Graphical Inference Methods for Fault Diagnosis based on Information from Unreliable Sensors

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Abstract—In this paper, we study the application of decoding algorithms to the multiple fault diagnosis (MFD) problem. Prompted by the resemblance between graphical representations for MFD problems and parity check codes, we develop a suboptimal iterative belief propagation algorithm (BPA) that is based on the graphical inference method for low density parity check codes. Our simulation results suggest that the algorithm performance strongly depends on the connection density and the reliability of the alarm network. In particular, when the connection density is low and when the alarms and/or connections are unreliable, the algorithm performs almost optimally, i.e., it converges to the solution with the highest posterior probability most of the times. We also provide analytical bounds on the performance of the algorithm for special classes of systems in our framework.

Keywords—Multiple fault diagnosis, belief propagation, unreliable sensors, alarm correlation.

I. INTRODUCTION

The increasing complexity of modern systems is imposing new demands on system maintenance tools, which are required to diagnose and repair the system as rapidly as possible in order to return it to correct operation [1]. Failure diagnosis is studied in applications ranging from aircraft systems [2] to medical diagnosis [3], and others. In these applications, the problem is to identify the set of potential failure sources based on available sensory information, such as flashing alarms.

Among several proposed formulations of the problem of fault diagnosis, graphical methods (see, for example, [4], [5]) have been among the most extensively studied ones. In this paper, we consider Rao’s directed system graph [4], which is presented by a set of vertices $V$ and a set of directed edges $E$, representing respectively the components of the system (i.e., failure sources and alarms) and the fault propagations between them. For diagnostic purposes, some of these components are equipped with an alarm that rings upon the arrival of one or more faulty or unacceptable conditions, as captured by the nodes connected to a particular component. This paper focuses on zero-time systems which essentially means that fault propagation among the nodes of this directed graph is instantaneous [4]. Traditionally, tests are commonly classified into two types: symmetrical tests, where the alarms are perfectly reliable, and asymmetrical tests, where the faulty condition does not always trigger the corresponding alarm [1]. In this paper, we consider unreliable tests and treat the reliability of the connections and the alarms separately. This allows our model to be more general than both symmetrical and asymmetrical tests.

In the MFD problem, we are given a directed graph which captures the first order cause-effect dependency between the components and the alarms as illustrated in Fig. 1(a). This graph can be used to construct a bipartite graph where one set of nodes represents the components, the other set represents the alarms, and the connections represent the dependencies between components and alarms; see Fig. 1(b). Note that this bipartite graph is similar to the graphical representation of parity check codes [6], except that the checking operation is the Boolean OR operation instead of the Boolean XOR operation used for parity check codes.

In this paper, we study the problem of multiple fault diagnosis by applying appropriately modified decoding algorithms. Using this decoding perspective as the starting point, we obtain novel insights, characterizations and results about the diagnostic problem on a given bipartite diagnosis graph. In particular, we use the belief propagation algorithm (BPA) to solve the problem of multiple fault diagnosis for general systems where both connections and alarms can be unreliable. Our approach allows us to obtain an upper bound on the probability that the BPA makes an erroneous diagnosis for a special class of asymmetric systems. Simulation results for
several systems suggest that, under certain conditions, the BPA can perform very closely to the optimal one (i.e., the approach that goes through all possible combinations of failure sources and returns the most likely one given the observations). Our theoretical analysis and simulations support our analysis of the BPA performance on the MFD problem and agree with the observations in [7].

The organization of this paper is as follows. Section II formulates the MFD problem and describes different types of systems under diagnosis, classified according to the reliability of underlying connections and alarms. In Section III, we develop MFD algorithms using graphical inference methods for general systems. We analytically obtain upper bounds on the probability of diagnostic error in Section IV and provide examples and simulation results in Section V. We conclude in Section VI.

II. MULTIPLE FAULT DIAGNOSIS PROBLEM

A. MFD Problem and Graphical Models

The fault propagation model of a system is given by the directed-graph \( G = (V, E) \), where \( V \) is the set of vertices corresponding to the set \( S \) of \( N \) components (or potential failure sources) \( \{S_1, ..., S_N\} \) of the system and a directed edge \((S_i, S_j) \) captures the propagation of a fault at component \( S_i \) to component \( S_j \). For diagnostic purposes, some of the components could be equipped with a binary outcome alarm that rings upon the arrival of the faulty or unacceptable conditions. Let \( \mathcal{A} = \{A_1, ..., A_M\} \) be the finite set of \( M \) available alarms. The testing capabilities of the alarms are presented by the test matrix \( T = [t_{mn}] \) where, for the simplest type of configuration we study (i.e., the case of symmetrical tests), \( t_{mn} = 1 \) if there exists a path from component \( S_n \) to alarm \( A_m \), and \( t_{mn} = 0 \) otherwise. For example, the symmetrical test matrix \( T \) of the system in Fig. 1 is presented in Table I.

<table>
<thead>
<tr>
<th>Alarms</th>
<th>Components</th>
<th>( S_1 )</th>
<th>( S_2 )</th>
<th>( S_3 )</th>
<th>( S_4 )</th>
<th>( S_5 )</th>
</tr>
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<tbody>
<tr>
<td>( A_1 )</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
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</table>

For the symmetrical test case, we assume that if at least one of the failure sources connected to alarm \( A_m \) is faulty, it will output one (\( A_m = 1 \)); otherwise, it will output zero (\( A_m = 0 \)). In other words, the alarm performs the OR operation of the states of the failure sources it is connected to. Associated with each component \( S_n \) are the prior probabilities of faulty \( (S_n = 1) \) and successful \( (S_n = 0) \) operation given by \( p_n^1 \) and \( p_n^0 \), respectively \( (p_n^1 + p_n^0 = 1) \). We also adopt the common assumption that a priori components fail independently. Then, the problem of the multiple fault diagnosis is to find the most likely combination \( S^* \) of the components given the observation \( \mathcal{A} \) of the alarms. This can be formulated as

\[
S^* = \arg\max_S \Pr(S | \mathcal{A}). \quad (1)
\]

Note that this MFD problem is very similar to the decoding problem of linear codes in that the alarms replace the parity bits and the OR operation of each alarm replaces the XOR operation of each linear constraint (the main difference is that in the coding case one observes the status –0 or 1– of both components and alarms). Before applying the belief propagation to the MFD problem, we introduce models and notation that allow us to capture the reliability of the connections and the alarms.

B. System Classification

This section introduces the various types of (connection and alarm) uncertainties that we would like to include in our study. In the case of asymmetric tests, each alarm cannot provide a false alarm but its detection probability may be nonunary, which means that the test matrix \( T \) is not binary [5]. The asymmetric test is illustrated in Fig. 3(a) where \( t_{mn} \) is the conditional probability that alarm \( A_m \) is triggered if a fault occurs in component \( S_n \). For convenience, we also call this the Z model since the connection model resembles the Z letter.

More generally, the alarm can also be falsely triggered although the connected component is still functional. In this case, the connection is modeled by the “binary channel” with two probabilities, \( t_{mn}^0 \) and \( t_{mn}^1 \), as indicated in Fig. 3(b). This model is called X model and is associated with two test matrices \( T^0 \) and \( T^1 \) of the form depicted in Table I.

<table>
<thead>
<tr>
<th>( S_n )</th>
<th>( A_m )</th>
<th>( t_{mn} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_n )</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>( S_n )</td>
<td>0</td>
<td>( t_{mn} )</td>
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<table>
<thead>
<tr>
<th>( S_n )</th>
<th>( A_m )</th>
<th>( t_{mn} )</th>
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<tr>
<td>( S_n )</td>
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<tr>
<td>( S_n )</td>
<td>0</td>
<td>( t_{mn} )</td>
</tr>
</tbody>
</table>

In the above X and Z models, the connections are unreliable but the alarms themselves are assumed to be reliable. More generally, alarm outcomes can also be erroneously transmitted to the observer; therefore, we can model each unreliable alarm \( A_m \) using the “binary channel” associated with two conditional probabilities \( Pa_m^0 \) and \( Pa_m^1 \), as in Fig. 3. This model is called Y model and allows both alarms and connections to be unreliable. Note that the X and Z models are special cases of the more general Y model. In the remainder of this paper, we study the BPA for this general Y model.

![Fig. 2. Different types of unreliable connections.](image)

In the above X and Z models, the connections are unreliable but the alarms themselves are assumed to be reliable. More generally, alarm outcomes can also be erroneously transmitted to the observer; therefore, we can model each unreliable alarm \( A_m \) using the “binary channel” associated with two conditional probabilities \( Pa_m^0 \) and \( Pa_m^1 \), as in Fig. 3. This model is called Y model and allows both alarms and connections to be unreliable. Note that the X and Z models are special cases of the more general Y model. In the remainder of this paper, we study the BPA for this general Y model.

![Fig. 3. Y model system: alarms and connections can be unreliable.](image)
III. MFD USING BELIEF PROPAGATION ALGORITHM

The bipartite graph representing the MFD problem resembles the graph of a linear code, except that (i) the checking operation is the Boolean OR operation (instead of the Boolean XOR operation used with parity check codes) and (ii) the information available involves only the status of the alarms. Given the alarm outcome \( A \), the MFD problem aims to find the most likely component configuration \( S^* \) as formulated in (1). Algorithms for the determination of \( S^* \) in belief networks can be found in [8]. In the next section, we adapt Gallager’s approach in [9] to determine the marginal posterior probabilities of each component when the bipartite graph is acyclic and under the Y model; then, we extend the results to develop the BPA for graphs that may contain cycles. The final algorithm is reminiscent to the suboptimal sum-product algorithm, which was proposed by MacKay [10] for the decoding of low-density parity check codes.

A. Probabilistic Inference in Acyclic Graphs

Following the notation in [10], let \( \mathcal{N}(m) = \{S_n : t^1_{mn} \neq 0\} \) denote the parents of alarm \( A_m \), and let \( \mathcal{M}(n) = \{A_m : t^0_{mn} \neq 0 \text{ or } t^1_{mn} \neq 0\} \) denote the children of component \( S_n \). We also use \( \mathcal{N}(m) \setminus n \) to denote the set \( \mathcal{N}(m) \) with component \( S_n \) excluded, and use \( \mathcal{M}(n) \setminus m \) to denote the set \( \mathcal{M}(n) \) with alarm \( A_m \) excluded. Given an acyclic bipartite graph, we can construct an equivalent tree-structure for each component \( S_n \) as in Fig. 4, where each line rising from \( S_n \) represents an alarm \( A_m \in \mathcal{M}(n) \). Besides component \( S_n \), this line contains all other components \( S_{n'} \in \mathcal{N}(m) \setminus n \) and forms the first tier from \( S_n \). Similarly, the lines rising from each component in the first tier form the second tier, and so on.

![Equivalent tree-structure of acyclic bipartite graph for \( S_n \).](image)

Assume now that all components are marginally independent and that the alarm outcomes are conditionally independent given the component combination. Using these assumptions, we can prove the following theorem for the Y model.

**Theorem 1:** Consider the equivalent tree-structure for component \( S_n \). For each alarm \( A_m \in \mathcal{M}(n) \), let \( P^0_{mn} \) and \( P^1_{mn} \) be the conditional probabilities that component \( S_{n'} \in \mathcal{N}(m) \setminus n \) in the first tier is 0 or 1 given the status of component \( S_n \), and let \( \mathcal{E}_n \) be the event that alarms \( A_m \in \mathcal{M}(n) \) are satisfied (i.e., \( A_m = 0 \) or 1 depending on the observed outcome of alarm \( A_m \)). Then

\[
\Pr(S_n = 0 | \mathcal{E}_n) = \frac{p^0_n}{p^1_n} \prod_{A_m \in \mathcal{M}(n)} A_m + (1 - 2A_m)[Pa^0_mX_{mn} + (1 - Pa^1_m)(1 - X_{mn})]
\]

\[
\Pr(S_n = 1 | \mathcal{E}_n) = \frac{p^1_n}{p^0_n} \prod_{A_m \in \mathcal{M}(n)} A_m + (1 - 2A_m)[Pa^1_mY_{mn} + (1 - Pa^0_m)(1 - Y_{mn})]
\]

where

\[
X_{mn} = t^0_{mn} \prod_{s_{n'} \in \mathcal{N}(m) \setminus n} [t^0_{mn'}P^0_{mn'} + (1 - t^1_{mn'})P^1_{mn'}],
\]

\[
Y_{mn} = (1 - t^1_{mn}) \prod_{s_{n'} \in \mathcal{M}(m) \setminus n} [t^0_{mn'}P^0_{mn'} + (1 - t^1_{mn'})P^1_{mn'}].
\]

**Proof:** Using Bayes’ rule and the assumption that alarm outcomes are conditionally independent, we have

\[
\frac{\Pr(S_n = 0 | \mathcal{E}_n)}{\Pr(S_n = 1 | \mathcal{E}_n)} = \frac{p^0_n}{p^1_n} \prod_{A_m \in \mathcal{M}(n)} \frac{\Pr(A_m | S_n = 0)}{\Pr(A_m | S_n = 1)}.
\]

For the OR operation to be zero, all of the inputs must be zero; therefore, if we let \( X_{mn} \) be the probability that \( A_m = 0 \) given \( S_n = 0 \) and ignore (for the time being) the effect of the binary channel from \( A_m \) to the observer, we obtain

\[
X_{mn} = t^0_{mn} \prod_{s_{n'} \in \mathcal{N}(m) \setminus n} [t^0_{mn'}P^0_{mn'} + (1 - t^1_{mn'})P^1_{mn'}].
\]

Including the effect of the binary channel from \( A_m \), we have

\[
\Pr(A_m = 0 | S_n = 0) = Pa^0_mX_{mn} + (1 - Pa^1_m)(1 - X_{mn}),
\]

and, of course,

\[
\Pr(A_m = 1 | S_n = 0) = 1 - \Pr(A_m = 0 | S_n = 0).
\]

Combining (7) and (8), we can write

\[
\Pr(A_m | S_n = 0) = A_m + (1 - 2A_m)[Pa^0_mX_{mn} + (1 - Pa^1_m)(1 - X_{mn})].
\]

Similarly, we get

\[
\Pr(A_m | S_n = 1) = A_m + (1 - 2A_m)[Pa^1_mY_{mn} + (1 - Pa^0_m)(1 - Y_{mn})],
\]

where

\[
Y_{mn} = (1 - t^1_{mn}) \prod_{s_{n'} \in \mathcal{M}(m) \setminus n} [t^0_{mn'}P^0_{mn'} + (1 - t^1_{mn'})P^1_{mn'}].
\]

The proof is completed by replacing (9), (10) into (5). \( \square \)

We can also use Theorem 1 to find the marginal probability of the components in the first tier; the only modification required is to exclude alarm \( A_m \) that is connected to \( S_n \) in the product of (2). By induction, this iteration process can be used to find the marginal posterior probabilities for all components as long as the graph does not close upon itself (i.e., it is acyclic). These probabilities are used in the belief propagation algorithm for the MFD problem, which is introduced in the next section.

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B. Belief Propagation Algorithm

Using the result in Theorem 1, we develop the BPA for the general MFD problem where the alarms and connections can be unreliable. This algorithm iteratively updates four quantities $Pd_{mn}^0$ and $Pu_{mn}^x$ associated with each connection $(m,n)$ between alarm $A_m$ and component $S_n$. The quantity $Pd_{mn}^0$ is the belief (probability) that component $S_n$ has value $S_n = x$, $x \in \{0,1\}$ given the information obtained from the alarms in $M(n) \setminus n$. The quantity $Pu_{mn}^x$ is the conditionally marginal probability that alarm $A_m$ is satisfied if component $S_n$ is fixed at value $S_n = x$. By marginal probability, we mean that this quantity is summed over all possible combinations of components in $N(m) \setminus n$. The BPA can be presented as follows.

Initialization: We initialize using the prior probabilities by setting

$$Pd_{mn}^0 = P_n^0$$  \hspace{1cm} (12)$$

$$Pd_{mn}^1 = P_n^1$$  \hspace{1cm} (13)$$

for every connection $(m,n)$.

Iteration:

- Downward step: In this step, we run through all alarms $A_m$'s and for each $S_n \in N(m)$, we compute the beliefs $Pu_{mn}^x$ by using (9) and (10) with $Pd_{mn}^0$ and $Pu_{mn}^1$, in (6) and (11) replaced by $Pd_{mn}^0$ and $Pd_{mn}^1$, respectively,

$$Pu_{mn}^x = \Pr(A_m \text{ is satisfied} | S_n = x).$$  \hspace{1cm} (14)$$

- Upward step: Assuming that all alarms are conditionally independent, from the remark after Theorem 1, we exclude $A_m$ in (2) and update $Pd_{mn}^0$ and $Pd_{mn}^1$ for each component $S_n$ by

$$Pd_{mn}^0 = \alpha_{mn}P_n^0 \prod_{A_m \in M(n) \setminus m} Pu_{m',n}^0,$$  \hspace{1cm} (15)$$

$$Pd_{mn}^1 = \alpha_{mn}P_n^1 \prod_{A_m \in M(n) \setminus m} Pu_{m',n}^1,$$  \hspace{1cm} (16)$$

where $\alpha_{mn}$ is the normalization constant chosen such that $Pd_{mn}^0 + Pd_{mn}^1 = 1$.

Termination: After an appropriate number of iterations, we compute the “pseudo” posterior probabilities of each component $S_n$ as

$$P_n^0 = \alpha_n P_n^0 \prod_{A_m \in M(n)} Pu_{m'n}^0,$$  \hspace{1cm} (17)$$

$$P_n^1 = \alpha_n P_n^1 \prod_{A_m \in M(n)} Pu_{m'n}^1,$$  \hspace{1cm} (18)$$

where $\alpha_n$ is again the normalization constant. Then the state of component $S_n$ is determined by the decision $P_n^0 \geq P_n^1$.

We now briefly discuss some implementation issues:

- Let $I$ be the number of iterations. In practice, our simulation shows that the BPA with $I = \min(M,N)$ yields acceptable results (see Section V).

- If $K \equiv \max_n(|M(n)|)$ and $L \equiv \max_m(|N(m)|)$ are the maximum numbers of connections from each component and alarm, the complexity of the proposed BPA can be easily determined to be $O(IML^2 + IKN^2)$. Clearly, since $K$ and $L$ are constant values and if $I$ is set to $\min(M,N)$, the complexity of the BPA is quadratic in $M$ or $N$, which makes this iterative algorithm very efficient compared to other multiple fault diagnosis algorithms.

It was shown in [8] that for an acyclic graph, the “pseudo” posterior probabilities in (17) and (18) would converge to the exact marginal posterior probabilities for each component. If we simply ignore cycles in the graph of a given MFD problem, then we have a suboptimal algorithm which uses the “pseudo” posterior probabilities to determine the most likely solution given the alarm observation; however, as we will see, the BPA still performs very well (in terms of finding the solution of (1)) under certain conditions. The following section gives some theoretical analysis of the performance of this suboptimal algorithm compared to the optimal one for a special class of systems.

IV. Probability of Diagnosis Error Using BPA

In this section, we focus on the asymmetric Z model which has been the focus of many earlier studies of the MFD problem [5]. In this case, $t_{mn}^0 = 1$ for all connections, and $Pd_{m}^0 = 1$, $Pu_{m}^1 = 1$ for all alarms (i.e., all alarms are reliable). In order to evaluate the performance of the BPA, we will say that the BPA has made an erroneous diagnosis when it does not produce the most likely solution given an alarm observation.

Lemma 1: If all components are conditionally independent given the alarm observation $A$ and if the BPA determines the exact values of the marginal posterior probabilities for each component, then the BPA produces the most likely solution.

Proof: If all components are conditionally independent given the alarm observation, then

$$\Pr(S|A) = \prod_n \Pr(S_n|A).$$  \hspace{1cm} (19)$$

From the correct marginal posterior probabilities, the BPA determines $S_n$ by maximizing $\Pr(S_n|A)$, which also maximizes $\Pr(S|A)$ following (19). Therefore, the BPA produces the most likely solution.

Lemma 2: Assume an arbitrary bipartite graph (not necessarily acyclic) under the Z model. If all alarms are zero ($A = 0^M$), then all components are conditionally independent given $A$. Moreover, the BPA always produces the most likely solution in this case.

Proof: Consider an unreliable connection $t_{mn}$ from component $S_n$ to alarm $A_m \in M(n)$; the output of this connection (before entering the alarm) is a binary random variable which is a function of $S_n$. Let us denote this variable $C_{mn} = C_{mn}(S_n)$. Since each alarm performs the OR operation of its inputs, in terms of this notation, we have

$$A_m = \text{OR}_{S_n \in N(m)} (C_{mn}).$$  \hspace{1cm} (20)$$
Then replacing into (17) and (18), we get that when marginal posterior probabilities for each component. In fact, the true marginal posterior probabilities \( \Pr(S_n) \) are conditionally independent given \( A = 0^M \).

To prove the second part of Lemma 2, we need to show that when \( A = 0^M \), the BPA returns the exact values of the marginal posterior probabilities. We are now ready to derive an upper bound on the probability that the diagnosis using BPA deviates from the most likely solution, which completes the proof.

Note that Lemma 2 is also true for the X model with a similar proof. We are now ready to derive an upper bound on the probability of diagnosis error of the BPA.

**Theorem 2**: For the Z model where all prior probabilities are equal \( p_0^n = p \) and all connections satisfy \( t_{mn} = t \), the probability that the diagnosis using BPA deviates from the most likely solution is upper bounded by

\[
P_e \leq 1 - [p(1 - t)^K + 1 - p]^N.
\]

**Proof**: Using Lemma 2, we get

\[
P_e \leq 1 - \Pr(A = 0^M).
\]

If \( K = \max_n(|M(n)|) \), we have

\[
\Pr(A = 0^M) = \sum_{S^N} \Pr(A = 0^M | S^N) \Pr(S^N)
\]

\[
\geq (1 - p)^N + \sum_{n=1}^{N} \left( \begin{array}{c} N \\ n \end{array} \right) p^n (1 - p)^{N-n} (1 - t)_{K_n}
\]

\[
= [p(1 - t)^K + 1 - p]^N,
\]

where the first part of (29) accounts for \( S^N = 0^N \), and the second part of (29) results from the fact that \( (1 - t)^{|M(n)|} \geq (1 - t)^K \). To obtain (26), we replace (30) into (27).

Although the bound in (26) is trivial since it only excludes the case when all alarms are zero, it helps us relate the performance of the BPA to the structure of the graph. In particular, the bound in (26) is an increasing function of \( K \) and \( t \); as a result, the probability of diagnostic error decreases when the graph is more sparse (i.e., when \( K \) decreases) or when the connections are less reliable (i.e., when \( t \) decreases). These trends are also verified by our empirical results in the following section.

**V. SIMULATION RESULTS**

In this section, we apply the BPA to different classes of systems to analyze the performance of the proposed algorithm. **Example 1**: Consider the Y model system A with 10 alarms, 10 components, test matrices \( T_0^A \), \( T_1^A \), and alarm parameters \( \alpha_0^0, \alpha_1^1 \) (Table II). The number of iterations of the BPA is set to \( I = 10 \).

**Table II**

<table>
<thead>
<tr>
<th>( T_0^A )</th>
<th>( T_1^A )</th>
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<tbody>
<tr>
<td>0.00</td>
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**Fig. 5.** BPA performance for system A.

We construct a histogram with 1000 sets of prior probabilities generated by assuming that each prior probability is an i.i.d. random variable uniformly distributed between 0 and 0.1. For each set of priors, we consider each of \( 2^M \) possible alarm outcomes and count the number of times when the BPA provides the most likely solution. The results are presented in Fig. 5. We observe that the BPA provides the most likely solution for the majority of the possible alarm combinations (note, however, that some of these combinations might be more likely than others). For comparison, we also include the result of running BPA on the same graph under the X model with test matrices as in Table II. It is obvious that the BPA works better for the Y model compared to the X model (recall that the X model has more reliable alarms).

**Example 2**: This example investigates the effect of connection density on the performance of the BPA under the Z model.
Consider two systems B and C with test matrices as in Table III chosen so that system B has lower density than system C. The histograms of the BPA performance on the two systems are presented in Fig. 6. We can see that the performance of the BPA is improved for the lower density system (system B).

From Examples 1 and 2 (and other experiments we performed), we observe that the BPA works better for systems with less reliable connections or lower density. Note that the bipartite graphs corresponding to systems A, B and C are not a tree. In the next example, by analyzing the probability of diagnosis error of the BPA, we show that the above observations are in agreement with Theorem 2.

### Example 3: Consider a special class of Z systems with equal prior probabilities \( p_k^z = p \) and similar connections \( t_{mn} = t \). For \( p \) varying from 0.001 to 0.01, we randomly construct 10 different systems with \( N = 10 \) components, \( M = 10 \) alarms and determine the probability of diagnosis error using the BPA. We also fix the reliability of each connection at \( t = 0.6 \). The theoretical upper bound in (26) and the empirical results for several systems are presented in Fig. 7. We see that when \( p \) increases, the BPA has higher probability of error. In addition, as \( K \) increases (i.e., with higher density), the BPA has higher probability of error. Note that these properties were also observed in [7].

### Table III: Z model systems B and C

<table>
<thead>
<tr>
<th>System</th>
<th>Parameters</th>
<th>( T_B )</th>
<th>( T_C )</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td></td>
<td></td>
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<tr>
<td>C</td>
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</tbody>
</table>

Example 3: Consider a special class of Z systems with equal prior probabilities \( p_k^z = p \) and similar connections \( t_{mn} = t \). For \( p \) varying from 0.001 to 0.01, we randomly construct 10 different systems with \( N = 10 \) components, \( M = 10 \) alarms and determine the probability of diagnosis error using the BPA. We also fix the reliability of each connection at \( t = 0.6 \). The theoretical upper bound in (26) and the empirical results for several systems are presented in Fig. 7. We see that when \( p \) increases, the BPA has higher probability of error. In addition, as \( K \) increases (i.e., with higher density), the BPA has higher probability of error. Note that these properties were also observed in [7].

### Fig. 7. BPA performance versus density.

**VI. CONCLUSIONS**

In this paper, we develop a polynomial complexity belief propagation algorithm (BPA) for a general fault diagnosis formulation where all connections and alarms can be unreliable. Our empirical studies show that (i) the BPA works better on graphs that are more sparse and (ii) the probability of diagnosis error is lower when the alarms and connections are less reliable, which makes the BPA more suitable for MFD applications with unreliable observations. These properties of the BPA are also verified by an upper bound on the probability of diagnosis error which we obtain for a special class of systems; however, this bound is relatively weak and remains to be improved.

### REFERENCES


