy (e.g., shortest path).

IV. SUB-GRADIENT ROUTING ALGORITHM

A. The Dual Problem

To solve the above optimization problem in a distributed way, we consider the Lagrange relaxation and the dual problem. We relax the energy constraints and maintain the flow constraints.

Let the set $\chi$ be the flows that satisfy the flow constraints where

$$\chi = \{x^i \in \mathbb{R}^{\|L\|} | A^i x^i = s^i \} \times \{x^2 \in \mathbb{R}^{\|L\|} | A^2 x^2 = s^2 \} \ldots \times \{x^{|S|} \in \mathbb{R}^{\|L\|} | A^{|S|} x^{|S|} = s^{|S|} \}.$$  \hspace{1cm} (10)

The partial Lagrangian $\mathcal{L}(x,p)$ is obtained by introducing Lagrange multiplier $p \in \mathbb{R}^{|S|}$ for the energy constraints, that is:

$$\mathcal{L}(x,p) = \sum_{i \in S} \sum_{j \in \text{out}(i)} w_{ij} U_{ij}(x_{ij}^i)$$

$$+ \sum_{i \in S} p_i E_i - \sum_{j \in \text{out}(i)} e_{ij}^k \sum_{k \in S} x_{ij}^k$$

$$- \sum_{j \in \text{in}(i)} e_{ji}^k \sum_{k \in S} x_{ji}^k.$$  \hspace{1cm} (11)

The dual function is defined as

$$D(p) = \max_{x \in \chi} \mathcal{L}(x,p),$$

and the dual problem of the primal is

$$\min \quad D(p)$$

$$s.t. \quad p \geq 0.$$  \hspace{1cm} (12) (13) (14)

We can easily show that Slater’s condition for the constraints qualification holds: that is, there exists a feasible solution $x, p$ such that the strict inequalities for the energy constraints hold. From this, and the convex nature of the optimization problem, the strong duality holds [14], and therefore, the optimal solution of the dual is also the optimal solution of the primal.

B. Sub-gradient Algorithm

The objective function in (9) is not strictly concave in all the variables $x_{ij}^k$. Notice that the objective function is only defined for the $x_{ij}^k$. The dual function is thus non-differentiable in some points. Instead of using a traditional gradient descent method such as in [16], a sub-gradient method [15] can be applied. The sub-gradient at $x \in \mathbb{R}^n$ of a non-differentiable convex function $f$ is a vector $d \in \mathbb{R}^n$ such that:

$$f(y) \geq f(x) + (y - x)^T d,$$  \hspace{1cm} (15)
for all \( y \in \mathbb{R}^n \). If the convex function is differentiable, the only choice of sub-gradient is the gradient of the function.

**Theorem 1:** Suppose \( x(p) \) is the maximizer of problem (12). The sub-gradient \( g \) corresponding to \( p \) is

\[
E - Px(p)
\]

**Proof:** For any point \( \tilde{p} \),

\[
D(\tilde{p}) = \max_{x \in x} \sum_{i \in S} \sum_{j \in \text{out}(i)} w_{ij} U_{ij}(x_{ij}^i) \\
+ \sum_{j \in \text{in}(i)} p_j \{ E_i - \sum_{e \in \text{in}(i)} e_{ij}^e \sum_{k \in S} x_{ij}^k \} \\
- \sum_{j \in \text{in}(i)} e_{ji}^e \sum_{k \in S} x_{ji}^k \}
\]

(17)

\[
= D(p) + \sum_{j \in \text{out}(i)} (\tilde{p}_j - p_j) \{ E_i - \sum_{e \in \text{in}(i)} e_{ij}^e \sum_{k \in S} x_{ij}^k \} \\
- \sum_{j \in \text{in}(i)} e_{ji}^e \sum_{k \in S} x_{ji}^k \}
\]

\[
\Rightarrow D(\tilde{p}) \geq D(p) + (\tilde{p} - p)^T (E - Px(p)).
\]

This proves the theorem.

In the sub-gradient method which is an iterative method, at each iteration step \( t = 1, 2, 3, \ldots \), we find the maximizer of problem (12) and evaluate the sub-gradient \( g = \{ g_i \} \), then the dual variable \( p \) is updated as follow:

\[
p_i(t + 1) = (p_i(t) - \gamma(t) g_i(t))^+
\]

(18)

where \( \gamma(t) \) is a positive scalar step size and the function \((\cdot)^+\) returns the maximum between its argument and 0. If \( \gamma(t) \) is a constant, it can be shown that if an optimal \( \tilde{p} \) exists, the sub-gradient method converges statistically to within \( \gamma G^2/2 \) of this optimal value where \( G \) is such that \(|g_i(t)| \leq G \) for all \( t \). The complete proof can be found in [12]. Alternatively, \( \gamma(t) \) can be obtained by using a diminishing update rule,

\[
\gamma(t) = \frac{\gamma(0)}{\sqrt{t}}
\]

(19)

where \( \gamma(0) \) is a fixed constants. In this case, the sub-gradient algorithm also converges to the optimum [15].

The remaining question is how can we find the optimal flow \( \bar{x} \) given the price vector \( p \).

If we assume the communication links to be bi-directional, which is the case in most practical cases as the medium access control protocol is often based on CSMA/CA with acknowledgements, that is \( \text{out}(i) = \text{in}(i) \). (Note that although the sinks are supposed to have outgoing links they never generate outgoing traffic.) We can rewrite problem (12) as

\[
D(p) = \max_{x \in x} \sum_{i \in S} \sum_{j \in \text{out}(i)} w_{ij} U_{ij}(x_{ij}^i) + \sum_{t \in S} p_t \{ E_t - \sum_{e \in \text{out}(t)} (e_{tv}^e \sum_{i \in S} x_{tv}^i + e_{ut}^e \sum_{i \in S} x_{ut}^i) \}
\]

(20)

with the cost being \( \sum_{i \in \text{out}(t)} (p_t e_{tv}^e + p_u e_{ut}^e) x_{tv}^i \) under the flow conservation constraint for commodity \( i \). Thus source \( i \) has to find a least cost path for each outgoing link to send out its own traffic so that the total cost is minimized. Therefore, the above problem can be decomposed into a least cost path problem for each source. The cost of each link \((i,j)\) is \( p_i e_{ij}^e + p_j e_{ji}^e \). We define \( SP(i,j) \) to be the least cost path for node \( i \) to send data via node \( j \). If there is no valid path to the sink, \( SP(i,j) = \emptyset \). Since \( U_{ij} \) is a strictly increasing concave function, the optimal value of \( x_{ij}^i \) can be obtained by

\[
\bar{x}_{ij}^i = U_{ij}^{-1}\left( \sum_{(u,i) \in SP(i,j)} \frac{p_u e_{iu}^s + p_v e_{vu}^r}{w_{ij}} \right).
\]

(21)

The complete algorithm (in its centralized representation) is shown in Algorithm 1, where convergence is reached when the price change is arbitrarily small from one iteration to the next one.

Note that since this is a least cost path algorithm, it can be implemented easily as a fully distributed algorithm. The implementation is out of the scope of this paper and will be reported later in an extended version of this paper.

**V. NUMERICAL RESULTS**

To illustrate numerically the behavior of the sub-gradient algorithm, we first start with the simple example of Fig. 1 where the topology is simple and predictable enough to allow us to study and understand the effect of applying different utility functions and adjusting the weights of the links to the data flow. In addition to this simple topology, we will also use a more realistic random topology to study the scalability of the algorithm.
Algorithm 1 Sub-gradient Routing Algorithm

1: Initial $p_i$ is set for each node $i$.
2: while No convergence do
3:     for each node $i \in S$ do
4:         for each neighbor node $j$ of node $i$ do
5:             Computes the link $(i, j)$ price by $p_i(t)e_{ij}^s + p_j(t)e_{ji}^r$
6:             Solves the least cost path problem of routing the data from node $i$ via link $(i, j)$
7:         Receives the sum of the link prices $\sum_{(u,v) \in SP(i,j)} p_u(t)e_{uv}^s + p_v(t)e_{vu}^r$
8:     Calculates the new flow $\bar{x}_{ij}^t(t)$ by (21)
9:     Communicates new flow value $\bar{x}_{ij}^t(t)$ to all the nodes in $SP(i, j)$
10:    Updates the price $p_i(t+1)$ by (18)
11:    Communicates the new price $p_i(t+1)$ to all the node $j$
12:    Optional step: If the diminishing update rule is used for the step size, updates $\gamma(t+1)$ by (19)
13:    Sets $t = t + 1$ /* next iteration */
14: end for
15: end for
16: end while

The topology in Fig. 1 is assumed to be fixed and the sending and receiving costs are uniform and equal to 2 Joules/byte and 1 Joule/byte respectively. Each source node starts with 10000J initial energy. A constant step size $\gamma$ is used here. The following known [11] concave utility functions are used:

$U_{ij}(x_{ij}) = \begin{cases} 
\log x_{ij}^t, & \text{if } k = 1 \\
(1-k)^{-1}x_{ij}^t, & \text{otherwise}
\end{cases}$  \hspace{1cm} (22)

Fig. 2 shows the individual flow value under different utility functions. We can see that nodes 4 and 6 get the highest flow values in all the cases. This is due to i) both nodes 4 and 6 have two routes to their sinks and ii) node 6 has a direct link to its sink. The figure also shows that larger values of $k$ lead to smaller flow values for bottleneck nodes 5 and 6 and thus more fairness. In addition, nodes 2 and 4 gain a larger portion of energy in the bottleneck nodes. Fig. 3 shows the aggregate flow value under different utility functions. As we can see, the flow value drops when $k$ increases and the algorithm converges within 40 iterations in all cases.

Fig. 4 and Fig. 5 show the impact of changing the weights $w_{ij}$ on the individual flow and the aggregate flow respectively. We compare three weight assignments, i) all the weights are equal to 1; ii) the weights of the direct links from sources to their corresponding sinks are equal to 0.6 and the rest of the links are equal to 1; and, iii) the weights of the direct links from sources to their corresponding sinks are equal to 0.3 and the rest of the links are equal to 1. The figures show that the smaller the weights of the direct links, the smaller the flow value of the bottleneck nodes and the larger the flow value of other nodes (i.e., better fairness). The total flow value decreases when the weights of the direct links decrease.
To illustrate the scalability of the algorithm, we use the topology shown in Fig. 6 which contains 15 nodes randomly distributed over a square area of 50m X 50m. There are 13 sources and 2 sinks. Each source forwards its traffic to the closest sink (in number of hops; ties are broken randomly). Each source has 10000J initial energy and a maximum communication range of 15m. The energy consumption model in [10] is used: that is, sending 1 byte from a node to a node $j$ distant by a distance of $d$ meters costs $e_{ij}^s = c_1 + c_2d^2$; receiving 1 byte costs $e_{ji}^r = c_1$ with $c_1 = 400\text{nJ/byte}$ and $c_2 = 800\text{pJ/byte/m}^2$.

The diminishing update rule for the step size is used here, with $\gamma(0)$ set to $10^{-7}$. The initial prices for all sources are set to 0.0005. Fig. 7 shows the individual flow value and Fig. 8 shows the total flow value. This latter confirms the convergence of the algorithm within a small number of iterations (30 iterations). Finally, by comparing this example with the simple example we notice that the diminishing update rule indeed leads the algorithm to converge faster.

### VI. CONCLUSION

In this paper, we have studied the problem of maximizing data collection with fairness consideration in wireless sensor networks. The problem is formulated as a convex optimization problem. A sub-gradient routing algorithm is proposed and is confirmed by numerical experiments to converge fast in different scenarios. Different price update rules have been investigated, and the effects of asymmetric hops has been taken into account through a simple spatial energy consumption model. Among the ongoing extensions of this work, are its implementation is a distributed environment using the distance vector approach, and the investigation of the impact of a MAC layer protocol on the routing algorithm and its convergence.

### REFERENCES