An Iterative Multiuser Detector for DS-CDMA Systems in Multipath Fading Channels.

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Abstract—We propose an iterative multiuser detector for Turbo coded Direct Sequence CDMA (DS-CDMA) systems in the uplink channel. The receiver is derived from the Maximum a-Posteriori (MAP) estimation of the single user’s transmitted data, conditioned on information about the estimate of the Multiple Access Interference (MAI) and the received signal from the channel. The complexity of the proposed receiver increases linearly with the number of users. Error rate performances of the receiver is found to be better than those of the receiver in which a direct subtraction of the estimated multiple access interference is performed. The performance of the developed iterative multiuser receiver is investigated in the multipath fading environment with and without the knowledge of the side information. It was observed that the performance of the proposed receiver improves in a multipath fading channel when the channel side information is considered in the development of the receiver.

Keywords—Turbo Codes, Multiuser Detection, CDMA, MAP.

I. INTRODUCTION

A significant amount of work has been done on the development of multiuser detectors (MUD) for CDMA since the publication of the novel work of Verdu [1]. The main focus of work on MUD development has been the search for suboptimal detectors because the optimum receiver of [1] has an implementation complexity that increases exponentially with the number of users.

Sub-optimal detectors that have been reported in the literature can be classified as linear or nonlinear detectors. In linear multiuser detection, linear filters are used in processing the received signal in order to extract the signal of the user of interest and suppress the multiple access interference. Nonlinear multiuser detection involves the subtraction of the estimate of the multiple access interference from the received signal [2].

Realizing that error correction alone cannot remove the effects of the multiple access interference effectively, a lot of emphasis is now being placed on designing multiuser detectors for channel coded CDMA systems. A pioneering work in this respect is the work of Giallorenzi and Wilson [3] where the optimum detector of [1] is combined with convolutional decoding. The complexity of the receiver of [3] increases exponentially with the product of the number of users and the constraint length of the convolutional encoder.

The advent of Turbo codes [4] and the generalization of the “Turbo principle” in many aspects of digital communication [5] have inspired the development of many “iterative” multiuser detectors. In [6], the “super trellis” of the joint convolutional coded and the time varying CDMA coded system was transverse based on the Maximum a-Posteriori (MAP) criterion. This is in contrast to the work of [3] where the Viterbi algorithm was used. The work of [6] has the same prohibitive complexity as the receiver designed in [3].

Work done on reducing the complexity of iterative detectors to levels that can be practically implemented has mainly focused on combining various sub-optimal multiuser detectors with iterative channel decoding in an integrated manner. In [8] an iterative interference canceller was proposed for convolutional coded CDMA. This scheme integrates the subtraction of the estimated multiple access interference and channel decoding. The iterative interference canceller was also studied in [9] and [10]. The partial interference canceller of [11] was combined with Turbo decoding in [12]. In a nutshell, iterative interference cancellation (and some of its variants) has received a wide acceptance. This could possibly be due to its low level of complexity.

Our work is different from the work of [8] in that we avoided a direct subtraction of the estimated multiple access interference from the received signal. Rather we used the estimated multiple access interference as added information in the MAP estimation of the transmitted bit of our user of interest. The motivation for this is the fact that the multiple access interference estimation error could lead to erroneous detection if subtracted directly from the received signal. The proposed iterative multiuser detector has a complexity that is linear with the number of users.

The rest of this paper is organized as follows. In Section 2 the CDMA system model is presented. The proposed iterative multiuser detectors are developed in Section 3. The performance of the proposed detector is investigated by simulation in Section 4. Section 5 concludes the paper.

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II. SYSTEM MODEL.

A Turbo coded asynchronous BPSK modulated DS/CDMA system is considered in this paper. The system transmits over an Additive White Gaussian Noise (AWGN) channel. In a multiple access system, the signal transmitted by a user \( k \) can be represented as:

\[
s_k(t) = \sqrt{2P_k}a_k(t)c_k(t)\cos(\omega_c t)
\]

where \( c_k(t) \) is the signal that represents the code bits of user \( k \). \( a_k(t) \) is the random signature waveform of user \( k \) of period equal to the coded bit interval, \( T_b \), and it is given by

\[
a_k(t) = \frac{1}{\sqrt{N}}\sum_{m=0}^{N-1} a_k[m]rect(t-mT_c)
\]

where \( rect(t) \) denotes the rectangular chip waveform, \( N \) is the processing gain, \( T_c \) chip duration \( T_c = T_b / N \). \( P_k \) is the power of the transmitted coded bit of user \( k \). \( P_k = R E_b / T_b \) where \( R \) is the coding rate and \( E_b \) is the energy of the uncoded information bit. \( \omega_c \) is the carrier frequency.

The summation of the signals that are transmitted by all users in the multiple access environment can be represented as

\[
S(t) = \sum_{k=1}^{K} \sqrt{2P_k}a_k(t)c_k(t)\cos(\omega_c t + \phi_k)
\]

where \( \phi_k \) is the modulation angle of user \( k \). The multipath fading channel is modeled as a time-delay impulse channel model [14]. The response of the channel can be represented as

\[
h_k(t) = \sum_{l=1}^{L} \beta_{lk} \delta(t-\tau_{lk})e^{j\theta_{lj}}
\]

where \( \beta_{lk} \), \( \tau_{lk} \) and \( \theta_{lj} \) are the \( l \)th path’s channel gain, delay and phase shift respectively for the \( k \)th user’s signal. \( \tau_{lk} \in [0,T_b] \) and is independent of the channel gain coefficient. \( T_b \) is the message bit duration. The received signal can therefore be represented as

\[
r(t) = \sum_{k=1}^{K} \sqrt{2P_k}a_k(t-\tau_{lk})c_k(t-\tau_{lk})\cos(\omega_c t + \psi_{lj}) + n(t)
\]

where \( \psi_{lj} = \theta_{lj} + \phi_k + \omega_c \tau_{lk} \) and \( \psi_{lj} \in [0,2\pi] \). Each user’s signals are taken to be propagated through \( L \) different paths.

The Maximum Ratio Combining (MRC) based RAKE receiver is used for diversity combining. The output of the RAKE receiver that has \( M \) fingers can be expressed as

\[
X_j = \sum_{m=1}^{M} \left\{ \int_{\tau_{mj}}^{\tau_{mj}+T} \beta_{mj}r(t)aj(t-\tau_{mj})\cos(\omega_c t + \psi_{mj}) dt \right\}
\]

\[
= \sum_{m=1}^{M} \left\{ U_{mj} + V_{mj} + W_{mj} + \eta_m \right\}
\]

where

\[
U_{jm} = \sqrt{\frac{P_j}{2}} c_{o,j}T(\beta_{mj}, j)^2
\]

\[
V_{jm} = \sum_{k=1}^{K} \sqrt{\frac{P_k}{2}} \sum_{l=1}^{L} \beta_{mj}R_{jk}(\tau_{lk} - \tau_{mj}) \cos(\psi_{lk} - \psi_{mj})
\]

\[
W_{jm} = \sqrt{\frac{P_j}{2}} \sum_{k=1}^{K} \sum_{l=1}^{L} \beta_{mj}R_{jk}(\tau_{lk} - \tau_{mj}) \cos(\psi_{lk} - \psi_{mj})
\]

\[
\eta_m = \int_{mT_c}^{(m+1)T_c} n(t)\beta_{mj}a_j(t-\tau_{mj})\cos(\omega_c t + \psi_{mj}) dt
\]

\[
U_{mj}, V_{mj}, W_{mj} \text{ and } \eta_m \text{ represent the desired user's signal, the MAI component, the Side Interference (SI) component and the AWGN component respectively.}
\]

\[
R_{kj}(\tau) = \int_{0}^{T_b} a_k(t-\tau)a_j(t)dt \text{ and } \hat{R}_{kj}(\tau) = \int_{0}^{T_b} a_k(t-\tau)a_j(t)dt.
\]

\[
c_{o,k} \text{ and } c_{-1,k} \text{ represent the message bit to be detected and the preceding bit respectively.}
\]

The relationship between the power of the direct signal and other reflected signals in the multipath fading channel has been modeled in a variety of ways. This relationship is referred to as the Multipath Intensity Profile (MIP) which is defined as the average power at the output of the channel as a function of path delay. The uniform, the exponential and the Gaussian MIP has been discussed in the literature [14]. A measure of the average power of a given path is the second moment of the fading gain coefficient for that path. In a multipath channel that has the uniform MIP, all signals on all the propagation paths has the same second moment, that is \( E(\beta_{lk}^2) = E(\beta_{2k}^2) = \cdots = E(\beta_{Lk}^2) \) for a multipath channel that has \( L \) paths. In the multipath channel that has the exponential MIP the second moment of the first incoming path is related to the second moment of other paths through the relationship \( E(\beta_{lk}^2) = E(\beta_{2k}^2)e^{-\delta l} \) with \( \delta \) being the decay factor, \( \delta > 0 \).

Experimental measurements made by Turin [15] have shown that the MIP in urban environment is exponential. Based on this understanding, the exponential MIP will be used throughout this paper.

The channel gain coefficients used in this paper are modeled as Rayleigh variates having the Probability Density Function (PDF) given as

\[
P(\beta) = 2\beta e^{-\beta^2} \text{ for } \beta > 0
\]
III. THE ITERATIVE MULTIUSER DETECTOR

A. No Channel side information.

Figure 1 illustrates the concept of the detector that is developed in this section. The estimate of the MAI is not subtracted directly from the received signal. The philosophy behind this approach is that the estimation noise in the estimated MAI can bias the resultant decision statistics after the cancellation adversely. Therefore, a Maximum a-Posteriori (MAP) estimation of the transmitted bit of the user of interest, given the received signal and the estimate of the MAI is done in this section. In designing the receiver, the following parameter definitions are made.

Let \( s' \) represent the immediately previous state on the trellis and let \( s \) represent the present state. Let the code bit that is desired to be estimated be represented as \( b_h \). Furthermore let the received sequence be represented by \( Y \), let the received sequence associated with the immediately previous transition be represented by \( Y_{j-1} \), let the received sequence associated with the present transition be represented by \( Y_j \), and let the received sequence associated with the transition immediately after the present transition be represented by \( Y_{j+1} \).

The MAP algorithm performs the estimation by selecting the value of the code bit that maximizes the probability \( P(Y_j | L_b) \). The log-likelihood ratio \( L(b_h | Y, L) \), stated in equation (6), is a reliable tool for this selection. \( L \) is the sequence of the estimated MAI. Let the following definition also be made about the sequence of the estimated MAI. Let the sequence of the estimated MAI associated with the immediately previous transition be represented by \( L_{j-1} \), let the sequence of the estimated MAI associated with the present transition be represented by \( L_j \) and let sequence of the estimated MAI associated with the transition immediately after the present transition be represented by \( L_{j+1} \). Therefore,

\[
L(b_h | Y, L) = \ln \left( \frac{P(b_h = +1 | Y, L)}{P(b_h = -1 | Y, L)} \right) = \ln \left( \frac{\sum_{b_{h-1}=+1} P(s, s', Y, L)}{\sum_{b_{h-1}=-1} P(s, s', Y, L)} \right)
\]

(12)

\( P(s, s', Y, L) \) can be simplified using the Baye’s rule as:

\[
P(s, s', Y, L) = \beta'_j(s) \gamma'_j(s, s') \alpha'_{j-1}(s')
\]

(13)

where \( \alpha'_{j-1}(s') \), \( \beta'_j(s) \) and \( \gamma'_j(s, s') \) are defined as:

\[
\alpha'_{j-1}(s') = P(Y_{j-1}, L_{j-1} | s') \quad \beta'_j(s) = P(Y_j, L_{j-1} | s)
\]

and \( \gamma'_j(s, s') = P(s, Y_j, L_j | s') \). It can be easily shown by using the procedure similar to the one used in [16] that

\[
\alpha'_j(s') = \sum_{all s} \alpha'_{j-1}(s') \gamma'_j(s, s') \beta'_j(s) = \sum_{all s} \beta'_{j+1}(s') \gamma'_j(s, s')
\]


\[
\text{and } \gamma'_j(s, s') = P(Y_j, L_j | x_j). P(b_h). \quad \alpha'_{j-1}(s') \text{ is the forward recursion coefficient, } \beta'_j(s) \text{ is the backward recursion coefficient and } \gamma'_j(s, s') \text{ is the transition coefficient. } x_j \text{ represents the code symbol. Implementing the MAP recursive algorithm as stated above leads to a numerically unstable algorithm [13]. To ensure stability, these quantities must be normalized as } \hat{\alpha}'_j(s') = \frac{\alpha'_j(s')}{\sum_{all s} \alpha'_j(s')} \text{ and } \hat{\beta}'_j(s) = \frac{\beta'_j(s)}{\sum_{all s} \beta'_j(s')}. \]

The log-likelihood ratio can thus be calculated from

\[
L(b_h | Y, L) = \ln \left( \frac{\sum_{b_{h-1}=+1} \hat{\alpha}'_{j-1}(s') \hat{\beta}'_j(s) \gamma'_j(s, s')}{\sum_{b_{h-1}=-1} \hat{\alpha}'_{j-1}(s') \hat{\beta}'_j(s) \gamma'_j(s, s')} \right)
\]

(14)

The estimated MAI sequence and the received signal sequence are not independent variables. They are mutually correlated. As the number of user increases, the two sequences can be taken to have a probability density function (PDF) that is jointly Gaussian. The joint PDF of the received sequence and the sequence of the estimated MAI given the transmitted coded sequence is therefore given as

\[
P(Y_j, L_j | x_k) = \prod_{l=1}^{n} \left\{ \frac{1}{2\pi \sigma_1 \sigma_2 \sqrt{1 - r^2}} \exp \left\{ \frac{-1}{2(r^2)} \frac{\left( Y_{jl} - X_{jl} \right)^2}{\sigma_1^2} - 2 \frac{r (Y_{jl} - X_{jl}) I_{jl}}{\sigma_1 \sigma_2} + \frac{I_{jl}^2}{\sigma_2^2} \right\} \right\}
\]

(15)
stands for the value of the correlation between the received signal\( (Y)\) and the estimate of the MAI\( (I)\), correlation, \(\sigma_1^2\) stands for the variance of the received signal and \(\sigma_2^2\) stands for the variance of the estimate of the MAI. For the case of a turbo coding with coding rate 1/3 that is considered in this paper, it can be shown (by using a procedure similar to the one used in [13]) that

\[
L(b_h) = L^e(b_h) + \frac{2Y_{j_1}}{1-\mu} + \frac{2r_{j_1}}{1-\mu} + \ln \left( \sum_{(s,s')} \frac{\alpha_j(s')}{b_h+1} \chi_{j}(s,s') \beta_j(s) \right)
\]

\[
+ \ln \left( \sum_{(s,s')} \frac{\alpha_j(s')}{b_h-1} \chi_{j}(s,s') \beta_j(s) \right)
\]

where \(L^e(b_h)\) is the extrinsic information from the previous detector,

\[
\chi_j(s,s') = \left( \exp \left( \frac{2Y_{jp}X_{jp}}{\sigma_1^2} + \frac{2r_{jp}I_{jp}}{\sigma_1^2} \right) \frac{1}{2(1-r^2)} \right)
\]

\[
\gamma_j(s,s') = \exp(b_h L^e(b_h)/2) \Delta
\]

where

\[
\Delta = \left( \exp \left( \frac{2Y_{j_1}b_h}{\sigma_1^2} + \frac{2r_{j_1}I_{j_1}}{\sigma_1^2} \right) \frac{1}{2(1-r^2)} \right) \chi_{j}(s,s')
\]

The algorithm is implemented to estimate the transmitted code bit and not the information bit. This is done so as to avoid re-encoding before interference cancellation. This log-likelihood ratio (taken for each user) is the reliable information that is used in the estimation of the multiple access interference sequence.

\[B. \text{ Receiver with channel side information.}\]

The development of the iterative multiuser receiver in the last section did not consider the Channel Side Information (CSI). In this section we shall develop the receiver on the assumption that the receiver has knowledge of the channel fading gain parameter. That essentially means that the log-likelihood ratio that will be computed will be \(L(b_h | Y, I, \beta)\).

By following a procedure that is similar to the one used in Section III-A, the transition probability can be obtained to be \(\gamma_j(s,s') = P(Y_{j_1} | L_{j_1}|, I_{j_1}, \beta | s_j) P(b_h)\). The joint PDF of the received sequence, the sequence of the estimated MAI and the channel fading gain parameter given the transmitted coded sequence is therefore given as

\[
P(Y_{j_1} | L_{j_1}|, I_{j_1}, \beta | x_k) = \prod_{l=1}^{n} \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-r^2}} 
\]

\[
\exp \left( \frac{(Y_{j_1} - \beta X_{j_1})^2}{2\sigma_1^2} - \frac{2r(Y_{j_1} - \beta X_{j_1}) I_{j_1}}{\sigma_1\sigma_2} + \frac{I_{j_1}^2}{\sigma_2^2} \right)
\]

\[
\exp \left( \frac{2Y_{jp} E(\beta) X_{jp}}{\sigma_1^2} + \frac{2r Y_{jp} I_{jp} + 2rE(\beta) X_{jp} I_{jp}}{\sigma_1\sigma_2} \right) \frac{1}{2(1-r^2)}
\]

where

\[
\Delta' = \left( \exp \left( \frac{2Y_{j_1} E(\beta) b_h}{\sigma_1^2} + \frac{2r Y_{j_1} I_{j_1} + 2r E(\beta) b_h I_{j_1}}{\sigma_1\sigma_2} \right) \frac{1}{2(1-r^2)} \right) \chi_{j}(s,s').
\]

\[E(\beta)\) is the expectation of the channel fading gain parameters over all the multiple paths.\]

\[IV. \text{ Performance Investigation.}\]

The performance results of the proposed system are discussed in this section. The developed system is first compared with the conventional iterative receiver system in an AWGN channel through simulations. By the conventional iterative receiver system we mean the approach in which the estimated interference is subtracted from the received signal prior to channel decoding. This type of receiver is discussed in [8], [9] and [10] among others. Throughout this paper, the constituent encoder for our turbo code is the RSC code with octal representation of (7,5)\(\otimes\) (7,5)\(\otimes\). The frame length is chosen as 200. The number of active users is 10. The unit energy constraint is observed throughout.

In Figure 2, the receiver that is developed in this paper (referred to as “Turbo IC” on the figure) and the conventional iterative interference cancellation receiver (referred to as "Conv Iter. IC" on the figure) are compared. The number of iterations is three. It will be observed that the receiver that is presented in this paper has better error rate.
performance than the conventional iterative interference cancellation receiver for the various values of Processing Gains (PG) that are used in the investigation.

In Figures 3 and 4 the effect of the CSI is investigated. Figure 3 shows results for 10 active users in a DS-CDMA system that has a processing gain of 15. A single fading path is used and the number of iteration is three. A dB gain of over 1.5dB is obtained by using the CSI in the receiver design at an error rate of 1.0E-3. Figure 4 shows results from the same DS-CDMA system but with three fading paths and three RAKE fingers. A dB gain of over 2dB is obtained by using the CSI in the receiver design at an error rate of 1.0E-2.

V. CONCLUSIONS.

In this paper, a low complexity iterative interference canceller for Turbo coded CDMA systems has been presented. The developed receiver was compared with the receiver of [8]. The performance of the proposed detector is found to be superior to those of the receiver of [8].

In the multipath fading channel, the performance of the proposed receiver can be enhanced when the CSI is used in the receiver design. The procedure for incorporating the CSI into the design of the proposed receiver is presented in this paper.

The complexity of the proposed receiver is linear with the number of users. This level of complexity of the proposed receiver and its performance advantage over the conventional iterative interference canceller makes the proposed receiver suitable for use in CDMA systems.

REFERENCE.

Figure 2. Comparison of the proposed iterative multiuser receiver with the conventional iterative interference cancellation receiver.

Figure 3. Performance of the proposed iterative multiuser detector with and without CSI. Single Rayleigh fading path.

Figure 4. Performance of the proposed iterative multiuser detector with and without CSI. Three Rayleigh fading paths.

Figure 5. Performance of the proposed iterative multiuser detector with CSI. Three Rayleigh fading paths.