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ORIGINAL ARTICLE

Solving a bi-objective cell formation problem with stochastic production quantities by a two-phase fuzzy linear programming approach

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Abstract We propose a bi-objective cell formation problem with demand of products expressed in a number of probabilistic scenarios. To deal with the uncertain demand of products, a framework of two-stage stochastic programming model is presented. The proposed model considers minimizing the sum of the miscellaneous costs (machine constant cost, expected machine variable cost, cell fixed-charge cost, and expected intercell movement cost) and expected total cell loading variation. Because of conflicting objectives, we develop a two-phase fuzzy linear programming approach for solving bi-objective cell formation problem. To show the effectiveness of the proposed approach, numerical examples are solved and the results are compared with the two existing approaches in the literature. The computational results show that the proposed fuzzy method achieves lower objective functions as well as higher satisfaction degrees.

Keywords Cellular manufacturing design · Cell formation problem · Multi-objective · Fuzzy linear programming · Stochastic programming model

1 Introduction

Cellular manufacturing (CM) is the application of group technology and has emerged as a promising alternative manufacturing system. CM entails the creation and operation of manufacturing cells. Parts are grouped into part families and machines grouped into cells. Various approaches have been suggested for forming manufacturing cells. Good discussions of cellular manufacturing systems can be found in Burbidge [1], Suresh and Meredith [2], and Selim et al. [3]. The design of cellular manufacturing systems has been called cell formation (CF). Given a set of part types, processing requirements, part type demand, and available resources (machines, equipment, etc.), a general design of cellular manufacturing consists of the following approaches: (a) part families are formed according to their processing requirements, (b) machines are grouped into manufacturing cells, and (c) part families are assigned to cells [4]. Because of the complexity and computational hardness of CF problem, majority of the methods in the literature attempt to optimize only one objective combine minimizing intercell movements, machine operation cost, machine set-up cost, subcontracting cost, and inventory holding cost in cell formation [5]. The design of manufacturing cells with respect to multiple criteria has been an attractive research which has not received as much attention as a single objective in the literature in the recent decades [6]. In this research, first, we survey the number of research works in the literature that proposed multi-objective mathematical models for the CF problem. Then well-known researches considering uncertainty concept in the CF problem are presented, and finally the concluding remarks are given.

Nei and Gaither [7] applied a capacity constrained method to multi-objective CF problem. The objective of the model is to minimize the bottleneck cost, intra/intercell load imbalances, and maximize the overage cell utilization. Venugopal and Narendran [8] developed a bi-criteria mathematical model for machine component grouping problem to minimize the volume of intercell movement and to total cell load variation. Zhao and Wu [9] suggested

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a genetic algorithm (GA) approach to apply certain operators to group the machines into manufacturing cells with multi-objective functions. The objectives of the model are: exceptional elements, inter/intracell parts movements cost and the total cell load variation. Mansouri et al. [6] reviewed the literature with respect to the multi-objective methodologies proposed for cellular manufacturing systems design. Baykasoghlu et al. [10] proposed an integer multiple objective nonlinear mathematical programming for cell formation problem with respect to minimizing the parts based on production requirements and processing sequences of parts, cell load imbalance, and extra-capacity requirements. Solimanpour et al. [11] presented a new mathematical model to solve the multi-objective cell formation problem with multiple process plans and independent manufacturing cells. The objectives considered in this model are: (1) to maximize the total similarity between the parts, (2) to minimize the total processing cost, (3) to minimize the total processing time, and (4) to minimize the total investment needed for the acquisition of machines. A genetic algorithm with multiple fitness functions is proposed to solve the formulation problem.

Lei and Wu [12] presented a Pareto-optimality-based multi-objective tabu search (MOTS) algorithm to the machine-part grouping problems with multiple objectives: minimizing the weighted sum of inter- and intracell moves and minimizing the total cell load variation. A new approach is developed to evaluate the nondominance of solutions produced by the tabu search. Comparisons between MOTS and the GA are done, and the results show that MOTS is quite promising in multi-objective cell design.

Aramoon Bajestani et al. [13] suggested a multi-objective dynamic cell formation problem to minimize the total cell load variation and sum of the miscellaneous costs which consist of machine cost, intercell material handling cost, and machine relocation cost. A new multi-objective scatter search proposes to find locally Pareto-optimal frontier and compares it with two salient multi-objective genetic algorithms.

Uncertainty can be considered essentially in two different types: randomness due to inherent variability in the system (i.e., in the population of outcomes of its stochastic process of behavior) and imprecision due to lack of knowledge and information on the system. The former type of uncertainty is often referred to as objective, stochastic whereas the latter is often referred to as subjective, epistemic, and state of knowledge [14, 15]. Most traditional cell formation procedures ignore any changes in demand over time caused by product redesign and uncertainties due to volume variation, part mix variation, and resource unreliability [16]. However in today's business environment, product life cycles are short, and demand volumes and product mix can vary frequently. Thus, the decision-making process in cellular manufacturing system often involves uncertainties. A number of researchers have applied uncertainties for the CF problem; the applications of uncertainties are classified as fuzzy theory and stochastic programming.

Kim et al. [17] considered a multi-objective machine cell problem, in which part types have several alternative part routings and the expected annual demand of each part type was known and total intercell part movements and total machine workload imbalances were simultaneously minimized. Liu et al. [18] proposed a mathematical model that incorporates multiple key real-life production factors simultaneously, namely, production volume; batch size, alternative process routings and perfect coefficient of each routing, cell size, unit cost of inter/intracell movements, and path coefficient of material flows and developed a heuristic algorithm. Ostrosi et al. [19] presented an approach to consensual cell formation in cellular manufacturing design. They proposed two models. The first model was devoted to the consensus conjecture which satisfies globally the set of measurable criteria and the second model addressed the problem of nucleus recognition. Tsai and Lee [20] presented a multi-functional MP model that incorporates the merits of related CF models based on the systematic study of MP models. The proposed model can offer the suitable modules that include the different objective functions and constraints for user to solve the related problem.

1.1 Fuzzy theory

A number of researchers have applied fuzzy clustering, fuzzy mathematics and fuzzy mathematical programming for the CF problem.

Masnate and Settineri [21] tailored a fuzzy c-means clustering algorithm for developing a nonbinary approach to group technology based on the capabilities of fuzzy logic. They also integrated fuzzy c-means with the strategy for minimum makespan scheduling. Shankar and Vrat [5] presented two fuzzy linear programming models for "post-clustering" stage and considered the fuzziness of part demand and budgetary limit on purchasing new machines and the aspiration level of the objective function. Susanto et al. [22] modified a fuzzy clustering approach presented by Chu and Hayya [23] and proposed a new fuzzy c-means and assignment technique able to perform part type clusters and machine-type clusters separately. A numerical example was illustrated and problems that arose in implementing this approach were discussed. Lozano et al. [24] also proposed a modified approach of the standard fuzzy c-means clustering algorithm by taking into account the effect of the weighting component on the fuzziness of the solution and the linking among the degree of membership of parts as well as machines and the prototypes of machine cells and part families.

Arikan and Gungor [25] proposed a fuzzy parametric programming model of cell formation by assuming fuzzy part demands, fuzzy machine capacity, and fuzzy machine duplication cost. Their objective minimizes three types of fuzzy costs associated with exceptional elements. Torkul et al. [26] employed fuzzy logic to study the design of part families and machine cells simultaneously. Their main aim was to compare manufacturing cell design made of fuzzy clustering algorithm (fuzzy c-means) with the crisp methods. The obtained computational results proved the superiority of fuzzy clustering solutions for selected datasets. Ravichandran and Rao [27] proposed a new fuzzy clustering algorithm and a new similarity coefficient for subgrouping parts/machines before the optimal grouping and for optimal grouping. The results showed that the new approach to fuzzy part family formation and grouping efficiency provided a more realistic solution methodology for part family formation in CM applications.

Pai et al. [28] developed a nonbinary part, machine, and cell matrix. In the matrix, each entry illustrates the degree of belonging of a part or a machine to a manufacturing cell. Both fuzzification and defuzzification procedures were used in dealing with the cell formation problem in a fuzzy environment. Szwarc et al. [29] were developed crisp and fuzzy mathematical models to optimally determine machine grouping and parts assignment under fuzzy demand and machine capacity. The object of these models was to minimize the processing and the material handling costs. Safaei et al. [30] developed a fuzzy programming-based approach to solve an extended mixed-integer programming model for a dynamic CF problem. Moreover, in real manufacturing systems some parameters such as part demands and the availability of manufacturing facilities in each period were regarded as piecewise fuzzy numbers. They proposed a fuzzy programming-based approach to design a dynamic cellular manufacturing system. The objective is to determine the optimal cell configuration in each period with maximum satisfaction degree of the fuzzy objective and constraint.

Papaioannou and Wilson [31] considered the fuzziness concept in a comprehensive mathematical programming formulation where parts are assigned to machines and machines to cells simultaneously by minimizing the number of distinct cells used by each part and set-up costs when allocating machines to cells and the number of times a part revisits a cell for a later machine operation.

1.2 Stochastic programming

In CMS, Tilsley and Lewis [32] addressed the issue of uncertainty in demand by using "cascading" strategy. A cascading system of cells is one where each cell is a child of another cell. The child cell consists of some machines similar to those of its parent cell along with some additional machines. If variable demand or mix changes result in the parent cell not being able to cope, the parts can be rerouted to one of the child cells giving the CMS flexibility.

Seiffodini [33] incorporated probabilistic demand in designing the CMS. Each product mix and the related part-machine matrix are assigned probabilities. For each product mix considered, the best cell configuration is determined. Subsequently for each of these best cell configurations, the expected intercell material handling cost based on possible product mixes id determined. Finally, that cell configuration with the lowest expected intercell material handling cost is selected as the preferred CMS. Later Seifoddini and Djassemi [34] conducted a simulation study of a CMS where the part mix changes to illustrate the sensitivity of the CMS to part mix changes. This sensitivity analysis can help the decision maker predict the performance of a CMS under uncertainty.

Harhalakis et al. [35] considered product demand changes during a multi-period planning horizon. They developed a mathematical programming model to minimize the expected intercell material handling cost. Cao and Chen [36] discussed a robust cell formation approach with demand of products expressed in a number of probabilistic scenarios. The model is to minimize machine cost and expected intercell material handling cost.

Ghezavati and Saidi-Mehrabad [37] addressed a new mathematical model for cellular manufacturing problem integrated with group scheduling in an uncertain space to minimize total expected cost consisting maximum tardiness cost among all parts, cost of subcontracting for exceptional elements, and the cost of resource underutilization. This model optimized cell formation and scheduling decisions, concurrently. It is assumed that processing time of parts on machines is stochastic and described by discrete scenarios enhances application of real assumptions in analytical process.

1.3 Concluding remarks

A number of conclusions from the survey of the literature review can be drawn such as:

- Most of the formulations for CF propose a mathematical programming model with the main objective of minimizing the total number of intercellular movements. Other mathematical programming formulations involve cost-related objective functions and only a few consider machine/load utilization as a goal parameter for CF.

- Fuzzy theory has been employed mainly for clustering purposes and within mathematical programming formulations for addressing uncertainty in certain model parameters.
- The number of research works dealing with stochastic programming in CF problem is fairly small.
- Most of the proposed methodologies in the last decade focus on a single criterion for CF. Only a few studies deal with multiple objectives.

Based on the above conclusions from the survey of the literature, we intend to propose probabilistic bi-objective CF problem with the minimal of two objectives simultaneously as follows: (1) machine constant/variable, intercell movements, and fixed cell costs and (2) cell loading variation costs.

The significance of this study is two folds. First, this is the first study which uses a two-stage stochastic programming model to face the bi-objective cell formation problem. Second, a two-phase fuzzy linear programming approach for the bi-objective cell formation problem is proposed and the superiority of the two-phase fuzzy linear programming approach over well-known solving approaches in the literature is shown.

The structure of this paper is as follows: Section 2 describes the background of the two-stage stochastic programming model. Section 3 presents a detailed description of the bi-objective stochastic cell formation problem. Section 4 proposes a two-phased fuzzy linear programming approach. In Section 5, experimental and comparison results are given. Finally, we present our conclusions in Section 6.

2 Framework of two-stage stochastic programming model

In the following text, the framework of the two-stage stochastic programming model is briefly described. For detail, the reader is refereed to Dantzing [38], Kall and Wallace [39], and Ruszczynski and Shapiro [40]. The most widely applied and studied stochastic programming models are two-stage linear programs [41]. Here, the decision maker takes some action in the first stage, after which a random event occurs affecting the outcome of the first-stage decision. A recourse decision can then be made in the second stage that compensates for any bad effects that might have been experienced as a result of the first-stage decision. The optimal policy from such a model is a single first-stage policy and a collection of recourse decisions (a decision rule) defining which second-stage action should be taken in response to each random outcome. Although two-stage stochastic linear programs are often regarded as the classical stochastic programming modeling paradigm, the discipline of stochastic programming has grown and broadened to cover a wide range of models and solution approaches. The approach of two-stage stochastic programming model has found various applications in different fields for example in electric energy producers [42], transportation network protection [43] and financial planning [44]. There is no study which applied the two-stage stochastic programming to the cell formation problem.

The general form of the model is expressed as follows:

$$\text{Min } c_1^{\mathrm{T}} x_1 + E_\psi\{\eta(x_1,\psi)\} \tag{1}$$

s.t.

$$Ax_1 = b \tag{2}$$

$$x_1 \in \hbar_+^{n_1} \tag{3}$$

where

$$\eta(x_1,\psi) = \text{Min } \mathrm{s}^{\mathrm{T}}\mathrm{y}_2 \tag{4}$$

s.t.

$$T(\psi)x_1 + wy_2 = h(\psi) \tag{5}$$

$$y_2 \in \hbar_+^{n_2} \tag{6}$$

and $E_\psi\{\bullet\}$ is the expected value function, Ψ is the random vector, and $h(\Psi)$ and $T(\Psi)$ depend on a random vector Ψ representing a particular sample from a multi-variant probability space π.

Equations 1–3 represent the first-stage model and Eqs. 4–6 represent the second-stage model. x_1 is the vector of first-stage decision variables. The optimal value of x_1 is not conditional on the realization of the uncertain parameters. c_1 is the vector of cost coefficients at the first stage. A is the first-stage coefficient matrix and b is the corresponding right-hand side vectors. y_2 is the vector of second-stage (recourse) decision variables. s is the vector of cost (recourse) coefficient vectors at the second stage. W is the second-stage (recourse) coefficient matrix and $h(\Psi)$ is the corresponding right-hand side vector. $T(\Psi)$ is the matrix that ties the two stage together. In the second-stage model, the random constraint defined in Eq. 5, $h(\psi) - T(\psi)x_1$, is the goal constraint: violations of this constraint are allowed, but the associated penalty cost, $s^{\mathrm{T}}y_2$, will influence the choice of x_1 . The function $\eta(x_1,\psi)$ is the recourse penalty cost or second-stage value function, and the notation $E_\psi\{\eta(x_1,\psi)\}$ denotes the expected value of recourse penalty cost (second-stage value function) with respect to the random vector Ψ.

Consider the special case when π is finite, which gives $\pi = \{\psi^1, \psi^2, ..., \psi^Q\}$, and $\Pr(\psi = \psi^q) = \mathrm{pr}_q$. $q = 1, ..., Q$ representing a scenario is its known distribution. By replacing $T(\Psi^q)$ with $T^q$and $h(\Psi^q)$ with h^q, the two-stage stochastic programming model can be reformulated as the following algebraic equivalent LP:

$$\text{Min } c_1^{\mathrm{T}} x_1 + \sum_{q=1}^{Q} \mathrm{pr}_q \left(s^T y_2^q\right) \tag{7}$$

s.t.

$$Ax_1 = b \tag{8}$$

$$T^q x_1 + w y_2^q = h^q \quad q = 1, ..., Q \tag{9}$$

$$x_1, y_2^q \geq 0 \quad q = 1, ..., Q \tag{10}$$

The vector of first-stage decision variables, x_1, is scenario independent. The vectors of second-stage decision variables, y_2^q are introduced to control the random constraints with minimal recourse penalty cost. In this special case where π is finite, the expected value of second-stage value function with respect to the random vector Ψ^q is $E_\psi\{\eta(x_1, \psi)\} = \sum_{q=1}^{Q} \mathrm{pr}_q \left(s^T y_2^q\right)$.

3 A two-stage stochastic programming approach for a bi-objective cell formation problem

In this section, we present the proposed bi-objective cell formation model as a mixed-integer programming. The problem is considered under the following assumptions:

3.1 Assumptions

1. The operating time for processing all part types on different machine types are known.
2. Each part must be processed according to a known sequence of operations.
3. The demand of product is not deterministic and described by several discrete scenarios with probabilities of their occurrences.
4. The capabilities and capacity of each machine type is known.
5. Parts are moved between cells in fixed cost and batch sizes.
6. The maximum number of cells must be specified in advance.
7. Lower and upper bounds of machines in each cell need to be specified in advance.
8. Each machine type can perform one or more operations (i.e., machine flexibility) without modification cost. Likewise, each operation can be done on one machine type with different times (i.e., routing flexibility).
9. The cell or cells to be constructed will be determined from the problem solution, depending on the fixed costs of cell construction, cell capacities are another related factor.
10. Intercell movement costs are constant for all moves regardless of the distance traveled.
11. Intercell batch sizes are constant for all productions.
12. Constraints relating to inventory control theory such as backorders, subcontracting, holding, etc. are not allowed.

3.2 Notations

C	Index for manufacturing cell (c=1,2,…,C)
m	Index for machine type (m=1,2,…,M)
p	Index of part type (p=1,2,…,P)
j	Index of operation of part (j=1,2,…,O_P)
q	Index of scenario (q=1, 2,…,Q)

3.3 Input parameters

3.3.1 Deterministic parameters

P	Number of part types.
O_P	Number operations of part P.
M	Number of machine types.
C	Maximum number of cells that can be formed in each period.
B	Batch size for intercell movements.
t_{jpm}	Time required to process operation j of part p on machine type m.
$a_{jpm}=1$	If operation j of part p can be done on machine type m; otherwise it is zero.
α_m	Constant cost of machine type m.
β_m	Variable cost of machine type m for each unit time.
F_c	Set-up and operating cost of cell c.
γ	Intercell movement cost per batch.
UB_c	Upper bound of cell c.
T_m	Capacity of machine type m.

3.3.2 Recourse parameters

D_p^q	Demand of part type p in scenario q.
Pr_q	Occurrence probability of scenario q.
$S_p^q = 1$	If part p is present in scenario q; otherwise it is zero.

$V^q = \left\{ i : S_p^q) = 1 \right\}$ Index set of parts present in scenario q.

3.4 Decision variables

3.4.1 First-stage decision variables

N_{mc} Number of machine type m in cell c.
Y_c=1 If cell c will be formed; otherwise it is zero.

3.4.2 Second-stage decision variables

$X_{jpmc}^q = 1$ If operation j of part p in cell c in scenario q performs by machine type m, otherwise it is zero.
$Z_{jpc}^q = 1$ If operation j of part p is done in cell c in scenario q, otherwise it is zero.

3.5 Objective functions

3.5.1 The objective function at the first stage

$$\text{Min}Z_1 = \sum_{c=1}^{C} F_c Y_c + \sum_{c=1}^{C}\sum_{m=1}^{M} \alpha_m N_{mc} \tag{11}$$

The first term in expression 11 is the machine cost or the machine depreciation cost which is calculated based on the number of different machine types used in a cell. The last term is the fixed-charge cost for setting up manufacturing cells.

3.5.2 The objective functions at the second stage

$$\begin{aligned}\text{Min}Z_1 = &\sum_{q=1}^{Q}\sum_{c=1}^{C}\sum_{p\in V^q}\sum_{j=1}^{OP}\sum_{m=1}^{M} S_p^q \text{pr}_q \beta_m D_p^q t_{jpm} x_{jpmc}^q \\ &+ \tfrac{1}{2}\sum_{q=1}^{Q}\sum_{p\in V^q}\sum_{j=1}^{OP-1}\sum_{c=1}^{C} S_p^q \text{pr}_q \left\lceil \tfrac{D_p^q}{B} \right\rceil \gamma \left| Z_{(j+1)pc}^q - Z_{jpc}^q \right|\end{aligned} \tag{12}$$

$$\text{Min}Z_2 = \sum_{q=1}^{Q}\sum_{c=1}^{C}\sum_{m=1}^{M}\sum_{p\in V^q} \left(\theta_{mpc}^q - \varepsilon_{mpc}^q\right)^2 \tag{13}$$

The first term in expression 12 is the expected variable cost of all machines required in all cells. The last term is the expected intercell material handling cost. The total expected cell load variation is considered as the objective function in expression 13. It is obvious that, expected intercell movement cost in expression 12 will equal 0 if all machines are assigned into one cell, and total expected cell load variation in expression 13 will be 0 provided that only one machine is assigned to each cell.

3.6 Constraints

3.6.1 The constraint at the first stage

$$\sum_{m=1}^{M} N_{mc} \le \text{UB}_c Y_c \quad \forall c \tag{14}$$

Constraint 14 specifies the lower bound for cell size.

3.6.2 The constraints at the second stage

$$\sum_{c=1}^{C}\sum_{m=1}^{M} a_{jpm} x_{jpmc}^q = 1 \qquad \forall j, q, p \in V^q \tag{15}$$

$$Z_{jpc}^q = \sum_{\text{m}=1}^{M} X_{jpmc}^q \qquad \forall j, p \in V^q, c, q \tag{16}$$

$$\sum_{q=1}^{Q}\sum_{p\in V^q}\sum_{j=1}^{OP} \text{pr}_q S_p^q D_p^q t_{jpm} x_{jpmc}^q \le T_m N_{mc} \quad \forall m, c \tag{17}$$

$$Y_c \ge Z_{jpc}^q \qquad \forall j, p \in V^q, c, q \tag{18}$$

$$\theta_{mpc}^q = \frac{\sum_{j=1}^{O_p} D_p^q \text{pr}_q S_p^q t_{jpm} X_{jpmc}^q}{T_m} \quad \forall m, p \in V^q, c, q \tag{19}$$

$$\varepsilon_{mpc}^q = \frac{\sum_{m=1}^{M} \theta_{mpc}^q N_{mc}}{\sum_{m=1}^{M} N_{mc}} \qquad \forall p \in V^q, c, q \tag{20}$$

Constraint 15 ensures that each operation is assigned only to one machine and one cell. Constraint 16 implies that if at least one operation of part p is performed in cell c in scenario q, then the value of Z_{jpc}^q will be equal to 1; otherwise, it will be set to zero. Constraint 17 guarantees that machine capacity is not exceeded. Constraint 18 ascertains whether cell c is formed. θ_{mpc}^q is the expected workload on machine m by part p in cell c under scenario q will be Compute by Eq. 19, and ε^{q}_{mpc}, the expected average intracell processing time for part p in cell c in scenario q will be obtained in Eq. 20.

4 Two-phase fuzzy linear programming approach

The development of various approaches, in order to solve multi-objective problems, has been an on-going effort by researchers. Various methods exist for optimizing the multi-objective optimization problems and were classified into five sets [39]: (1) scalar methods, (2) interactive methods, (3) fuzzy methods, (4) method with use a meta-heuristic, and (5) decision aid methods.

In this study, a two-phase fuzzy linear programming approach is employed that belongs to the third classified set.

4.1 Phase 1

First the general bi-objective model for cell formation problem is presented and then appropriate operators for this decision-making problem are discussed.

A general linear bi-objective model can be presented as: Find a vector x written in the transformed $u^{\mathrm{T}} = [u_1, u_2, \ldots, u_{\mathrm{n}}]$ which minimizes objective function Z_k with

$$Z_k = \sum_{i=1}^{n} c_{ki} u_i \; k = 1, 2. \tag{21}$$

and constraints:

$$u \in U_d, \quad U_d = \left\{ u/g(u) = \sum_{i=1}^{n} a_{ri} u_i \leq b_r, r = 1, 2, \ldots, l, u \geq 0 \right\} \tag{22}$$

where c_{ki}, a_{ri} and u_r are crisp or fuzzy values.

Zimmermann [45] has solved problems 21 and 22 by using fuzzy linear programming. He formulated the fuzzy linear program by separating every objective function Z_j into its negative ideal solution (Z_k^{NIS}) and positive ideal solution (Z_k^{PIS}) by solving:

$$Z_k^{\mathrm{NIS}} = \max Z_k, \quad u \in U_d, \quad Z_k^{\mathrm{PIS}} = \min Z_k, \quad \mathrm{u} \in U_d \tag{23}$$

$Z_k^{\mathrm{PIS}}$is obtained through solving the bi-objective problem as a single objective using, each time, only one objective and $u \in U_d$ means that solutions must satisfy constraints.

Since every objective function Z_k, the value of it changes linearly from Z_k^{PIS} to Z_k^{NIS}, it may be considered as a fuzzy number with the linear membership function $\mu_{Z_k}(U)$ as shown in Fig.1. It was shown that a linear programming problem (Eqs. 21 and 22) with fuzzy goal may be presented as follows:

Find a vector u to satisfy:

$$\widetilde{Z}_k = \sum_{i=1}^{n} c_{ki} u_i \underset{\sim}{\leq} Z_k^0 \qquad k = 1, 2, \ldots, K \tag{24}$$

s.t.

$$g_r(u) = \sum_{i=1}^{n} a_{Ki} u_i \leq b_r \; r = 1, \ldots, l \tag{25}$$

$$u_i \geq 0 \;\; i = 1, 2, \ldots, n. \tag{26}$$

In this model, the sign ~ indicates the fuzzy environment. $Z_k{}^0$ is the satisfaction degree that the decision maker wants to reach.

Assuming that membership function, based on preference or satisfaction is the linear membership for minimization goals (Z_k) is given as follows:

$$\mu_{Z_k}(u) = \begin{cases} 1 & \text{for } Z_k \leq Z_k^{\mathrm{PIS}}, \\ \frac{\left(Z_k^{\mathrm{NIS}} - Z_k(u)\right)}{\left(Z_k^{\mathrm{NIS}} - Z_k^{\mathrm{PIS}}\right)} & \text{for } Z_k^{\mathrm{PIS}} \leq Z_k(u) \leq Z_k^{\mathrm{NIS}}, \quad k = 1, 2, \\ 0 & \text{for } Z_k \geq Z_k^{\mathrm{NIS}}. \end{cases} \tag{27}$$

In fuzzy programming modeling, using Zimmermann's approach, a fuzzy solution is given by the intersection of all the fuzzy sets representing either fuzzy objective. The fuzzy solution for all fuzzy objectives may be given as

$$\mu_D(u) = \left\{ \bigcap_{j=1}^{p} \mu_{Z_j}(u) \right\} \tag{28}$$

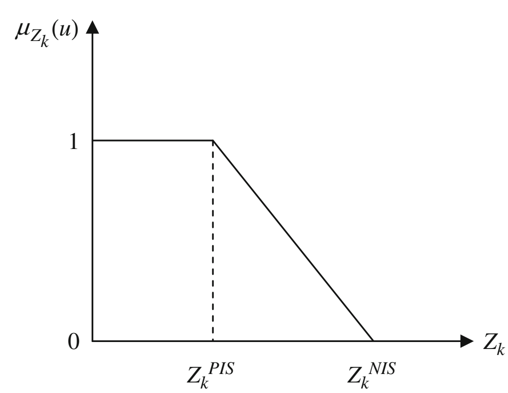

Fig. 1 Objective function Z_k as a fuzzy number

The optimal solution (u*) is given by (Bellman and Zadeh [46]):

$$\mu_D\left(u^*\right) = \max_{u\in U_d}\mu_D(u) = \max_{u\in U_d}\left[\min_{j=1,\dots,K}\mu_{Z_j}(u)\right] \tag{29}$$

With the "max–min" operator and α satisfaction degree, the fuzzy linear programming problem can be solved as a single-objective problem:

$$\text{Maximize } \alpha \tag{30}$$

s.t.

$$\alpha \le \frac{\left(Z_k^{\text{NIS}} - Z_k(u)\right)}{\left(Z_k^{\text{NIS}} - Z_k^{\text{PIS}}\right)} \qquad k = 1,2 \tag{31}$$

$$g_r(u) \le b_r \qquad r = 1,\dots,m \tag{32}$$

$$x_i \ge 0 \qquad i = 1,2,\dots,n \quad \text{and} \quad \alpha \in [0,1] \tag{33}$$

The "max–min" obtains the best solution (due to max operator) among the set of the worst objective values (due to min operator), each determined by a feasible solution. An alternative approach may decide a different operator, in place of the min operator. Indeed quite reasonably, it may sometimes be desirable for a compensatory operator to be used instead of the min operator (Lee and Li. [47]).

4.2 Phase 2

Here, we make use of the result of phase 1 to overcome disadvantages of one-phase approach.

Lee and Li [47], Guu and Wu [48], and Li and Li [49] used two-phased approaches to fix situations where the max–min operator is not efficient. The two-phase method uses the max–min operator in its first phase. It is well known that the optimal solution obtained by phase 1 may not be an efficient solution in the sense that there may exist another solution in the feasible space dominating the obtained solution by the max–min operator in phase 1 (a solution a is said to dominate solution b if: (1) a is at least as good as b regarding all objectives, and (2) a is strictly better than b for at least one objective). In the second phase, the solution is forced to improve upon and dominate the one obtained by the max–min operator, adding constraints and a new auxiliary objective function to phase 2 to achieve at least the satisfaction degree obtained in phase 1. An arithmetic operator$\overline{\lambda}$is proposed to obtain new satisfaction degrees that represent the bi-objective linear programming objectives' satisfaction degrees. Thus, the proposed phase 2 problem is as follows:

$$\text{Maximize } \overline{\lambda} = \min\left(\mu_{Z_k}(u)\right) + \frac{1}{K}\sum_{k=1}^{K} w_k(\lambda_k - \alpha) \tag{34}$$

s.t.

$$\alpha \le \lambda_k \le \frac{\left(Z_k^{\text{NIS}} - Z_k(u)\right)}{\left(Z_k^{\text{NIS}} - Z_k^{\text{PIS}}\right)} \qquad k = 1,2 \tag{35}$$

$$g_r(u) \le b_r \qquad r = 1,\dots,m \tag{36}$$

$$u_i \ge 0 \; i = 1,2,\dots,n \quad \text{and} \quad \alpha,\lambda \in [0,1] \tag{37}$$

where w_k, the given weighting coefficients presenting the relative importance among the fuzzy goals, are so that $\sum_{k=1}^{K} w_k = 1, 0 < w_k \le 1$. We note that the constraints 35 enforce a better solution in phase by the requirements $\lambda_k \ge \alpha$, $k = 1,2$ while maximizing the weighted mean squares of the improvements. A good starting point for solving the phase 2 problem is $\lambda_k = \alpha$, $k = 1,2$. Then, the general steps of our algorithm are outlined.

1. {Step 1} Compute the negative ideal solutions, i.e., the Z_k^{NIS} as given in Eq. 23; use these as the initial point for the "max–min" model (Eqs. 30–33) to compute an optimal solution (the obtained value of α is to be used in phase 2.
2. {Step 2} Let $\overline{\lambda} = \alpha$ in Eq. 34 and solve Eqs. 34–37 to get an optimal solution.

The bi-objective fuzzy linear programming for the proposed cell formation problem formulation is presented as follows:

$$\begin{aligned}\text{Min } \tilde{Z}_1 = & \sum_{c=1}^{C} F_c Y_c + \sum_{c=1}^{C}\sum_{m=1}^{M}\alpha_m N_{mc} \\ & + \sum_{q=1}^{Q}\sum_{c=1}^{C}\sum_{p\in V^q}\sum_{j=1}^{OP}\sum_{m=1}^{M} S_p^q \text{pr}_q \beta_m D_p^q t_{jpm} x_{jpmc}^q \\ & + \frac{1}{2}\sum_{q=1}^{Q}\sum_{p\in V^q}\sum_{j=1}^{OP-1}\sum_{c=1}^{C} S_p^q \text{pr}_q \left\lceil \frac{D_p^q}{B}\right\rceil \gamma \left| Z_{(j+1)pc}^q - Z_{jpc}^q \right| \tilde{\le} Z_1^0\end{aligned}$$

$$\text{Min } \tilde{Z}_2 = \sum_{q=1}^{Q}\sum_{c=1}^{C}\sum_{m=1}^{M}\sum_{p\in V^q}\left(\theta_{mpc}^q - \varepsilon_{mpc}^q\right)^2 \tilde{\le} Z_2^0$$

s.t.

Constraints 14 to 20.

The following solution procedure is employed to solve the fuzzy bi-objective linear programming for the proposed stochastic cell formation problem.

4.2.1 Algorithm: two-phase fuzzy bi-objective linear programming approach

Phase 1

Step 1 Construct the crisp fuzzy bi-objective linear programming for the bi-objective stochastic cell formation problem formulation.

Step 2 Solve the kth objective function with an optimization technique such as branch-and-bound (B&B) method embedded in Lingo 8 software, and set Z_k^{PIS} to the objective function value of the found minimum solution (lower bound for the kth objective Z_k).

Step 3 Determine the values of the other objective functions of the obtained solution in the previous step and set Z_k^{NIS}= the maximum value among the obtained values (upper bound for the kth objective Z_k).

Step 4 Repeat steps 2 and 3 for all the objective functions.

Step 5 Define the membership function of each goal in the fuzzy bi-objective linear programming.

Step 6 Construct the equivalent crisp formulation of fuzzy bi-objective linear programming cell formation problem according to Eqs. 30 to 33.

Step 7 Solve the equivalent crisp formulation in previous step; then calculate the relative membership α of each objective value's satisfaction degrees.

Phase 2

Step 8 Set $\lambda_k = \alpha$, $k = 1, 2, \ldots, K$, and solve the problem (Eqs. 34–37) to get an optimal solution.

From the above algorithm, it should be evident that the approach is effective in generating a preferred compromise solution and the corresponding feasible satisfaction degree of each objective function. In addition, we should highlight an important issue which is the relationship between the preferred compromise solution and the satisfaction degrees. Figure 2 illustrates the block diagram of the proposed approach.

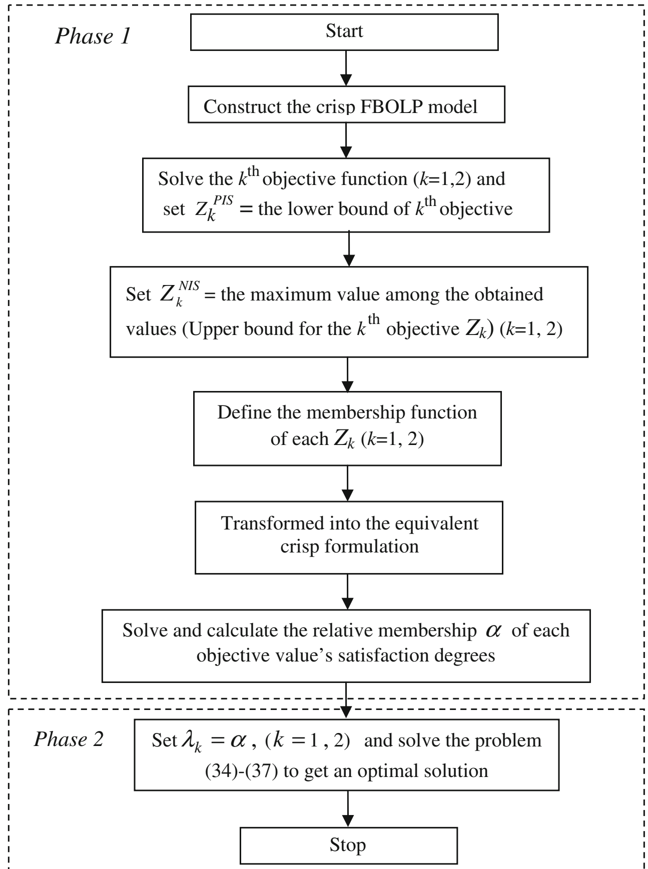

Fig. 2 Block diagram of the proposed approach

5 An application example and performance analysis of fuzzy bi-objective linear programming approach

5.1 A solution example

In this paper, the problem is considered for five parts, seven machines, and three scenarios. A typical dataset for the proposed model shown in Table 1. Each part has two operations that must be done respectively. For simplicity, we assume that all machine types have same capacity (i.e., 300 h/period) independent of machine type. Dataset is randomly generated in terms of the uniform distribution by inspiration of literature. Table 2 shows different part types to be processed in the three possible scenarios.

The linear membership function is used for fuzzifying the objective functions for the above problem. The dataset for the values of the lower bounds and upper bounds of the objective functions and fuzzy number for the demands are given in Table 3 (steps 1 to 4).

The below membership functions for two-objective functions are provided to minimize the sum of the miscellaneous costs (machine constant cost, expected machine variable cost, cell fixed-charge cost, and expected intercell movement cost) and the expected total cell loading variation (step 5).

$$\mu_{Z_1}(x) = \begin{cases} 1 & Z_1 \le 3,471.96, \\ \frac{7,010 - Z_1}{3,538.04} & 3,471.96 \le Z_1 < 7,010, \\ 0 & Z_1 \ge 7,010. \end{cases} \quad (38)$$

$$\mu_{Z_2}(x) = \begin{cases} 1 & Z_2 \le 0.1021, \\ \frac{6.8749 - Z_2}{6.7728} & 0.1021 \le Z_2 < 6.8749, \\ 0 & Z_2 \ge 6.8749. \end{cases} \quad (39)$$

Table 1 Typical test problem

Machine information			Process	P1		P2		P3		P4		P5	
$\alpha_m$$	β_m $	T_m		1	2	1	2	1	2	1	2	1	2
715	0.43	300	M1									0.94	
356	0.5	300	M2	0.27			0.47				0.63		0.5
314	0.62	300	M3							0.76		0.68	
587	0.12	300	M4	0.42			0.23		0.81				
177	0.56	300	M5		0.95			0.86			0.16		
818	0.39	300	M6		0.12	0.21		0.27					
775	0.75	300	M7			0.01			0.43	0.07			0.52

B=50; γ=40; F_1=50; F_2=50; F_3=50; UB_1=4; UB_2=4; UB_3=4

The crisp formulation of fuzzy bi-objective linear programming stochastic cell formation problem for the numerical example can be formulated as follows (step 6):

$$\text{Maximize } \alpha \tag{40}$$

s.t.

$$\alpha \leq \frac{7,010 - Z_1}{3,538.04} \tag{41}$$

$$\alpha \leq \frac{6.8749 - Z_2}{6.7728} \tag{42}$$

$$\alpha \in [0, 1] \tag{43}$$

Constraints 14 to 20.

The Lingo software was used to run this fuzzy bi-objective linear programming model, obtaining the results for the objectives as Z_1=4,790.713, Z_2=0.8988, and the overall degree of satisfaction with the DM's multiple fuzzy goals as 0.6267.

After getting the optimal solution from previous step, according to the optimal objective function values, the step 6 satisfaction degree, α, can be used in this step (step 7). Below equation represents the fuzzy bi-objective linear programming stochastic cell formation problem that is transferred from previous step. Furthermore, the decision maker provided the relative importance of objectives linguistically as: $w = (w_1, w_2) = (0.65, 0.35)$.

$$\text{Maximize } \lambda = \min\left(\frac{7,010 - Z_1}{3,538.04}, \frac{6.8749 - Z_2}{6.7728}\right) + \frac{1}{2}\left(w_1(\lambda_1 - 0.6267) + w_2(\lambda_2 - 0.6267)\right) \tag{44}$$

s.t.

$$0.6267 \leq \lambda_1 \leq \frac{7,010 - Z_1}{3,538.04} \tag{45}$$

$$0.6267 \leq \lambda_2 \leq \frac{6.8749 - Z_2}{6.7728} \tag{46}$$

$$\lambda_1, \lambda_2 \in [0, 1] \tag{47}$$

Constraints 14 to 20.

This experiment is carried out by a B&B method by using the Lingo 8 software, which is executed on a Pentium 4, 3 GHz, and Windows XP using 512 MB of RAM. The results of applying the fuzzy bi-objective linear programming to the problem of stochastic cell formation problem are shown in Table 4.

In the first phase of the solution procedure in this study, the acceptable DM satisfaction degree, α, is 0.6272 in the fuzzy environment. Then, in phase 2, the DM satisfaction degree is improved by adding lower limits, which all objective functions have to exceed and new auxiliary objective function. As a result of this modification, the

Table 2 Part demand

Scenario	Probability	Demand				
1	0.65	300	500	0	1,000	900
2	0.25	0	200	100	600	200
3	0.10	1,033	918	0	0	1,153

Table 3 The dataset for membership functions

	μ=0	μ=1 (PIS)	μ=0 (NIS)
Z_1 (sum of the miscellaneous costs)	–	3,471.96	7,010
Z_2 (expected total cell loading)	–	0.1021	6.8749

PIS positive ideal solution, *NIS* negative ideal solution

Table 4 Obtained result of proposed approach

Z_1*	Z_2*	λ_1*	λ_2*	O.F.V.
3,949.401	0.4763	0.8650	0.9447	0.9978

satisfaction degree is increased to 0.9978 and all objective function values are decreased. The best cell configuration obtained of the proposed approach is shown in Fig. 3.

5.2 Performance analysis

To evaluate the performance of the suggested approach, let us consider the solution of the illustrated example by using different approaches. The proposed approach was applied to the test problem and its performance was compared with the two-phase approach (LZL) proposed by Li and Li [49]. The two-phase approach proposed by Li and Li [49] is:

$$\text{Maximize } \sum_{k=1}^{2} w_k \mu_{Z_k}(x) \tag{48}$$

s.t.

$$\lambda \leq \frac{7,010 - Z_1}{3,538.04} = \mu_{Z_1}(x) \tag{49}$$

$$\lambda \leq \frac{6.8749 - Z_2}{6.7728} = \mu_{Z_2}(x) \tag{50}$$

$$\lambda, \mu_{Z_1}(x), \mu_{Z_2}(x) \in [0, 1] \tag{51}$$

Constraints 14 to 20.

In the above formulation, λ denotes the minimum satisfaction degree of the objective function which is found by solving the Zimmermann's max–min [45] approach, as follows:

$$\text{Maximize } \lambda \tag{52}$$

s.t.

$$\lambda \leq \frac{7,010 - Z_1}{3,538.04} \tag{53}$$

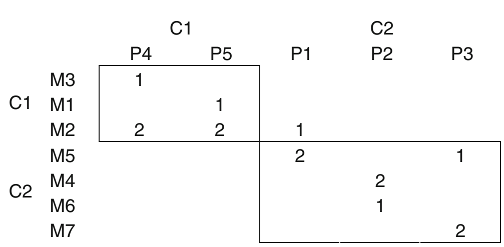

Fig. 3 Best cell configuration

$$\lambda \leq \frac{6.8749 - Z_2}{6.7728} \tag{54}$$

$$\lambda \in [0, 1] \tag{55}$$

Constraints 14 to 20.

The results of applying the LZL approach to the problem of stochastic cell formation problem are shown in Table 5.

To determine the degree of closeness of the fuzzy bi-objective linear programming approach results to the ideal solution, let us define the following family of distance functions [50]:

$$D_p(w, K) = \left[\sum_{k=1}^{K} w_k^p (1 - d_k)^p \right]^{\frac{1}{p}} \tag{56}$$

where d_k represents the degree of closeness of the preferred compromise solution vector to the optimal solution vector with respect to the kth objective function. $\lambda = (\lambda_1, \lambda_2, ..., \lambda_k)$ is the vector of objectives satisfaction degrees. The power p represents a distance parameter $1 \leq p \leq \infty$. Assuming $\sum_{k=1}^{K} \lambda_k = 1$, we can write $D_p(w,K)$ with p=1.2 and ∞as follows:

$$D_1(w, K) = 1 - \sum_{k=1}^{K} w_k d_k \quad \text{(the Manhattan distance)} \tag{57}$$

$$D_2(w, K) = \left[\sum_{k=1}^{K} w_k^2 (1 - d_k)^2 \right]^{\frac{1}{2}} \quad \text{(the Euclidean distance)} \tag{58}$$

$$D_\infty(w, K) = \max_k \{\lambda_k (1 - d_k)\} \quad \text{(the Tchebycheff distance)} \tag{59}$$

where, in minimization problems, d_k takes the form: $d_k =$ (the *optimal solution of* Z_k)/(the preferred compromise solution Z_k).

Thus, we can state that the approach which can derive a preferred compromise solution is better than the others if: Min $D_p(\lambda,K)$ is achieved by its solution with respect to some p as discussed in Abd El-Wahed and Lee [50]. Comparison of the results obtained using the max–min

Table 5 Obtained results of LZL approach

Z_1*	Z_2*	O.F.V.
3,921.866	0.5105	0.8962

approach and Two-phase approach with the proposed fuzzy bi-objective linear programming approach is summarized in Table 6. From Table 6, it is clear that the suggested fuzzy bi-objective linear programming approach gave a preferred compromise solution which is better than the solution by the approaches in [50] for all the distance functions D_1, D_2, and D_∞. This comparison shows the proposed fuzzy bi-objective linear programming approach is superior to max–min and LZL approach in the above test problem.

Figure 4 shows sum of the miscellaneous costs (machine constant cost, expected machine variable cost, cell fixed-charge cost, and expected intercell movement cost) and expected total cell loading variation according to applied approaches for solving proposed multi-objective stochastic cell formation problem.

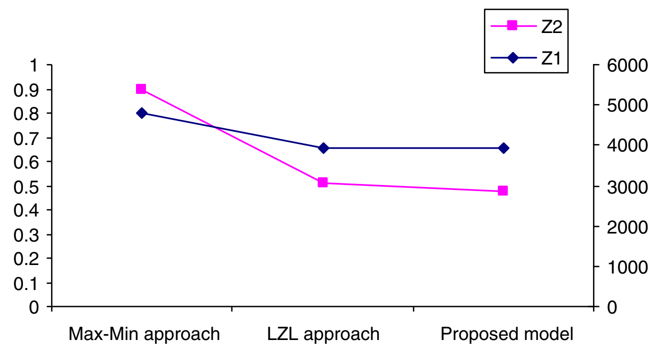

Fig. 4 Objective functions values

6 Discussion of results for future researches

In this paper, we proposed a two-phase fuzzy programming approach and we solve the problem with Lingo 8 software. Although it is an effective approach in practice, but we can consider some weaknesses as following:

1. In Lingo software and with fuzzy programming, the time of problem solving is very much, in the other words this approach is a time consuming one.
2. The Lingo 8 solver and proposed algorithm are not proper for large-sized problems.
3. The Lingo 8 solve the problem by an exact algorithm (branch and bound), but meta-heuristic algorithms can produce better results.
4. We cannot solve the problem with different approaches in Lingo 8, so the software limits the users with one approach.
5. The algorithm in Lingo 8 has a same procedure, but in a meta-heuristic algorithm, we can change the procedure with changing the conditions like problem size and so on for achieving to high quality results.

These are only some limits of Lingo 8 software, future researches can consider these weaknesses and find a solution for each of them, although in literature some papers propose the approaches which do not have the above problems. Guo ZX et al. [51, 52] are the best examples in the literature. Guo ZX [51] has presented a genetic optimization process to solve this model, in which a new chromosome representation, a heuristic initialization process and modified crossover and mutation operators are proposed. Also, Guo ZX [52] has presented a new chromosome representation to tackle the operation assignment by assigning one operation to multiple machines as well as assigning multiple operations to one machine. Furthermore, a heuristic initialization process and modified genetic operators are proposed. The time of problem solving has been presented in these papers; the time is logical in these papers. These approaches with the modifications are effective in large-sized problems. The heuristic initialization process and modified operators can adjust the algorithm with our models and the problem conditions like size and so on. As we discussed, these approaches can solve the problems which we have with Lingo 8 software. Although, these papers are two proper ones for above weaknesses, but this topic is a broad one and the future researches can present new approaches which do not have these weaknesses.

Table 6 Comparison of solutions by three different approaches

	Max–min approach [40]	LZL approach [44]	Proposed approach	Optimal solution
Z_1	4,790.713	3,921.866	3,949.401	3,471.961
Z_2	0.8988	0.5105	0.4763	0.1021
D_1	0.4891	0.3545	0.3535	–
D_2	0.3580	0.2897	0.2859	–
D_∞	0.3102	0.28	0.2749	–

7 Conclusions

A fuzzy bi-objective linear programming approach is developed for the cell formation problem with demand of products expressed in a number of probabilistic scenarios. Two crucial objectives of the machine constant cost, expected machine variable cost, cell fixed-charge cost, expected intercell movement cost, and the expected total cell loading variation are considered for minimization. We proposed a two-phase approach for the fuzzy bi-objective linear programming. In the first phase, the problem is solved using a max–min approach. The max–min solution not being efficient, in general, we proposed a new model in the second phase to maximize a composite satisfaction degree at least as good as the degrees obtained by phase one. To show the effectiveness of the proposed approach, a numerical example is solved to compare the performance of the fuzzy bi-objective linear programming with the max–min approach and LZL approach. The numerical results show that the proposed approach achieves lower objective functions as well as higher satisfaction degrees. The proposed approach of this study showed satisfactory result for small sized problems. Moreover, time complexity is not addressed in this paper; however, this issue might be important in large-sized problems, therefore, developing efficient exact or heuristic solution methods can be appealing in this area.

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