Control of Hybrid Machines With 2-DOF for Trajectory Tracking Problems

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Abstract—There are two types of drivers in production machine systems: constant velocity (CV) motor and servo-motor. If a system contains two or more drivers, among which some are of the CV motor while the other are the servo-motor, the system has the so-called hybrid driver architecture and is called hybrid machine for short. The hybrid system has the advantage of high payload and application flexibility. In this brief, we propose a control algorithm and show that the controlled hybrid machine is stable. A simulation is performed to verify the proposed controller. The CV motor has the velocity fluctuation due to the change of its workload. The common approach to attenuate the velocity fluctuation is via a flywheel which is attached on the shaft of the CV motor. We show that this can further improve the tracking performance of the hybrid system. A five-bar linkage with two degrees of freedom is used for illustration throughout the brief.

Index Terms—Flywheel, hybrid machine, trajectory tracking, velocity fluctuation.

I. INTRODUCTION

THERE ARE TWO types of drivers in production machine systems: constant velocity (CV) motor and servo-motor. If a system contains two or more drivers, among which some are of the CV motor while the other are the servo-motor, the system has the so-called hybrid driver architecture and is called hybrid machine for short. The hybrid system has the advantage of high payload and application flexibility, because the CV motor can undertake a high constant-part workload while the servo-motor can be real-time regulated to meet the change of a task [1]–[5].

It is known that the CV motor has the velocity fluctuation when it carries the time-varying workload. Such fluctuation will propagate to the end-effector or executive component of the hybrid machine. Control of the hybrid machine is then a challenge. Previous published studies on the hybrid machine usually substituted the CV motor by the servo-motor with the CV profile [1]–[5]. This treatment will inevitably bring in errors in modeling of the dynamics of the hybrid machine system and control of the system.

II. DYNAMIC MODEL OF A FIVE-BAR HYBRID MECHANISM WITH 2-DOF

A. Dynamic Model of the Five-Bar Mechanism With 2-DOF

As suggested in [9] and [10], a planar mechanism can be viewed to consist of free systems to which constraints are applied. In the planar five-bar mechanism (Fig. 1), the free system is two open-chain serial links, each of which contains two links, and the constraints are two independent scleronomous holonomic constraint equations as shown in (1) at the bottom of the next page where \( \mathbf{q}' = [q_1, q_2, q_3]^T \) is the vector of the generalized coordinates of the free system. Let \( r_i \) and \( \delta_i \) denote, for link \( i \), the location of the center of mass, \( m_i \) and \( L_i \) denote the mass and the length, respectively, and \( J_i \) the moment of inertia with respect to the centroid. Applying the Lagrangian method leads to the dynamic model of the free system as follows:

\[
D(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = Br
\]

This brief reports a study that considers the CV motor as it is and develops a control method based on the general idea that the disturbance on the trajectory of the end-effector due to the velocity fluctuation in the CV motor can be compensated by the controller for the servomotor. A five-bar hybrid mechanism with 2-degrees of freedom (2-DOF) is taken as an example in this study. Section II outlines the dynamic model of the five-bar hybrid mechanism including its electric motors. Section III presents the proposed control method with its stability analysis. Section IV gives simulation results. Section V is a conclusion.
where $D'(q')$ is the inertia matrix defined as follows:

$$
D'(q') = \begin{bmatrix}
  d_{11} & 0 & d_{13} & 0 \\
  0 & d_{22} & 0 & d_{24} \\
  d_{31} & 0 & d_{33} & 0 \\
  0 & d_{42} & 0 & d_{44}
\end{bmatrix}
$$

where

$$
\begin{align*}
d_{11} &= m_1 \frac{r_1^2}{2} + m_3 \left( L_1^2 + r_3^2 \right) + 2L_1r_3 \cos(q_1 + \delta_3) + J_1 + J_3 \\
d_{13} &= d_{31} = m_3 \left( \frac{r_1^2}{2} + L_1r_3 \cos(q_1 + \delta_3) \right) + J_3 \\
d_{22} &= m_2 \frac{r_2^4}{2} + m_4 \left( \frac{r_4^2}{2} + L_2r_4 \cos(q_1 + \delta_4) \right) + J_2 + J_4 \\
d_{24} &= d_{42} = m_4 \left( \frac{r_4^2}{2} + L_2r_4 \cos(q_1 + \delta_4) \right) + J_2 \\
d_{33} &= m_3 r_3^2 + J_3 \\
d_{44} &= m_4 r_4^2 + J_4
\end{align*}
$$

and $C'(q', \dot{q'})'\dot{q'}$ contains the centrifugal and Coriolis terms, and it is defined as follows:

$$
C'(q', \dot{q'}) = \begin{bmatrix}
  h_1 \dot{q}_3 & 0 & h_1(\dot{q}_1 + \dot{q}_3) & 0 \\
  0 & h_2 \dot{q}_4 & h_2(\dot{q}_2 + \dot{q}_4) & 0 \\
  -h_1 \dot{q}_1 & 0 & 0 & 0 \\
  0 & -h_2 \dot{q}_2 & 0 & 0
\end{bmatrix}
$$

where $h_1 = -m_3 L_1 r_3 \sin(q_3 + \delta_3)$, and $h_2 = -m_4 L_2 r_4 \sin(q_4 + \delta_4)$.

$g'(q')$ is the gravity vector, and it is defined as shown in (5) at the bottom of the page where $g = 9.81 \text{ m/s}^2$ is the gravitational acceleration constant.

$B\tau$ is the input torque. Noticing that the actuated joints are Joint 1 and 2, respectively, we have

$$
B = \begin{bmatrix}
  1 & 0 \\
  0 & 1 \\
  0 & 0 \\
  0 & 0
\end{bmatrix}, \quad \tau = \begin{bmatrix}
  \tau_1 \\
  \tau_2
\end{bmatrix}
$$

where $\tau_1$ and $\tau_2$ are the torques applied to Joint 1 and 2, respectively. The following mapping can be obtained:

$$
q = \begin{bmatrix}
  q_1 \\
  q_2
\end{bmatrix}, \quad q' = \alpha(q'),
$$

The dynamic model of the 2-DOF five-bar mechanism is then given as follows [9], [10]:

$$
\begin{align*}
D(q')\ddot{q} + C(q', \dot{q'})\dot{q} + g(q') &= \tau \\
\dot{q'} &= \rho(q')\dot{q} \\
q' &= \sigma(q)
\end{align*}
$$

where

$$
\begin{align*}
D(q') &= \rho^T(q')D'(q')\rho(q') \\
C(q', \dot{q'}) &= \rho^T(q')C'(q', \dot{q'})\rho(q') \\
g(q') &= \rho^T(q')g'(q')
\end{align*}
$$

where

$$
\rho(q') = \Psi_q^{-1}(q') = \begin{bmatrix}
  0 & 0 \\
  0 & 0 \\
  0 & 1
\end{bmatrix}
$$

and $\Psi_q(q')$ is the Jacobian matrix of the following function

$$
\Psi(q') = \begin{bmatrix}
  \phi_1(q') \\
  \phi_2(q')
\end{bmatrix}
$$

with respect to the vector $q'$. The mapping $\rho(q', \dot{q'})$ is the derivative of $\rho(q')$ with respect to time $t$. For the particular five-bar mechanism under consideration, $q' = \sigma(q)$ is expressed by

$$
q_1 = \tan^{-1}(\sqrt{A^2 + B^2 - C^2/C} + \tan^{-1}(A/B) - q_2
$$

where $A = 2 L_4 \lambda B = 2 L_4 \mu$, and $C = L_3^2 - L_2^2 - \lambda^2 - \mu^2$, and $\lambda = 2 L_2 \cos(q_2) - L_1 \cos(q_1) + L_5$

$$
q_3 = \tan^{-1}(\mu + L_4 \sin(q_2 + q_4))/\lambda + L_4 \sin(q_2 + q_4)) - q_1
$$

B. Dynamic Model of Electric Motors

Based on a Newtonian kinetics law, sum of torques acting upon the motor shaft gives [12], [13]

$$
t_d = ki = t_L + bw + j \frac{dw}{dt}
$$

where

$$
\phi(q') = \begin{bmatrix}
  \phi_1(q') \\
  \phi_2(q')
\end{bmatrix} = \begin{bmatrix}
  L_1 \cos(q_1) + L_3 \cos(q_1 + q_3) - L_2 \cos(q_2) - L_4 \cos(q_2 + q_4) - L_5 \\
  L_1 \sin(q_1) + L_3 \sin(q_1 + q_3) - L_2 \sin(q_2) - L_4 \sin(q_2 + q_4)
\end{bmatrix} = 0
$$

$$
g'(q') = g \begin{bmatrix}
  m_1 r_1 \cos(q_1 + \delta_1) + m_3 (L_1 \cos(q_1) + r_3 \cos(q_1 + q_3 + \delta_3)) \\
  m_2 r_2 \cos(q_2 + \delta_2) + m_4 (L_2 \cos(q_2) + r_4 \cos(q_2 + q_4 + \delta_4)) \\
  m_3 r_3 \cos(q_1 + q_3 + \delta_3) \\
  m_4 r_4 \cos(q_2 + q_4 + \delta_4)
\end{bmatrix}
$$
where \( t_d \) is the magnetic motor torque, \( b \) the viscous damping coefficient, \( j \) the moment of inertia of the motor, \( t_L \) the load torque, and \( k \) the motor constant. Rearranging (15) leads to
\[
t_d - t_L = bw + j \frac{dq}{dt}.
\]
(16)

When \( t_d - t_L = bw \) the motor is at the steady state and runs at a constant speed. For the CV motor, during the operating process, the load torque \( t_d \) varies periodically, so the speed fluctuation will be present in the motor.

**C. Dynamic Model of the Hybrid Machine**

As shown in Fig. 1, the CV motor drives the actuated joint 1 while the servomotor drives the actuated joint 2. In addition, Link 1 and Link 2 are directly mounted to the motor shafts. Integration of (8) and (15) leads to the dynamic model of the hybrid machine as follows:
\[
\begin{align*}
\{ & \textbf{D}(q') \ddot{q} + C(q, q') \dot{q} + Bq + g(q') = KI \\
& \dot{q}' = \rho \dot{q} \\
& \dot{\rho} = \sigma(q)
\end{align*}
\]
(17)

where \( \textbf{D}(q') = (D(q') + J), J = \text{diag}[j_1 j_2], B = \text{diag}[b_1 b_2], K = \text{diag}[k_1 k_2], I = [i_1 i_2], \) and \( j_i, b_i, k_i, i_i (i = 1, 2) \) are the moment of inertia, the viscous damping coefficient, the motor constant, and the armature current of the \( i \)th motor, respectively.

**III. CONTROLLER DESIGN AND ANALYSIS**

Notice that the dynamic model derived as having a coupling relation between two motor variables, \( q_1 \) and \( q_2 \). Therefore the speed fluctuation in the CV motor should affect the motion \( q_2 \). It is then possible that the controller for the servomotor may offset the effect due to the speed fluctuation in the CV motor. Thus, the control problem can be stated as: Given the operating speed of the CV motor, \( q (\text{rad/s}) \), and the desired trajectory \( q_{d1}, q_{d2}, q_{d3} \), determine a control law for the servomotor to follow the desired trajectory with the consideration of the speed fluctuation in the constant speed motor. Furthermore, we assume the desired \( q_{d1}, q_{d2}, q_{d3} \) are bounded and notice that \( i_1 \) is a constant.

A control method proposed by Slotine and Li [15] is adapted to the control problem here. Let \( \dot{q}_{d1} = [q_{d1} q_{d2} q_{d3}]^T, \dot{q}_{d1} = [\dot{q}_{d1} \dot{q}_{d2} \dot{q}_{d3}]^T \). Further we have \( \dot{q}_{d1} = a \) and \( \dot{q}_{d2} = at \) (for simplicity). Also let \( e = q - q_{d1}, \dot{q}_i = \dot{q}_{d1} - \Lambda e \), and
\[
\dot{s} = \dot{q} - \dot{q}_i = \dot{e} + \Lambda e = [s_1 \ s_2]^T
\]
(18)

where the diagonal matrix \( \Lambda = \text{diag}[0 \ \lambda] \), and \( \lambda \) is a positive constant number.

The dynamic model of the hybrid mechanism can be written in terms of \( s \) as shown in (19) at the bottom of the page. Let
\[
\begin{align*}
f_1 &= g_1(q') + \ddot{q}_{d1}(q') \dot{q}_{d1} + \ddot{q}_{d2}(q') \dot{q}_{d2} \\
&+ c_{11}(q', q') \dot{q}_{d1} + c_{12}(q', q') \dot{q}_{d2} \\
f_2 &= g_2(q') + \ddot{q}_{d1}(q') \dot{q}_{d1} + \ddot{q}_{d2}(q') \dot{q}_{d2} \\
&+ c_{21}(q', q') \dot{q}_{d1} + c_{22}(q', q') \dot{q}_{d2}.
\end{align*}
\]
(20)

Assume the armature current in the CV motor is
\[
i_1 = b_1 a / k_1
\]
(22)

and let
\[
i_2 = (b_2 \dot{q}_{d2} + f_2 - k_d s_2 + \dot{i}_r) / k_2
\]
(23)

Then, we have
\[
\textbf{D}(q') s + C(q', \dot{q}') s + B s = [-f_1 \ \dot{i}_r]
\]
(24)

where the matrix \( B = \text{diag}[b_1 b_2 + k_d], k_d > 0 \) is a design parameter, and \( \dot{i}_r \) is a design signal.

**Remark 1:** The matrix \( \Lambda \) in (18) is a non-negative–definite matrix while its counterparts in [14]–[16] are the positive–definite matrices.

The stability analysis of the proposed controller is presented as follows. The dynamic model of a five-bar linkage should have the following properties:

**Lemma 1** [9], [10]:
1) The mass matrix \( \textbf{D}(q') \) is symmetric and positive–definite.
2) By a suitable arrangement of elements of the matrix, \( \textbf{C}(q', \dot{q}') \), \( (1/2) \textbf{D}(q') - \textbf{C}(q', \dot{q}') \) is skew.

**Proof:** Since the matrix \( \textbf{D}(q') \) and the matrix \( \textbf{J} \) are symmetric and positive–definite, the matrix \( \textbf{D}(q') \) is also symmetric and positive–definite. The elements of the matrix \( \textbf{C}(q', \dot{q}') \) are made up of the partial derivates of the matrix \( \textbf{D}(q') \), so \( (1/2) \textbf{D}(q') - \textbf{C}(q', \dot{q}') \) is also skew.

**Theorem 1:** If i) the current in the CV motor is given by (22), ii) the design signal \( \dot{i}_r \) in (23) is defined as follows:
\[
\dot{i}_r = \begin{cases} 
-|s_1|/s_2, & \text{if } s_2 \neq 0 \\
0, & \text{if } s_2 = 0
\end{cases}
\]
(25)

and iii) the control law is given by (23) and (25), then the trajectory tracking errors \( \dot{e}_1(t), \dot{e}_2(t) \), and \( \dot{e}_3(t) \) will exponentially converge to zero when time \( t \) tends to infinite large, and \( \dot{e}_1(t) \) is bounded for any time \( t \geq 0 \).

**Proof:** Consider the following energy term as the Lyapunove function
\[
V = \frac{1}{2} s^T \textbf{D}(q') s.
\]
(26)
We can find along the solution of (24)
\[
\dot{V} = s^T D(q') s + \frac{1}{2} s^T \dot{B}(q') s
\]
\[
= s^T \left( \frac{1}{2} \dot{B}(q') - C(q', \dot{q}') - \dot{B} \right) s + s^T \left[ -f_1 \right]
\]
\[
= -s^T \dot{B} s - s_1 f_1 + s_2 \dot{e}_r
\]

(27)
where the identical relation \( s^T \left((1/2) \dot{B}(q') - C(q', \dot{q}')\right) s = 0 \) is used to derive (27). Furthermore, using (25) we can verify
\[
-s_1 f_1 + s_2 \dot{e}_r = -s_1 f_1 - |s_1 f_1| \leq 0.
\]
(28)
Substituting (28) into (27) yield
\[
\dot{V} \leq -s^T \dot{B} s
\]

According to the Lyapunov’s stability theorem, \( s(t) \) will exponentially tend to zero when \( t \) tends to infinite large. From the standard stable filter theory, this then implies that errors \( e_1(t), e_2(t), \) and \( e_3(t) \) will exponentially converge to zero when \( t \) tends to infinite large. Furthermore, since \( e_3(t) \) exponentially converges to zero, \( e_1(t) \) will be normally bounded. This then concludes the proof.

**Remark 2:** The control law given by (22), (23), and (25) is discontinuous and needs to be smoothed at the implementation level. For this, we replace (25) by
\[
\dot{e}_r = \frac{|s_1 f_1| s_2}{\delta + s_2^2}
\]
(29)
where \( \delta > 0 \) is the boundary layer thickness. Such a smoothing method [14] generally leads to the conclusion that trajectory tracking error is globally uniformly ultimately bounded. In fact, the simulation results in the next section confirm the boundedness with this method.

**IV. SIMULATION VERIFICATION**

The parameters of the five-bar mechanism and two motors are listed in Tables I and II, respectively. Noticing that the desired trajectory in Link 1 should be a line, while the desired trajectory in Link 2 can be any curve. Assume the following trajectories:
\[
q_{1d} = at, \quad q_{2d} = 2\pi \left( \frac{6 t^5}{t_f^5} - 15 \frac{t^4}{t_f^4} + 10 \frac{t^3}{t_f^3} \right)
\]
(30)
where \( t_f = 4(s) \), and \( a = 15 \text{(rad/s)} \). The simulation starts from \( t_0 = 0(\text{s}) \) and stops at \( t_f = 4(\text{s}) \).

In the first simulation, the controller parameters are assumed to be \( \Lambda = \text{diag}[0, 15], k_d = 30 \). Fig. 2 depicts the error curves and torques. Specifically, Fig. 2(a) and (b) shows that the maximum of the angular displacement error and the maximum of the angular velocity error of Link 1 are, respectively, 0.08 and 0.25, and they look bounded. Fig. 2(c) and (d) shows that the errors of the angular displacement and the angular velocity of Link 2 are very small, specifically \( 10^{-4} \) and \( 10^{-3} \), respectively. Theoretically, the velocity error of Link 1 is related to the boundary layer thickness \( \delta > 0 \). The less the boundary layer thickness the less the velocity error of Link 1 is. However, in implementation, reduction of the boundary layer thickness can be hindered due to the computational problem. Fig. 2(e) and (f) depicts the curves of the torques (where the torques are the product of the motor constant \( k_i \) and the armature current \( i_a = 1, 2 \)). Fig. 3(a) and (d) depicts the curves of the desired (solid line) and simulated (dashed line) trajectories, respectively. The results in these figures are excellent except for the one shown in Fig. 3(b), which is expected, as this is associated with the speed fluctuation in the CV motor.

In the second simulation, a flywheel (9 kg-m²) is attached to the shaft of the CV motor because the flywheel is notably a
means of reduction of the speed fluctuation in the CV motor. The controller parameters are the same as those in the first simulation. The simulation results are depicted in Fig. 4. Comparing the simulation results in Fig. 4 with those in Fig. 2, it is observed that after a flywheel is attached, the overall tracking performance of the hybrid machine is improved significantly.

Fig. 4(a) and (b) shows that the angular displacement error and the angular velocity error of Link 1 are reduced to the order of $10^{-2}$. However, there is no significant improvement in the angular displacement and velocity of the servomotor [see Fig. 4(c) and (d)]. Fig. 4(e) and (f) shows the torques in the CV motor and the servomotor, respectively, which do not increase with the inclusion of the flywheel in the system, compared to the results of the torques depicted in Fig. 2(e) and (f). A further simulation can show the following trend, i.e., with the increase of the inertia of flywheel, (1) the angular displacement error and the angular velocity error of Link 1 continue to reduce, (2) the angular displacement error and the angular velocity error of Link 2 keep the orders of $10^{-4}$ and $10^{-3}$, respectively, and (3) the control energy remains the same.

V. CONCLUSION

The proposed controller for the five-bar hybrid mechanism with two DOF appears effective. In this case, the CV motor is not controllable while the servomotor is controllable. The proposed controller is adapted from the one proposed by Slotine and Li in [15] and [16], and is asymptotically convergent. The inclusion of a flywheel on the shaft of the CV motor has a significantly positive effect on the control performance of the hybrid machine system.

REFERENCES