Highlights

• We developed a set of valid reasoning postulates in CTLKC$^+$. 

• We proved Soundness and Completeness of CTLKC$^+$ using the Correspondence Theory.

• We used NetBill as a concrete application example to illustrate the postulates.
On the Soundness, Completeness and Applicability of the Logic of Knowledge and Communicative Commitments in Multi-Agent Systems

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Abstract

Benthem’s correspondence theory is one of the most important tools of the theory of modal logics developed in the last three decades. Correspondence theory, a subfield of the model theory, reflects a systematic study of relations between classes of frames and modal language. In this paper, we use correspondence theory for modal logics to solve a problem not addressed yet in the literature, namely the soundness and completeness of a logic combining two different, yet related modalities: agents’ knowledge and commitments. The paper proves the soundness and completeness of this logic called CTLKC+. This work is highly significant as it proves that combining the two agents’ modalities resulted in a consistent logic that can be used to design reliable systems. The methodology is as follows: we develop a set of reasoning postulates (axioms) that reflect the interaction between agents’ knowledge and social commitments in Multi-Agent System (MAS) using the CTLKC+ logic and correspond them to certain classes of frames. In particular, we first give a name, formalization and meaning for each postulate. Then, we correspond the postulates to certain classes of frames and provide the required proofs. Thereafter, we present a discussion that illustrates the importance of the proposed postulates in MASs using a concrete application example called the NetBill protocol taken from the business domain. Finally, we show how the postulates were addressed in the literature. The existence of such a correspondence allows us to prove that the logic generated by any subset of these

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postulates is sound and complete with respect to models that are based on the corresponding frames. The ultimate goal of this paper is to further assess the logic of knowledge and commitments (CTLKC\(^+\)) from a new perspective (i.e., the soundness and completeness). Consequently, this work advances the literature of logics in MASs and closes a gap that has not been explored before.

**Keywords:** Multi-Agent Systems, Knowledge, Social Commitments, Soundness, Completeness, Correspondence Theory

### 1. Introduction

#### 1.1. Motivations and Background

Multi-Agent Systems (MASs) paradigm is a popular distributed approach to solve complex computational problems that are out of the capabilities of individual software systems. Typically, a Multi-Agent System consists of a collection of independent autonomous software agents that communicate and coordinate their actions in order to achieve some objectives (Wooldridge, 2009). The increasing adoption of MASs in various real life applications, for instance, e-commerce, e-health and web services (Jennings & Wooldridge, 1998), highlighted the importance of a large dimension of agents’ aspects that need to be properly addressed. Among the agents’ units that proved to have a vital role in the development of effective MASs are knowledge, commitments, trust, reputation, uncertainty, etc. (Sultan, Bentahar, & El-Menshawy, 2014a; Bentahar, Khosravifar, Serhani, & Alishahi, 2012b; Marey, Bentahar, Dssouli, & Mbarki, 2014). Each of these significant aspects has been extensively studied. However, the widespread use of MASs has also encouraged researchers to explore the interaction between some elements of these dimensions. Among others, the interaction between knowledge and commitments has received a considerable attention in the recent years (Al-Saqqar, Bentahar, Sultan, & El-Menshawy, 2014; Sultan, Bentahar, Wan, & Al-Saqqar, 2014b; Schmidt, Tishkovsky, & Hustadt, 2004). Although plenty of research work has been carried out to define the semantic of knowledge (Halpern & shore, 2004; Lomuscio & Penczek, 2012; van Ditmarsch, Halpern, van der Hoek, & Kooi, 2015) and social commitments (Bentahar, El-Menshawy, Qu, & Dssouli, 2012a; El-Menshawy, Bentahar, K holy, & Dssouli, 2013b) independently, there is a crucial need to have a consistent, sound and
complete logic that can reason about the interaction between knowledge and social commitment simultaneously.

In a previous work (Al-Saqqar et al., 2014), we studied the relationship between knowledge and communicative social commitments from semantic and model checking perspectives. We focused primarily on the notion of “social commitment” (Singh, 2000), which has been used as a means of communication between interacting agents in MASs. Unlike internal commitments, which are private and refer to commitments form an agent to itself, social commitments are public and noticeable engagements from one agent to another agent to bring about something (Sultan et al., 2014b). Generally, the object of a commitment can be an action to be performed, a goal to be achieved, a condition to be fulfilled, etc. (Sultan et al., 2014a). In particular, we proposed CTLKC⁺, the logic of knowledge and commitments, which allows us to reason about social commitments and knowledge simultaneously in a consistent manner. Later in (Al-Saqqar, Bentahar, Sultan, Wan, & Asl, 2015), we advanced our research by developing a fully-automated reduction-based technique for the model checking problem of the CTLKC⁺ logic that was proposed in (Al-Saqqar et al., 2014). More precisely, we first transformed the problem of model checking CTLKC⁺ into the problem of model checking an existing logic of action called Action-Restricted CTL (ARCTL) (Pecheur & Raimondi, 2006) to get benefit from the extended version of NuSMV symbolic model checker of ARCTL. After that, we analyzed the complexity of the proposed model checking technique. The results of this analysis revealed that the complexity of our reduction-based procedure is PSPACE-complete for local concurrent programs with respect to the size of these programs and the length of the formula being checked. From the time perspective, we proved that the complexity of the proposed approach is P-complete with regard to the size of the model and length of the formula, which makes it efficient. Finally, we implemented our model checking approach on top of extended NuSMV and reported some experimental results by verifying the well known NetBell payment protocol (Sirbu, 1997) against some desirable properties. The obtained results show the effectiveness of our model checking approach when the system scales up. Though CTLKC⁺ logic has been already evaluated from different perspectives such as the model checking and complexity (Al-Saqqar et al., 2014, 2015), the soundness and completeness for CTLKC⁺ have not been investigated before. In this paper, we aim to prove its soundness and completeness using Correspondence theory for modal logic, which was introduced by van Benthem (van Benthem, 1984).
On the one hand, the soundness of a given logic is the statement that this logic is semantically sound, i.e., it allows deriving truth theorems from true statements. On the other hand, the completeness of a given logic is the statement that the logic is complete (for its language), i.e., any true deduction that can be expressed in the language can be written as a formal proof in the logic (Blackburn, Benthem, & Wolter, 2006). Formally, the soundness theorem is the theorem stating $\Sigma \vdash \varphi$ implies $\Sigma \models \varphi$. Here $\Sigma \vdash \varphi$ expresses that there is a formal proof of the statement $\varphi$ from assumptions in $\Sigma$, where the proof obeys some formally defined and precise rules. The conclusion $\Sigma \models \varphi$ expresses that every interpretation making $\Sigma$ true also makes $\varphi$ true. The completeness theorem states $\Sigma \models \varphi$ implies $\Sigma \vdash \varphi$ (van Ditmarsch, 2012). Thus, our objective in this paper is to ensure that for the CTLKC$^+$ logic, any provable formula is a true formula (soundness), and every true formula is provable (completeness).

Correspondence theory is a subfield of the model theory of modal logic (Conradie, Fomatatai, Palmigiano, & Sourabh, 2015). Some features of correspondence theory are the discoveries of the class of first-order definable and canonical Sahlqvist formulas (Sahlqvist, 1975) in addition to some significant cases of non first-order definability of modal principles such as the axioms of Gödel, Mckinsey and Segerberg (Conradie, Goranko, & Vakarelov, 2009). Though correspondence approaches have become principal technical tools in pure and applied modal logic, they did not form an area of research in their own right. However, correspondence analysis is viewed as a major part in the philosophical search for logical “core theories” of semantics in languages and computations (van Benthem, 1984). One of the most attractive aspects of possible worlds semantics and modal logic is highlighted by the fact that modal axioms are kind of representations for the properties of the accessibility relations (Blackburn et al., 2006). For example, a typical modal completeness theorem read as follows:

“A formula is provable in S4 iff it is true in all models based on frames whose accessibility relation is transitive and reflexive” (Blackburn et al., 2006).

This means that, the theorems of S4 (i.e., the axioms K, T and 4 are traditionally been called S4 (Fagin, Halpern, Moses, & Vardi, 1995) must hold in all models with a transitive & reflexive relations (Blackburn et al., 2006). Many techniques for proving such completeness results are available, ranging from simple viewing of the canonical model constructed from all complete theories in the logic, to various types of model manipulation (such
as filtration, unraveling, and taking bounded morphic images) (Blackburn et al., 2006).

The early completeness theorems in modal logic were presented by Segerberg (1971) as follows: “modal logic $L$ is determined by a class $\mathcal{R}$ of Kripke frames”, i.e., $L$ axiomatises the modal theory of $\mathcal{R}$ (on the basis of the minimal logic $K$) (van Benthem, 1984). In this context, two perspectives emerge here. First, one may start with a given class $\mathcal{R}$ of Kripke frames, asking for an axiomatisation $L$ of its modal theory. Second, one has a certain logic $L$, asking for a class $\mathcal{R}$ of Kripke frames with respect to which it is complete. Note that, the latter perspective represents the current direction in modal logic. The typical example of this perspective is the proof system $GL$, that is $K$ augmented with all instances of the L"ob axiom schema $\Box(\Box \varphi \to \varphi) \to \Box \varphi$ (Blackburn et al., 2006).

Generally speaking, correspondence theory exhibits a systematic study of relations (correspondences) between classes of frames and modal languages (Indrzejczak, 2008). Such correspondence illustrates the relationship between the semantics and reasoning. Furthermore, the existence of this correspondence results in having a set of soundness and completeness theorems (Singh, 2008).

1.2. Paper Contribution

As mentioned above, in (Al-Saqqar et al., 2014), we integrated knowledge and commitment operators in a single language called $CTLKC^+$ to be able to formulate formulas that express combinations of the two concepts. Then, we addressed the model checking, complexity and implementation issues in (Al-Saqqar et al., 2015). In this paper, we use correspondence theory for modal logic to prove the soundness and completeness of the $CTLKC^+$ logic. This process can be seen as a step towards demonstrating and evaluating the efficiency and consistency of $CTLKC^+$ from a new perspective. In particular, as depicted in Figure 1, we develop a set of reasoning postulates in $CTLKC^+$ and correspond them to certain classes of frames providing the required proofs. Consequently, we prove that the logic generated by any subset of those postulates is sound and complete with respect to the models that are based on the corresponding frames.

The main strength of the proposed method lies in the fact that all the results are obtained through the use of solid formal theories. Moreover, the developed axioms are associated with their corresponding proofs based on models defined over the frames. This shows that the proposed logical
model is correct and any application built on it would not suffer from any inconsistency. However, the fact that the model is being built using a purely theoretical foundation could be seen as a potential weakness as the model could be considered abstract and far from any real and concrete application. To handle this issue and facilitate the reader’s understanding of the paper, we provide a concrete application as a running example throughout the paper over which the axioms are illustrated.

1.3. Paper Organization

The rest of this paper is organized as follows. In Section 3, we review the evolution of reasoning about knowledge and commitments in MASs together with their modeling and verification techniques. Thereafter, we present the interpreted systems formalism as an effective way for modeling MASs and
underlying interactions. Finally, we illustrate the concept of frame definability (correspondence). In Section 4, we introduce the syntax, semantics and properties of CTLKC$^+$. In Section 5, we first present the Netbill protocol as an application example to illustrate our proposed reasoning postulates. After that, we address the problem of corresponding the reasoning postulates to certain classes of frames and then we prove the soundness and completeness of the CTLKC$^+$ logic. Finally, we conclude the paper in Section 6.

2. Related Work

Schmidt et al. (2004) formalized the interaction between knowledge, action and commitment. In this work, the commitment adopted was not the social commitment but rather an internal commitment which refers to a commitment from an agent to itself (Castelfranchi, 1995). The authors developed a new logic called Agent Dynamic Logic (ADL) and utilized it to capture some interactions between knowledge and internal commitments as in the following formula: $\text{Comm}_i(\alpha) \rightarrow K_i \text{Comm}_i(\alpha)$, which expresses that whenever agent $i$ commits to perform action $\alpha$, this implies that agent $i$ knows ($K_i$) about its internal commitment ($\text{Comm}_i$). Furthermore, they proved that their deductive systems are sound and complete with respect to a Kripke-style semantics. However, from a communication perspective, internal commitment is neither communicative nor public.

Bentahar, Moulin, Meyer, & Chaib-draa (2004) developed a modal approach for dialectical and practical commitments. The focus of their work was on how commitments are updated in conversations. They formulated several reasoning postulates but did not pursue the soundness and completeness of their proposed logic.

Singh (2008) introduced a logic to reason about practical and dialectical conditional commitments and developed a set of reasoning postulates together with a set of semantics constraints. Thereafter, he used Bentham’s correspondence theory for modal logic to prove the correspondence between the reasoning postulates and the semantics constraints. Consequently, he was able to prove the soundness and completeness of his proposed logic. However, Singh’s approach is mainly related to conditional commitments. Further, instead of corresponding the reasoning postulates to classes of frames, Singh (2008) corresponded the reasoning postulates to a set of semantic constraints stated in terms of set theory.
Bentahar, Meyer, & Wan (2009) proposed a model checking algorithm aimed at verifying systems designed as a set of autonomous interacting agents. These software agents are equipped with knowledge and beliefs and interact with each other according to protocols controlled by a set of logical rules. In this work, instead of using correspondence theory for modal logics, the authors developed a tableau-based algorithm for the model checking of their logic and proved the soundness and completeness of the verification procedure, but not of the logic itself. Moreover, they did not investigate the interaction between commitments and knowledge from the semantics perspective.

Chesani, Mello, Montali, & Torroni (2013) presented a commitment modeling language that enables metric temporal reasoning, and allow the domain modeler to freely specify which roles the debtor and creditor agents shall play in the manipulation of commitments. Furthermore, they proposed an axiomatization of commitment operations in a first order Event Calculus framework, that accommodates reasoning with data and metric time. However, they did not address soundness and completeness.

EL Kholy, Bentahar, El Menshawy, Qu, & Dssouli (2014) put forward a new temporal logic, CTL\textsuperscript{cc}, which extends CTL (Emerson, 1990) with modalities to represent conditional commitments and their fulfillments using the formalism of interpreted systems. Furthermore, they introduced a set of rules to reason about conditional commitments and their fulfillments. Nevertheless, they did not investigate the soundness and completeness of their new logic.

Woźna-Szczęsniaś (2014) proposed CDCTL\textsuperscript{*}K, a new temporal logic that extends CTL\textsuperscript{*} (Clarke, Grumberg, & Peled, 1999) with modalities to reason about knowledge, correct functioning behaviour, and different social commitments in MASs. In this work, Communication Deontic Interpreted Systems (CDIS) was used to interpret the formulas of the logic. Furthermore, a SAT-based bounded model checking technique was developed for the existential fragments of CDCTL\textsuperscript{*}K and for CDIS. However, neither the interaction between knowledge and commitment in MASs nor the soundness and completeness of the proposed logic were tackled.
3. Preliminaries

3.1. Knowledge and Commitments in MASs

Knowledge in MASs has long been modeled and reasoned about (Fagin et al., 1995; Lomuscio & Penczek, 2012; Halpern & Régó, 2013; van Ditmarsch et al., 2015). Informally, it is captured through the interpretation that an agent knows something (say a fact) if it is true in all the worlds that the agent thinks possible (Halpern & Shore, 2004). Knowledge is formally denoted by $K_i\varphi$ meaning that agent $i$ knows $\varphi$ where $\varphi$ is the content of knowledge. Furthermore, the logic of knowledge has been successfully applied in many disciplines including computer science, philosophy, logic and economics (Lomuscio & Penczek, 2012). The first systematic attempts towards defining epistemic framework were started by Hintikka (1962) and later by Lenzen (1978). Their efforts were concentrated on defining some axioms of modal logic to capture certain properties of knowledge in a reasonable way. The typical framework is an $S5_n$ system built on top of the propositional calculus, where $n$ is the number of agents (Lomuscio & Penczek, 2012). In this framework, in addition to normal rules of necessitation: “From $\varphi$ infer $K_i\varphi$”, and modus ponens, the following axioms are considered (Fagin et al., 1995):

- **K:** $(K_i p \land K_i(p \rightarrow q)) \rightarrow K_i q$
- **T:** $K_i p \rightarrow p$
- **4:** $K_i p \rightarrow K_i K_i p$
- **5:** $\neg K_i p \rightarrow K_i \neg K_i p$
- **D:** $\neg K_i (false)$

To capture the semantics of knowledge in epistemic systems, the formalism of Kripke models (Kripke, 1963) was initially introduced. Kripke models are of the form $\mathcal{M} = (S, \{R_i\}_{i \in A}, \mathcal{V})$, where $S$ is the set of “possible worlds”, $R_i \subseteq S \times S$ is a binary relation that expresses the epistemic indistinguishability (or accessibility) between two possible worlds for each agent $i$ in the set of all agents $A$, and $\mathcal{V} : S \rightarrow 2^p$ is a function interpreting a set of propositional variables (Lomuscio & Penczek, 2012). Later, to model the evolution of a system composed of autonomous components with respect to epistemic and temporal properties effectively, the “Interpreted systems” formalism was
introduced (Fagin et al., 1995). Details about this formalism will be given later in Section 3.2.

Another area of research in the domain of agent-based systems was focussed on agent communication, a key element of any dynamic system of agents (Bentahar, Moulin, Meyer, & Chaib-draa 2004; Chopra, Artikis, Bentahar, Colombetti, Dignum, Fornara, Jones, Singh, & Yolum, 2013). Agents interact with each other using ACLs to achieve their goals (Wooldridge, 2009). The two main approaches for defining semantics for those ACLs are mental (cognitive) and social approaches. The first attempt to define such a semantics was influenced by the Searle’s *speech acts theory* (Searle, 1969). Speech act theory treats communication as actions. This semantics is known as the mental approach which tries to capture core communication concepts such as: beliefs, desires, intentions, and goals. However, a major weakness in these approaches is being built based on the assumption that agents can access each other mind (Singh, 1998). Therefore, mental approaches cannot verify whether an agent is acting according to a given semantics. This problem is commonly known as ACL semantics verification problem (Wooldridge, 2009). Another drawback associated with this kind of semantics is that it does not allow ACLs to have enough interoperability among heterogeneous agents (Singh, 1998).

Therefore, a shift in MASs community towards social approaches has been occurred to overcome the shortcomings of ACLs semantics defined using mental approaches (Singh, 2000). Social approaches are hired to define a formal semantics for ACLs in which no assumptions on the mental states of agents are made (Singh, 2000; Fornara & Colombetti, 2004; Yolum & Singh, 2004). Social commitments are exploited in some of these social approaches that successfully provide a robust representation to model multi-agent interactions (Singh, 2000; Gûnay & Yolum, 2013). In such approaches, commitments are considered as abstractions used to represent contracts among autonomous and possibly heterogeneous agents that should be discharged, but could be violated or cancelled (Bentahar, Meyer, & Wan, 2010). Social commitments, in the last decade, have been used successfully in a wide range of areas ranging from developing artificial institutions (Fornara, Viganò, Verdicchio, & Colombetti, 2008), business contracts (Desai, Chopra, & Singh, 2009), Web-based applications (Singh & Huhns, 2005) to specifying and modeling multi-agent interaction protocols, called commitment-based protocols (Baldoni, Baroglio, & Marengo, 2010). Commitment-based social approaches for agent communication have the ability to treat commitments through a
set of actions called commitment actions, such as creation, (discharge) fulfillment, violation, cancellation, release, delegation and assignment (Singh, 1999). This ability to manipulate commitments is in fact an important characteristic that makes commitment-based approaches flexible and powerful.

In this paper, we use communicative commitments, introduced in (Bentahar et al., 2012a), as a means of communication by conveying information via message passing. Those commitments are formally expressed as $C_{i \rightarrow j} \varphi$ meaning that agent $i$, the debtor, commits to agent $j$, the creditor, that the content of the commitment $\varphi$ holds.

### 3.2. Interpreted Systems

The formalism of Interpreted Systems (ISs) was introduced by Fagin et al. (1995) to model the temporal evolution of a system of agents in order to reason about temporal properties and knowledge. Interpreted systems models different classes of MASs such as synchronous and asynchronous systems. Moreover, it is a popular formalism to model autonomous and heterogeneous agents interacting within a global system (EL-Menshawy, Bentahar, El Kholy, & Dssouli, 2013a).

This formalism consists of a set of $n$ agents $\mathcal{A} = \{1, \ldots, n\}$. Each agent $i \in \mathcal{A}$ is described by a nonempty set of local states $L_i$. At any given time in the system, each agent is in a specific local state. Each local state of an agent captures the complete information about the system that the agent has at a given moment. The local state of agent $i$ is denoted by $l_i \in L_i$. Global state $g$ represents a snapshot of all agents in the system at a given time. The set of all global states in the system is represented by $G$. A global state $g \in G$ is a tuple $g = (l_1, \ldots, l_n)$. The set of all global states $G = L_1 \times \cdots \times L_n$ is the Cartesian product of all local states of $n$ agents. The local state of agent $i$ in the global state $g$ is characterized by the notion $l_i(g)$. $I \subseteq G$ is the set of initial global states for the system. Moreover, each agent $i$ is identified by a set of local actions $\text{Act}_i$ to model the temporal evolution of the system. Furthermore, as in (Lomuscio, Penczek, & Qu, 2010), we assume that $\text{null} \in \text{Act}_i$ for each agent $i$ where $\text{null}$ refers to the silence action (i.e., the fact of doing nothing). Each agent $i \in \mathcal{A}$ has a local protocol $R_i : L \rightarrow 2^{\text{Act}_i}$ to determine the set of enabled actions that could be carried out in a given local state. The global evolution function can be defined as follows: $\tau : G \times ACT \rightarrow G$, where $ACT = \text{Act}_1 \times \cdots \times \text{Act}_n$ and each component $a \in ACT$ is a joint action, which is a tuple of actions (one for each agent). $\tau_i$ is a local evolution function that points out the transitions
for an individual agent $i$ between his local states and it is defined as follows: $	au_i : L_i \times \text{Act}_i \rightarrow L_i$.

In this paper, we have adopted the extended communicative version of Interpreted systems presented in (Bentahar et al., 2012a). The idea of this model is that, two agents $i$ and $j$ should share a communication channel (shared variable) to communicate. In this model, each agent is linked with a set of local variables $\text{Var}_i$. In particular, a communication channel between $i$ and $j$ does exist iff $\text{Var}_i \cap \text{Var}_j \neq \emptyset$. This intuitively represents the existence of a communication channel between $i$ and $j$ (in their global states $g$ and $g'$) through which the shared variables have been sent by one of the two agents to the other. Therefore, $i$ and $j$ will share the same values for these variables. It is worth noticing that shared variables only motivate the existence of communication channels, not the creation of communication. Consequently, having one shared variable between two agents is enough to model such a communication channel.

**Definition 1 (Model of CTLKC$^+$).** A model $\mathcal{M} = (S, I, R_t, \{\approx_i \mid i \in A\}, \{\approx_{i \rightarrow j} \mid (i, j) \in A^2\}, V)$ that belongs to the set of all models $\mathcal{M}$ is a tuple, where:

- $S \subseteq L_1 \times \cdots \times L_n$ is the set of reachable $^1$ global states for the system.
- $I \subseteq S$ is a set of initial global states for the system.
- $R_t \subseteq S \times S$ is the transition relation defined by $(s, s') \in R_t$ iff there exists a joint action $(a_1, \ldots, a_n) \in \text{ACT}$ such that $\tau(s, a_1, \ldots, a_n) = s'$.
- For each agent $i \in A$, $\approx_i \subseteq S \times S$ is the epistemic accessibility relation defined by $s \approx_i s'$ iff $l_i(s) = l_i(s')$.
- For each pair $(i, j) \in A^2$, $\approx_{i \rightarrow j} \subseteq S \times S$ is the social accessibility relation defined by $s \approx_{i \rightarrow j} s'$ iff $\text{Var}_i \cap \text{Var}_j \neq \emptyset$ such that $\forall x \in \text{Var}_i \cap \text{Var}_j$ we have $l^x_i(s) = l^x_i(s') = l^x_j(s')$.
- $V : S \rightarrow 2^{\Phi_p}$ is a valuation function where $\Phi_p$ is a set of atomic propositions.

$^1$ $S$ contains only states that are reachable from $I$ by means of $R_t$
A path \( \pi = (s_0, s_1, \ldots) \) in a model \( \mathcal{M} \) is an infinite sequence of reachable global states in \( S \) such that for all \( i \geq 0, (s_i, s_{i+1}) \in R_i \). \( \pi(i) \) denotes the \((i + 1)^{th}\) state of \( \pi \) (i.e., \( \pi(i) = s_i \)) (Baier & Katoen, 2008).

In the model \( \mathcal{M} \), we assume that each accessible state is reachable. Furthermore, the epistemic accessibility relation \( \approx_i \) captures the intuition that two global states \( s, s' \) are indistinguishable for agent \( i \) (Penczek & Lomuscio, 2003). Formally, \( s \approx_i s' \) iff \( l_i(s) = l_i(s') \). The epistemic accessibility relation is an equivalence relation.

The intuition behind the social accessibility relation \( \approx_{i \rightarrow j} \) from one global state \( s \) to another global state \( s' \) (\( s \approx_{i \rightarrow j} s' \)) is that there are some communication channels (shared variables) between \( i \) and \( j \) such that agent \( i \) fills the channel in \( s \), and agent \( j \) receives the channel’s content in \( s' \). After receiving the channel’s content, all the shared variables between \( i \) and \( j \) will have the same values (i.e., \( l^x_i(s) = l^x_i(s') = l^x_j(s') \forall x \in \text{Var}_i \cap \text{Var}_j \)). There is no constraint on the content of the unshared variables for both agents, as they can both receive, simultaneously, information from other agents through other channels involving other variables for each one of them. Thus, those variables can be different from \( s \) to \( s' \). This idea is illustrated in Figure 2, where two agents \( i \) and \( j \) are communicating through a communication channel and their shared and unshared variables are as follows: Agent \( i \): \( \text{Var}_i = \{ x_1, x_2 \} \); Agent \( j \): \( \text{Var}_j = \{ x_1, x_2' \} \). The variable \( x_1 \) is the shared variable between the two agents which represents the existence of a communication channel between \( i \) and \( j \). The variables \( x_2, x_2' \) are the unshared variables between the two agents. When the communication channel is created, the value of \( x_1 \) for agent \( j \) in \( s \) is changed to be equal the value of variable \( x_1 \) for agent \( i \) in \( s' \). This illustrates the message passing through the channel.

Notice that only the shared variables should be the same as both the creditor \( j \) (towards which the commitment is made) and the debtor \( i \) (that makes the commitment) could gain more information when moving from one state to an accessible one (i.e., they could obtain new information through other communications).

3.3. Frame Definability (Correspondence)

In this section, we aim to answer the following semantic questions: “what can modal formulas say about the frames, and how do they say it?” (Blackburn et al., 2006).

To answer these questions, let us first define frame, model, frame validity, frame definability, and then show how to use frame definability in capturing
the correspondence between a given modal formula and a class of frames.

**Definition 2** (Frame). A tuple \((W, R_1, ..., R_n)\) with \(W\) is a nonempty set of states (worlds) and for each \(i\) (\(1 \leq i \leq n\)), \(R_i\) is a binary is a binary relation on \(W\) is called a frame.

**Definition 3** (Model). Given a frame \(F = (W, R_1, ..., R_n)\), we say the model \(M\) is based on the frame \(F = (W, R_1, ..., R_n)\) if \(M = (W, R_1, ..., R_n, V)\) for some valuation function \(V\), where \(V\) is defined as follows: \(V : W \times \Phi_p \rightarrow \{T, F\}\) over the set of atomic propositions \(\Phi_p\).

**Definition 4** (Frame Validity). Given a frame \(F = (W, R_1, ..., R_n)\), we say that a modal formula \(\varphi\) is valid on \(F\), denoted by \(F \models \varphi\), if \(M \models \varphi\) for all models \(M\) based on \(F\). A modal formula \(\varphi\) is valid on a class of frames \(\mathcal{F}\) if it is valid on each frame \(F\) in \(\mathcal{F}\) (Blackburn et al., 2006).

**Remark 1**
"Note that if \(F \models \varphi\) where \(\varphi\) is some modal formula, then \(F \models \varphi^*\) where \(\varphi^*\) is any substitution instance of \(\varphi\). That is, \(\varphi^*\) is obtained by replacing sentence letters in \(\varphi\) with modal formulas. In particular, this means, for example, that in order to show that \(F \not\models \Box \varphi \rightarrow \varphi\) it is enough to show that \(F \not\models \Box p \rightarrow p\) where \(p\) is a sentence letter" (Pacuit, 2009).

**Definition 5** (Frame Property). Suppose that \(P\) is a property of relations (e.g., reflexivity or transitivity). We say a frame \(F = (W, R_1, ..., R_n)\) has property \(P\) w.r.t. a particular \(R_i\) (\(1 \leq i \leq n\)) provided \(R_i\) has property \(P\).
Thus, we introduce the following frames:

- $\mathcal{F} = (W, R_1, ..., R_n)$ is called a serial frame w.r.t. a particular $R_i$ ($1 \leq i \leq n$) provided $R_i$ is serial, i.e., for all $w \in W$ there exists $v \in W$, such that $wR_iv$.

- $\mathcal{F} = (W, R_1, ..., R_n)$ is called a reflexive frame w.r.t. a particular $R_i$ ($1 \leq i \leq n$) provided $R_i$ is reflexive, i.e., for all $w \in W$, $wR_iw$.

- $\mathcal{F} = (W, R_1, ..., R_n)$ is called a transitive frame w.r.t. a particular $R_i$ ($1 \leq i \leq n$) provided $R_i$ is transitive, i.e., for all $w, x, v \in W$, if $wR_ix$ and $xR_iv$ then $wR_iv$.

- $\mathcal{F} = (W, R_1, ..., R_n)$ is called an Euclidean frame w.r.t. a particular $R_i$ ($1 \leq i \leq n$) provided $R_i$ is Euclidean, i.e., for all $w, x, v \in W$, if $wR_iv$ and $wR_ix$ then $vR_ix$.

- $\mathcal{F} = (W, R_1, ..., R_n)$ is called a symmetric frame w.r.t. a particular $R_i$ ($1 \leq i \leq n$) provided $R_i$ is symmetric, i.e., for all $w, v \in W$, if $wR_iv$ then $vR_iw$.

We also introduce a new particular frame

- $\mathcal{F} = (W, R_1, R_2)$ is called epistemic social frame ($\mathcal{ES}$) provided $R_1$, $R_2$ are the epistemic and social accessibility relations such that for all $w, v, x \in W$ if $wR_1v$ and $vR_2x$ then $wR_2x$.

**Definition 6** (Frame Correspondence). “A modal formula $\varphi$ defines a class of frames $\mathcal{F}$ if it is valid on precisely the frames in $\mathcal{F}$. That is, not only must $\varphi$ be valid on every frame in $\mathcal{F}$, it must also be possible to falsify $\varphi$ on any frame that is not in $\mathcal{F}$.” (Blackburn et al., 2006).

Hereafter are some examples, from (Pacuit, 2009), of what classes of frames can a modal language define.

**EXAMPLE 1.** $\Box \varphi \rightarrow \varphi$ defines the class of reflexive frames.

**Proof.** ($\Leftarrow$) Suppose that $\mathcal{F} = (W, R)$ is reflexive and let $\mathcal{M} = (W, R, V)$ be any model based on $\mathcal{F}$. Given $w \in W$, we must show $(\mathcal{M}, w) \models \Box \varphi \rightarrow \varphi$. Suppose that $(\mathcal{M}, w) \models \Box \varphi$. Then for all $v \in W$, if $wRv$ then $(\mathcal{M}, v) \models \Box \varphi$.

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Since $R$ is reflexive, we have $wRw$. Hence, $(\mathcal{M}, w) \models \varphi$. Therefore, $(\mathcal{M}, w) \models \Box \varphi \rightarrow \varphi$, as desired.

$(\Rightarrow)$ We argue by contraposition. Suppose that $\mathcal{F}$ is not reflexive. We must show $\mathcal{F} \not\models \Box \varphi \rightarrow \varphi$. Using Remark 1, it is enough to show $\mathcal{F} \not\models \Box p \rightarrow p$ for some sentence letter $p$. Since $\mathcal{F}$ is not reflexive, there is a state $w \in W$ such that it is not the case that $wRw$. Consider the model $\mathcal{M} = (W, R, V)$ based on $\mathcal{F}$ with $V(v, p) = T$ for all $v \in W$ such that $v \neq w$. Then $(\mathcal{M}, w) \models \Box p$ since, by assumption, for all $v \in W$ if $wRv$, then $v \neq w$ and so $V(v, p) = T$. Also, notice that by the definition of $V$, $(\mathcal{M}, w) \not\models p$. Therefore, $(\mathcal{M}, w) \models \Box p \land \neg p$, and so, $\mathcal{F} \not\models \Box p \rightarrow p$. □

EXAMPLE 2. $\Box \varphi \rightarrow \Box \Box \varphi$ defines the class of transitive frames.

Proof. ($\Leftarrow$) Suppose that $\mathcal{F} = (W, R)$ is transitive and let $\mathcal{M} = (W, R, V)$ be any model based on $\mathcal{F}$. Given $w \in W$, we must show $(\mathcal{M}, w) \models \Box \varphi \rightarrow \Box \Box \varphi$. Suppose that $(\mathcal{M}, w) \models \Box \varphi$. We must show $(\mathcal{M}, w) \models \Box \Box \varphi$. Suppose that $v \in W$ and $wRv$. We must show $(\mathcal{M}, v) \models \Box \varphi$. To that end, let $x \in W$ be any state with $vRx$. Since $R$ is transitive and $wRv$ and $vRx$, we have $wRx$. Since $(\mathcal{M}, w) \models \Box \varphi$, we have $(\mathcal{M}, x) \models \varphi$. Therefore, since $x$ is an arbitrary state accessible from $v$, $(\mathcal{M}, v) \models \Box \varphi$, Hence $(\mathcal{M}, w) \models \Box \Box \varphi$, and so $(\mathcal{M}, w) \models \Box \varphi \rightarrow \Box \Box \varphi$ as desired.

($\Rightarrow$) We argue by contraposition. Suppose that $\mathcal{F}$ is not transitive. We must show $\mathcal{F} \not\models \Box \varphi \rightarrow \Box \Box \varphi$. Using Remark 1, it is enough to show $\mathcal{F} \not\models \Box p \rightarrow \Box \Box p$ for some sentence letter $p$. Since $\mathcal{F}$ is not transitive, there are states $w, v, x \in W$ with $wRv$ and $vRx$ but it is not the case that $wRx$. Consider the model $\mathcal{M} = (W, R, V)$ based on $\mathcal{F}$ with $V(y, p) = T$ for all $y \in W$ such that $y \neq x$. Since $(\mathcal{M}, x) \neq p$ and $wRv$ and $vRx$, we have $(\mathcal{M}, w) \neq \Box p$. Furthermore, $(\mathcal{M}, w) \models \Box p$ since the only state where $p$ is false is $x$ and it is assumed that it is not the case that $wRx$. Therefore, $(\mathcal{M}, w) \models \Box p \land \neg \Box p$, and so, $\mathcal{F} \not\models \Box p \rightarrow \Box \Box p$, as desired. □

In Section 5, we will follow a similar approach in proving the correspondence between certain classes of frames and our reasoning postulates.

4. CTLKC$^+$ Logic

In this section, we recall CTLKC$^+$, the logic of knowledge and commitments that has been introduced in (Al-Saqqar et al., 2014). We first present the syntax and semantics of the logic, then we proceed with some logical properties.
4.1. Syntax and Semantics of CTLKC+

The syntax of CTLKC+ logic, which is a combination of branching time CTL (Emerson, 1990) with knowledge and social commitments is defined as follows:

**Definition 7** (Syntax of CTLKC+).

\[ \varphi ::= p \mid \neg \varphi \mid \varphi \lor \varphi \mid EX \varphi \mid EG \varphi \mid E(\varphi U \varphi) \mid K_i \varphi \mid C_{i \rightarrow j} \varphi \mid Fu(C_{i \rightarrow j} \varphi). \]

Where:

- \( p \in \Phi \) is an atomic proposition;
- \( E \) is the existential quantifier on paths;
- \( X, G \) and \( U \) are CTL path modal connectives standing for "next", "globally", and "until" respectively;
- The Boolean connectives \( \neg \) and " \lor " are defined in the usual way;
- The modal connective \( K_i \) stands for "knowledge for agent \( i \)";
- The modal connective \( C_{i \rightarrow j} \) stands for "commitment from \( i \) to \( j \)"; and
- The modal connective \( Fu \) stands for "fulfillment".

In CTLKC+, \( K_i \varphi \) is read as “agent \( i \) knows \( \varphi \)”. \( C_{i \rightarrow j} \varphi \) is read as “agent \( i \) commits towards agent \( j \) to bring about \( \varphi \)”. \( Fu(C_{i \rightarrow j} \varphi) \) is read as “the commitment \( C_{i \rightarrow j} \varphi \) is fulfilled”. Other temporal modalities, e.g., \( F \) (future), and the universal path quantifier \( A \) (“for all paths”) can be defined in terms of the above as usual (see for example (Emerson, 1990)). Moreover, we use the abbreviations — “implies” (\( \rightarrow \)), “equivalent” (\( \leftrightarrow \)) — and “constant false proposition” (\( \bot \)) as follows:

- \( \varphi \rightarrow \psi \triangleq \neg \varphi \lor \psi \),
- \( \varphi \leftrightarrow \psi \triangleq (\varphi \rightarrow \psi) \land (\psi \rightarrow \varphi) \),
- \( \bot \triangleq \varphi \land \neg \varphi \).

**Definition 8** (Satisfiability and Validity). *A formula \( \varphi \) is said to be satisfied* (written as \( (M,s) \models \varphi \)), *when there exists a model \( M \) and a state \( s \) for which \( \varphi \) holds. A formula \( \varphi \) is valid* (written as \( \models \varphi \)) *iff \( (M,s) \models \varphi \) for all models \( M \in \mathcal{M} \) and for all \( s \in S \).*
Definition 9 (Satisfaction of CTLKC+).
Given the model $\mathcal{M}$, the satisfaction of a CTLKC+ formula $\varphi$ in a global state $s$, denoted by $(\mathcal{M}, s) \models \varphi$, is recursively defined as follows:

- $(\mathcal{M}, s) \models p$ iff $p \in \mathcal{V}(s)$;
- $(\mathcal{M}, s) \models \neg \varphi$ iff $(\mathcal{M}, s) \not\models \varphi$;
- $(\mathcal{M}, s) \models \varphi \lor \psi$ iff $(\mathcal{M}, s) \models \varphi$ or $(\mathcal{M}, s) \models \psi$;
- $(\mathcal{M}, s) \models EX\varphi$ iff there exists a path $\pi$ starting at $s$ such that $(\mathcal{M}, \pi(1)) \models \varphi$;
- $(\mathcal{M}, s) \models E(\varphi U \psi)$ iff there exists a path $\pi$ starting at $s$ such that for some $k \geq 0$, $(\mathcal{M}, \pi(k)) \models \psi$ and $(\mathcal{M}, \pi(j)) \models \varphi$ for all $0 \leq j < k$;
- $(\mathcal{M}, s) \models EG\varphi$ iff there exists a path $\pi$ starting at $s$ such that $(\mathcal{M}, \pi(k)) \models \varphi$ for all $k \geq 0$;
- $(\mathcal{M}, s) \models K_i\varphi$ iff for all global states $s' \in S$ such that $s \approx_i s'$, we have $(\mathcal{M}, s') \models \varphi$;
- $(\mathcal{M}, s) \models C_{i\rightarrow j}\varphi$ iff for all global states $s' \in S$ such that $s \approx_{i\rightarrow j} s'$, we have $(\mathcal{M}, s') \models K_i\varphi$ and $(\mathcal{M}, s') \models K_j\varphi$;
- $(\mathcal{M}, s) \models Fu(C_{i\rightarrow j}\varphi)$ iff there exists $s' \in S$ such that $s' \approx_{i\rightarrow j} s$ and $(\mathcal{M}, s') \models C_{i\rightarrow j}\varphi$ or there exists $s \in S$ such that $s' \approx_{i\rightarrow j} s$ and $(\mathcal{M}, s) \models C_{i\rightarrow j}\varphi$ or there exists $s'' \in S$ such that $s'' \approx_{i\rightarrow j} s'$ and $(\mathcal{M}, s'') \models C_{i\rightarrow j}\varphi$.

The semantics of CTLKC+ state formulas is defined in the model $\mathcal{M}$ as usual (semantics of CTL, see for example (Emerson, 1990)) with modalities for reasoning about knowledge and social commitments and their fulfillments respectively. The state formula $K_i\varphi$ is satisfied in the model $\mathcal{M}$ iff the content $\varphi$ holds in every accessible state $s'$ gained by the epistemic accessibility relation $\approx_i$. The state formula $C_{i\rightarrow j}\varphi$ is satisfied in the model $\mathcal{M}$ iff the modalities $K_i\varphi$ and $K_j\varphi$ hold in every accessible state $s'$ gained by the social accessibility relation $\approx_{i\rightarrow j}$. The intuition of this semantics is as follows: when $i$ commits to $j$ using a communication channel, the two agents become aware
of the content in the resulting states, which correlate with the states where the commitment is supposed to be satisfied. The state formula $Fu(C_i \rightarrow_j \varphi)$ is satisfied in the model $\mathcal{M}$ iff, there exists a state $s'$ satisfying the commitment from which $s$ can be seen using the social accessibility relation $\approx_{i \rightarrow_j}$.

The intuition behind this semantics is as follows: a state $s$ is a fulfillment state if it is socially accessible from the commitment state. Once fulfillment states are determined, fulfillment will be propagated to all the states that are equivalent for each agent (i.e., states that are accessible from the fulfillment states using epistemic accessibility relation). This is because when a fulfillment holds, the situation should be reflected in all equivalent states for the two interacting agents. It is worth mentioning that the semantics of the $Fu(C_i \rightarrow_j \varphi)$ is redefined in such a way to avoid the recursion that appeared in the second and third options of the semantics of the fulfillment in (Al-Saqqar et al., 2014).

4.2. Logical Properties

In this section, we investigate the logical properties that the models $\mathcal{M}$ of $\mathfrak{M}$ inherit from the axiomatic point of view.

**Proposition 1.** $\approx_i$ is reflexive, symmetric and transitive (i.e., equivalence) (Al-Saqqar et al., 2015).

**Proposition 2.** $\approx_{i \rightarrow j}$ is serial, transitive and Euclidean (Al-Saqqar et al., 2015).

Thus, the resulting logic of knowledge and commitments is $S5$ with respect to $\approx_i$ and $KD45$ with respect to $\approx_{i \rightarrow j}$. Further more, from the properties of $\approx_{i \rightarrow j}$ and $\approx_i$ accessibility relations the following theorem holds (Al-Saqqar et al., 2014).

**Theorem 1.** If $s_1 \approx_i s_2$ then $s_1 \approx_{i \rightarrow j} s_3$ iff $s_2 \approx_{i \rightarrow j} s_3$.

**Proof.** See (Al-Saqqar et al., 2014). □

Fulfillment propagation property is shown in the following theorem and corollary.

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2The semantics of $Fu(C_i \rightarrow_j \varphi)$ in (Al-Saqqar et al., 2014) was defined as follows:

$(\mathcal{M}, s) \models Fu(C_i \rightarrow_j \varphi)$ iff there exists $s' \in S$ such that $s' \approx_{i \rightarrow j} s$ and $(\mathcal{M}, s') \models C_i \rightarrow_j \varphi$ or there exists $s'' \in S$ such that $(\mathcal{M}, s'') \models Fu(C_i \rightarrow_j \varphi)$ or there exists $s'' \in S$ such that $(\mathcal{M}, s'') \models Fu(C_i \rightarrow_j \varphi)$.

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Theorem 2. Let ϱ ∈ {i, j}. If (M, s) |= Fu(Ci→jϕ) then ∀s′ s.t. s ≈ ϱ s′, we have (M, s′) |= Fu(Ci→jϕ)

Proof. We prove the theorem for ≈i, the proof for ≈j is similar. Assume that (M, s) |= Fu(Ci→jϕ) and ∃s′ ∈ S such that s ≈i s′ and (M, s′) |= ¬Fu(Ci→jϕ). From the semantics of Fu(Ci→jϕ), the three or-conditions are not satisfied. Therefore, ∀s″ such that s′ ≈i s″, we have (M, s″) |= ¬Fu(Ci→jϕ). Since s′ ≈i s as ≈i is symmetric, we have contradiction with the assumption when s = s″, so we are done. □

Corollary 1. Let ϱ ∈ {i, j}. If (M, s) |= Fu(Ci→jϕ) then ∀s′ s.t. s ≈ ϱ s, we have (M, s′) |= Fu(Ci→jϕ)

Proof. The result is direct from Theorem 2 since the accessibility relation ≈ϱ is symmetric. □

Lemma 1. If s1 ≈i s2 and s3 ≈i→j s2 then s1 ≈i→j s2.

Proof. Assume s1 ≈i s2 and s3 ≈i→j s2 for any pair i, j ∈ A. According to the definition of ≈i, it is the case that l_i(s_1) = l_i(s_2). Therefore, l^\phi_i(s_1) = l^\phi_i(s_2) ∀x ∈ Var_i ∩ Var_j. From the definition of ≈i→j, it is the case that l^\phi_i(s_3) = l^\phi_i(s_2) = l^\phi_j(s_2) ∀x ∈ Var_i ∩ Var_j. Consequently, s_1 ≈i→j s_2. □

5. Corresponding Reasoning Postulates

In this section, we introduce the NetBill payment protocol (Sirbu, 1997) as an application example, taken from the e-commerce domain, to illustrate the proposed reasoning postulates. After that, we consider the reasoning postulates along with their corresponding classes of frames. Finally, we illustrate how to use the correspondence results in proving the soundness and completeness of CTLKC^+.

5.1. Application Example

The NetBill payment protocol, depicted in Figure 3, is developed for buying and selling encrypted software goods on the Internet (Sirbu, 1997). It has been applied to show how commitments can specify and verify protocols in business settings (Yolum & Singh, 2000, 2004). This protocol involves two interacting agents: the customer (cus) and the merchant (mer).
NetBill consists of eight steps. First, both the customer and merchant authenticate each other using a public-key certificate. Thereafter, the customer asks for a special quote from the merchant. The merchant sends the requested quote to the customer. If the customer accepts the quote (i.e., commits to pay), then the merchant promises to send the digital information (i.e., commits to deliver the goods) in an encrypted format and withholds the key. In the meanwhile, the customer creates an electronic payment order (EPO) including a description for the received goods to the merchant (i.e., fulfills its commitment of paying). Then, the merchant verifies the EPO and conveys it to the NetBill server. The NetBill server checks the customer bank account and credits the payment on the merchant account. After that, a receipt including the key to decrypt the goods is sent to the merchant first and then to the customer. Finally, upon receiving the key with the receipt, the customer can decrypt the purchased information.

In this paper, we consider the following commitments to represent the interactions between the agents in NetBill:

1. $C_{\text{cus} \rightarrow \text{mer}} \ Pay$, which means that the customer (\text{cus}) commits towards the merchant (\text{mer}) to send the agreed amount of payment (\text{Pay}) (\text{Pay} is an atomic proposition that represents the content of the commitment).

2. $C_{\text{mer} \rightarrow \text{cus}} \ Deliver$, which means that the merchant (\text{mer}) commits towards the customer (\text{cus}) to deliver the required goods (\text{Deliver}) (\text{Deliver} is an atomic proposition that represents the content of the commitment).

3. $C_{\text{mer} \rightarrow \text{cus}} \ Receipt$, which means that the merchant (\text{mer}) commits towards the customer (\text{cus}) to send the receipt (\text{Receipt}) (\text{Receipt} is an atomic proposition that represents the content of the commitment).

5.2. Reasoning Postulates and Corresponding Frames

In this section, we follow correspondence theory for modal logic (van Benthem, 1984) to prove the correspondence between our proposed reasoning postulates and their related classes of frames. In our approach, we first give a name, formalization and meaning for each postulate. Thereafter, we correspond the postulates to certain classes of frames and provide the required proofs. Then, we present a discussion that illustrates the importance of the postulates in MASs and show how they are exemplified with the NetBill protocol, and how they were addressed in the literature. It is worth
mentioning that for valid postulates in any frame, we will not discuss their correspondence since they simply correspond to all possible frames.

P1. [Fulfillment]

**Formalization:** \( Fu(C_{i \rightarrow j} \varphi) \rightarrow \varphi \).

**Meaning:** When a commitment is fulfilled, its content holds.

**Correspondence:** For any frame \( F = (W, \approx_i, \approx_j, \approx_{i \rightarrow j}) \), \( F \models Fu(C_{i \rightarrow j} \varphi) \rightarrow \varphi \) iff \( F \) is reflexive and symmetric with respect to \( \approx_i \) or \( \approx_j \).

**Proof.** \((\Rightarrow)\) Suppose that \( F = (W, \approx_i, \approx_j, \approx_{i \rightarrow j}) \) is reflexive and symmetric with respect to \( \approx_i \) or \( \approx_j \) and let \( M = (W, \approx_i, \approx_j, \approx_{i \rightarrow j}, V) \) be any model based on \( F \). Given \( s_1 \in W \), we must show \( (M, s_1) \models Fu(C_{i \rightarrow j} \varphi) \rightarrow \varphi \). Suppose that \( (M, s_1) \models Fu(C_{i \rightarrow j} \varphi) \). We must show \( (M, s_1) \models \varphi \). From the semantics of \( Fu(C_{i \rightarrow j} \varphi) \), three options are to be considered. According to the first option, there exists \( s_2 \in W \) such that \( s_2 \approx_{i \rightarrow j} s_1 \) and \( (M, s_2) \models C_{i \rightarrow j} \varphi \). From the semantics of \( C_{i \rightarrow j} \varphi \), we have \( (M, s_1) \models K_i \varphi \wedge K_j \varphi \). From the semantics of \( K_i \varphi \), for all \( s_3 \in W \) such that \( s_1 \approx_i s_3 \) we have \( (M, s_3) \models \varphi \). Since \( F \) is reflexive, we have \( s_1 \approx_i s_1 \). Thus, \( (M, s_1) \models \varphi \). So, we are done for the first option.
For the second and third options, there exists two states $s_2$ and $s_3 \in W$ such that $s_1 \approx_i s_2$, $s_3 \approx_i s_2$, and $(M, s_3) \models (C_{i \rightarrow j} \phi)$. From the semantics of $C_{i \rightarrow j} \phi$, we have $(M, s_2) \models K_i \phi \land K_j \phi$. Since $F$ is symmetric, then $s_2 \approx_i s_1$. Therefore, from the semantics of $K_i \phi$, $(M, s_1) \models \varphi$ as desired.

$(\Rightarrow)$ We argue by contraposition. Suppose that $F$ is not reflexive and not symmetric. We must show $F \not\models Fu(C_{i \rightarrow j} \phi) \rightarrow \varphi$. Using Remark 1, it is enough to show $F \not\models Fu(C_{i \rightarrow j} \phi) \rightarrow p$ for some sentence letter $p$. Consider the model $M = (W, \approx_i, \approx_j, \approx_{i \rightarrow j}, V)$ based on $F$. Assume $(M, s_1) \models Fu(C_{i \rightarrow j} \phi)$. From the semantics of $Fu(C_{i \rightarrow j} \phi)$, there exists $s_2 \in W$ such that $s_2 \approx_{i \rightarrow j} s_1$ and $(M, s_2) \models C_{i \rightarrow j} \phi$. From the semantics of $C_{i \rightarrow j} \phi$, we have $(M, s_1) \models K_i \phi \land K_j \phi$. From the semantics of $K_i \phi$, for all $s_3 \in W$ such that $s_1 \approx_i s_3$ we have $(M, s_3) \models \neg p$. Assume $(M, s_1) \models \neg p$. Since $F$ is not reflexive, then it might not be the case that $s_1 \approx_i s_1$ and so $(M, s_1) \models Fu(C_{i \rightarrow j} \phi) \land \neg p$. Thus, $F \not\models Fu(C_{i \rightarrow j} \phi) \rightarrow p$.

Discussion: It is obvious that this postulate is reasonable and realistic because in general when an agent fulfills its commitment, the content of this commitment holds at the same state. For example, with respect to the NetBill protocol, once the customer pays the agreed amount of money (i.e., fulfills its commitment), then the payment (i.e., the content of the commitment) holds. Formally, $Fu(C_{\text{cust-pay Pay}}) \rightarrow \text{Pay}$. This postulate is incorporated in the axioms of fulfillment introduced by (Bentahar et al., 2012a). Furthermore, a similar postulate is also incorporated by (Yolum & Singh, 2004; Chesani et al., 2013; EL Kholy et al., 2014; Chopra & Singh, 2015).

P2. **[Knowing the content of its own fulfilled commitment]**

Formalization: $Fu(C_{i \rightarrow j} \phi) \rightarrow K_i \phi$.

Meaning: An agent knows the content of its fulfilled commitment.

Correspondence: For any frame $F = (W, \approx_i, \approx_j, \approx_{i \rightarrow j})$, $F \models Fu(C_{i \rightarrow j} \phi) \rightarrow K_i \phi$ iff $F$ is symmetric and transitive with respect to $\approx_i$.

Proof. $(\Leftarrow)$ Suppose that $F = (W, \approx_i, \approx_j, \approx_{i \rightarrow j})$ is symmetric and transitive with respect to $\approx_i$ and let $M = (W, \approx_i, \approx_{i \rightarrow j}, V)$ be any model based on $F$. Given $s_1 \in W$, we must show that $(M, s_1) \models Fu(C_{i \rightarrow j} \phi) \rightarrow K_i \phi$. Suppose that $(M, s_1) \models Fu(C_{i \rightarrow j} \phi)$. We must show $(M, s_1) \models K_i \phi$. From the semantics of $Fu(C_{i \rightarrow j} \phi)$, three options
are to be considered. According to the first option, there exists \( s_2 \in W \) such that \( s_2 \approx_{i \rightarrow j} s_1 \) and \( (M, s_2) \models C_{i \rightarrow j} \varphi \). From the semantics of \( C_{i \rightarrow j} \varphi \), we have \( (M, s_1) \models K_i \varphi \wedge K_j \varphi \). So, we are done for the first option.

For the second and third options, there exists two states \( s_2 \) and \( s_3 \) \( \in W \) such that \( s_1 \approx_i s_2 \), \( s_3 \approx_{i \rightarrow j} s_2 \), and \( (M, s_3) \models (C_{i \rightarrow j} \varphi) \). From the semantics of \( C_{i \rightarrow j} \varphi \), we have \( (M, s_2) \models K_i \varphi \wedge K_j \varphi \). Assume, \( (M, s_1) \models \neg K_i \varphi \). From the semantics of \( K_i \varphi \), there exists \( s_4 \in W \) such that \( s_1 \approx_i s_4 \) and \( (M, s_4) \models \neg \varphi \). Since \( F \) is symmetric, then \( s_2 \approx_i s_1 \). Further, since \( F \) is transitive, then \( s_2 \approx_i s_4 \). Therefore, from the semantics of \( K_i \varphi \), we have \( (M, s_4) \models \varphi \). Thus, we have contradiction and so \( (M, s_1) \models K_i \varphi \) as desired.

(\( \Rightarrow \)) Suppose that \( F \) is not symmetric and not transitive. We must show \( F \models \neg Fu(C_{i \rightarrow j} \varphi) \rightarrow K_i \varphi \). We argue by contraposition. Consider the model \( M = (W, \approx_i, \approx_{i \rightarrow j}, V) \) based on \( F \). Suppose that \( (M, s_1) \models Fu(C_{i \rightarrow j} p) \). From the semantics of fulfillment, there exists two states \( s_2 \) and \( s_3 \in W \) such that \( s_1 \approx_i s_2 \), \( s_2 \approx_i s_3 \approx_{i \rightarrow j} s_2 \), and \( (M, s_3) \models (C_{i \rightarrow j} p) \). From the semantics of \( C_{i \rightarrow j} p \), we have \( (M, s_2) \models K_i p \wedge K_j p \). Assume \( (M, s_1) \models \neg K_i p \). From the semantics of \( K_i p \), there exists \( s_4 \in W \) such that \( s_1 \approx_i s_4 \) and \( (M, s_4) \models \neg p \). Since \( F \) is not symmetric, then it might not be the case that \( s_2 \approx_i s_1 \). Further, since \( F \) is not transitive, then it might not be the case that \( s_2 \approx_i s_4 \). Therefore, \( (M, s_1) \models Fu(C_{i \rightarrow j} p) \wedge \neg K_i p \). Consequently, \( F \not\models Fu(C_{i \rightarrow j} p) \rightarrow K_i p \).

\[ \square \]

**Discussion:** This postulate highlights the fact that an agent knows the content of its fulfilled commitment. Otherwise, it could happen that the agent does not know about its action. Hence, the agent might repeat it again and re-fulfill its commitment. For instance, from the NetBill protocol, assume that the merchant delivers the required goods to the customer (i.e., fulfills its commitment of delivering the goods). Applying the postulate, \( Fu(C_{mer \rightarrow cus} Deliver) \rightarrow K_{mer} Deliver \), reveals that the merchant should be aware about the content of its commitment after having fulfilled it. This postulate is incorporated in our work in (Al-Saqqar et al., 2014).

**P3.** [**Knowing the content of the fulfilled commitment**]

**Formalization:** \( Fu(C_{i \rightarrow j} \varphi) \rightarrow K_j \varphi \).

**Meaning:** An agent knows the content of the fulfilled commitment.
Correspondence: For any frame $F = (W, \approx_i, \approx_j, \approx_{i \rightarrow j})$, $F \models Fu(C_{i \rightarrow j} \varphi) \rightarrow K_j \varphi$ iff $F$ is symmetric and transitive with respect to $\approx_j$.

Proof. The proof is similar to the previous one, but with respect to $j$.

Discussion: This postulate conveys the fact that the creditor knows the content of the fulfilled commitment. The postulate is reasonable as the creditor should be aware of the satisfaction of the commitment directed to it once this fulfillment occurs. Otherwise, the creditor might require the debtor to re-discharge the commitment. In the previous example, assume that the merchant delivers the required goods to the customer (i.e., fulfills its commitment). Thus, the customer should know that the goods has been delivered. Formally, $Fu(C_{\text{mer} \rightarrow \text{cus Deliver}}) \rightarrow K_{\text{cus Deliver}}$. Otherwise, the customer could argue that no goods has been delivered, so it will require that a new delivery be performed, and the situation can be repeated. This postulate is incorporated in our work in (Al-Saqqar et al., 2014).

P4. [Knowing the fulfillment of its own commitment]

Formalization: $Fu(C_{i \rightarrow j} \varphi) \rightarrow K_i Fu(C_{i \rightarrow j} \varphi)$.

Meaning: An agent knows that it fulfills its own commitment.

Correspondence: For any frame $F = (W, \approx_i, \approx_j, \approx_{i \rightarrow j})$, $F \models Fu(C_{i \rightarrow j} \varphi) \rightarrow K_i(Fu(C_{i \rightarrow j} \varphi))$ iff $F$ is symmetric with respect to $\approx_i$.

Proof. ($\Leftarrow$) Suppose that $F = (W, \approx_i, \approx_j, \approx_{i \rightarrow j})$ is symmetric with respect to $\approx$ and let $M = (W, \approx_i, \approx_j, \approx_{i \rightarrow j}, V)$ be any model based on $F$. Given $s_1 \in W$, we must show $(M, s_1) \models Fu(C_{i \rightarrow j} \varphi) \rightarrow K_i(Fu(C_{i \rightarrow j} \varphi))$. Suppose that $(M, s_1) \models Fu(C_{i \rightarrow j} \varphi)$. We must show $(M, s_1) \models K_i(Fu(C_{i \rightarrow j} \varphi))$. From the semantics of $Fu(C_{i \rightarrow j} \varphi)$, three options are to be considered. According to the first option, there exists $s_2 \in W$ such that $s_2 \approx_{i \rightarrow j} s_1$ and $(M, s_2) \models C_{i \rightarrow j} \varphi$. From the semantics of $C_{i \rightarrow j} \varphi$, we have $(M, s_1) \models K_i \varphi \land K_j \varphi$. Assume $(M, s_1) \models \neg K_i(Fu(C_{i \rightarrow j} \varphi))$. From the semantics of knowledge, there exists $s_3 \in W$ such that $s_1 \approx_i s_3$ and $(M, s_3) \models \neg Fu(C_{i \rightarrow j} \varphi)$. Using the semantics of $Fu(C_{i \rightarrow j} \varphi)$, the three or-conditions are not satisfied. Therefore, using Theorem 2, $\forall s_4 \in W$ such that $s_3 \approx_i s_4$, we have $(M, s_4) \models \neg Fu(C_{i \rightarrow j} \varphi)$. Since $F$ is symmetric, then $s_3 \approx_i s_4$. Thus, we have contradiction with the assumption when $s_1 = s_4$, and so $(M, s_3) \models Fu(C_{i \rightarrow j} \varphi)$. Consequently, $(M, s_1) \models K_i(Fu(C_{i \rightarrow j} \varphi))$.}

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For the second and third options, there exists two states \( s_2 \) and \( s_3 \) \( \in W \) such that \( s_1 \approx_i s_2, s_3 \approx_{i\rightarrow j} s_2 \), and \( (M, s_3) = (C_{i\rightarrow j} \varphi) \). From the semantics of \( C_{i\rightarrow j} \varphi \), we have \( (M, s_2) = K_i \varphi \land K_j \varphi \). Assume, \( (M, s_1) = \neg K_i (Fu(C_{i\rightarrow j} \varphi)) \). According to the semantics of knowledge, there exists \( s_4 \in W \) such that \( s_1 \approx_i s_4 \) and \( (M, s_4) = \neg Fu(C_{i\rightarrow j} \varphi) \). From the semantics of \( Fu(C_{i\rightarrow j} \varphi) \), the three or-conditions are not satisfied. Therefore, using Theorem 2, \( \forall s_5 \in W \) such that \( s_4 \approx_i s_5 \), we have \( (M, s_5) = \neg Fu(C_{i\rightarrow j} \varphi) \). Since \( F \) is symmetric, then \( s_1 \approx_i s_1 \). Thus, we have contradiction with the assumption when \( s_1 = s_5 \), and so \( (M, s_4) = Fu(C_{i\rightarrow j} \varphi) \). Consequently, \( (M, s_1) = K_i (Fu(C_{i\rightarrow j} \varphi)) \) as desired.

\( (\Rightarrow) \) We argue by contraposition. Suppose that \( F \) is not symmetric. We must show \( F \not\equiv Fu(C_{i\rightarrow j} \varphi) \rightarrow K_i (Fu(C_{i\rightarrow j} \varphi)) \). Consider the model \( M = (W, \approx_i, \approx_{i\rightarrow j}, \approx_{j\rightarrow i, V}) \) based on \( F \). Assume \( (M, s_1) = Fu(C_{i\rightarrow j} p) \). From the semantics of \( Fu(C_{i\rightarrow j} p) \), there exists \( s_2 \in W \) such that \( s_2 \approx_{i\rightarrow j} s_1 \) and \( (M, s_2) = C_{i\rightarrow j} p \). Using the semantics of \( C_{i\rightarrow j} p \), we have \( (M, s_1) = K_j p \land K_i p \). To this end, assume \( (M, s_1) = \neg K_i (Fu(C_{i\rightarrow j} p)) \). According to the semantics of knowledge, there exists \( s_3 \in W \) such that \( s_1 \approx_i s_3 \) and \( (M, s_3) = \neg Fu(C_{i\rightarrow j} p) \). From the semantics of \( Fu(C_{i\rightarrow j} \varphi) \), the three or-conditions are not satisfied. Therefore, using Theorem 2, for all \( s_4 \) such that \( s_3 \approx_i s_4 \), we have \( (M, s_4) = \neg Fu(C_{i\rightarrow j} \varphi) \). Since \( F \) is not symmetric, then it might not be the case that \( s_3 \approx_i s_1 \). Thus, \( (M, s_1) = Fu(C_{i\rightarrow j} p) \land \neg K_i (Fu(C_{i\rightarrow j} p)) \). So, we are done.

\( \square \)

**Discussion:** It is clear that this postulate has certain importance in open MASs. Actually, it reflects the agent’s intentionality of its action to fulfill its own commitment. The validity of this postulate is significant as violating this postulates would mean that the agent is not aware about its action of discharging (fulfilling) its own commitment. Consequently, the agent could discharge the commitment many times. For example, assume that the customer fulfills its commitment of paying the required amount of money to the merchant, then the customer should know that. This is formally denoted as: \( Fu(C_{cus\rightarrow mer} Pay) \rightarrow K_{cus} Fu(C_{cus\rightarrow mer} Pay) \). This postulate is incorporated in our work in (Al-Saqqar et al., 2014).

P5. [Knowing the fulfillment of the commitment]
Formalization: $Fu(C_{i \to j} \varphi) \to K_j Fu(C_{i \to j} \varphi)$.

Meaning: An agent knows the fulfillment of the commitment.

Correspondence: For any frame $F = (W, \approx_i, \approx_j, \approx_{i \to j})$, $F \models Fu(C_{i \to j} \varphi) \to K_j Fu(C_{i \to j} \varphi)$ iff $F$ is symmetric with respect with $\approx_j$.

Proof. The proof is similar to the previous one, but with respect to $j$.

Discussion: This postulate emphasizes the awareness of the creditor about the fulfillment of the debtor’s commitment. In particular, it captures the intuition that, in open MASs, fulfilling the commitment is public. For example, assume that the merchant fulfills its commitment of delivering the required goods to the customer. This fulfillment should be public so that the customer can recognize it. It can be formally expressed as: $Fu(C_{mer \to cus Deliver}) \to K_{cus} Fu(C_{mer \to cus Deliver})$. If this postulate is violated, then serious problems could happen. Indeed, being aware about the fulfillment of the commitment, prevents the creditor to ask the debtor to fulfill it again. This postulate is incorporated in our work in (Al-Saqqar et al., 2014).

P6. [Knowing its own commitment]

Formalization: $C_{i \to j} \varphi \to K_i (C_{i \to j} \varphi)$.

Meaning: An agent knows about its own commitment.

Correspondence: For any frame $F = (W, \approx_i, \approx_j, \approx_{i \to j})$, $F \models C_{i \to j} \varphi \to K_i (C_{i \to j} \varphi)$ iff $F$ is $\mathcal{ES}$.

Proof. ($\Rightarrow$) Suppose that $F = (W, \approx_i, \approx_j, \approx_{i \to j})$ is $\mathcal{ES}$ and let $\mathcal{M} = (W, \approx_i, \approx_j, \approx_{i \to j}, V)$ be any model based on $F$. Given $s_1 \in W$, we must show $(\mathcal{M}, s_1) \models C_{i \to j} \varphi \to K_i (C_{i \to j} \varphi)$. Suppose that $(\mathcal{M}, s_1) \models C_{i \to j} \varphi$. We must show $(\mathcal{M}, s_1) \models K_i (C_{i \to j} \varphi)$. Further, we must show $(\mathcal{M}, s_2) \models C_{i \to j} \varphi$ for any $s_2 \in W$ and $s_1 \approx_i s_2$. Assume $(\mathcal{M}, s_2) \models \neg C_{i \to j} \varphi$. From the semantic of $C_{i \to j} \varphi$, there exists $s_3 \in W$ such that $s_2 \approx_{i \to j} s_3$ and $(\mathcal{M}, s_3) \models \neg K_i \varphi \vee \neg K_j \varphi$. Based on the definition of $\mathcal{ES}$, we have $s_1 \approx_{i \to j} s_3$ and so $(\mathcal{M}, s_3) \models K_i \varphi \wedge K_j \varphi$ which is a contradiction. Thus, $(\mathcal{M}, s_2) \models C_{i \to j} \varphi$ and therefore, $(\mathcal{M}, s_1) \models K_1(C_{i \to j} \varphi)$. Consequently, $(\mathcal{M}, s_1) \models C_{i \to j} \varphi \to K_1 (C_{i \to j} \varphi)$ as desired.

($\Rightarrow$) Suppose that $F$ is not $\mathcal{ES}$. We must show $F \not\models C_{i \to j} \varphi \to K_1 (C_{i \to j} \varphi)$.

Consider the model $\mathcal{M} = (W, \approx_i, \approx_j, \approx_{i \to j}, V)$ based on $F$. Using contraposition, suppose that $(\mathcal{M}, s_1) \models \neg C_{i \to j} \varphi$. Furthermore, assume there exists $s_2 \in W$ such that $s_1 \approx_i s_2$ and $(\mathcal{M}, s_2) \models \neg C_{i \to j} \varphi$. From the
semantics of commitment, there exists $s_3 \in W$ such that $s_2 \approx_{i \rightarrow j} s_3$ and $(M, s_3) \models \neg K_ip \lor \neg K jp$. Since $F$ is not $ES$, then it might not the case that $s_1 \approx_{i \rightarrow j} s_3$. Therefore, $(M, s_1) \not\models K_i(C_{i \rightarrow j}p)$. Consequently, $(M, s_1) \models C_{i \rightarrow j}p \land \neg K_i(C_{i \rightarrow j}p)$ and so, $F \not\models C_{i \rightarrow j}p \rightarrow K_i(C_{i \rightarrow j}p)$, as desired.

Discussion: Being committed to do something, the agent (i.e., the debtor) has to be aware about this action. This postulate is reasonable to be applied in MASs as agents should realize their own placed commitments. To illustrate the importance of this postulate, let us assume that the merchant commits towards the customer to deliver the required goods. Applying this postulate, $C_{mer \rightarrow cus \, Deliver} \rightarrow K_{mer} (C_{mer \rightarrow cus \, Deliver})$, it is obvious that the committing agent (i.e., the merchant) should know about this particular commitment as it doesn’t make sense for an agent to create a commitment and in the same time it is not aware about the consequences of this action. Otherwise, this would mean that the commitment is made accidently. This postulate is incorporated in our work in (Al-Saqqar et al., 2014). Further, a similar postulate is also incorporated by (Schmidt et al., 2004).

P7. [R-Conjoin]

Formalization: $(C_{i \rightarrow j}\varphi_1) \land (C_{i \rightarrow j}\varphi_2) \rightarrow C_{i \rightarrow j}(\varphi_1 \land \varphi_2)$.

 Meaning: An agent $i$ would become committed to bring about both $\varphi_1$ and $\varphi_2$ if $i$ individually commits to bring about $\varphi_1$ and commits to bring about $\varphi_2$.

Proof. Given $s_1 \in W$, we must show $(M, s_1) \models (C_{i \rightarrow j}\varphi_1) \land (C_{i \rightarrow j}\varphi_2) \rightarrow C_{i \rightarrow j}(\varphi_1 \land \varphi_2)$. Assume that $(M, s_1) \models ((C_{i \rightarrow j}\varphi_1) \land (C_{i \rightarrow j}\varphi_2)) \land \neg(C_{i \rightarrow j}(\varphi_1 \land \varphi_2))$. From the semantics of $C_{i \rightarrow j}\varphi$, for all global states $s_2 \in S$ such that $s_1 \approx_{i \rightarrow j} s_2$, we have $(M, s_2) \models (K_i\varphi_1 \land K_j\varphi_1) \land (K_i\varphi_2 \land K_j\varphi_2) \land (\neg K_i(\varphi_1 \land \varphi_2) \lor \neg K_j(\varphi_1 \land \varphi_2))$. Further, from the semantics of knowledge, there exists $s_3 \in S$ such that $s_2 \approx_{i \rightarrow j} s_3$ and $(M, s_3) \models (\varphi_1 \land \neg \varphi_1) \lor (\varphi_2 \land \neg \varphi_2)$. Thus, we have contradiction. Consequently, $(C_{i \rightarrow j}\varphi_1) \land (C_{i \rightarrow j}\varphi_2) \rightarrow C_{i \rightarrow j}(\varphi_1 \land \varphi_2)$. So, the postulates.

Discussion:
The validity of this postulate is captured from the fact that agent $i$ has the ability to have more than one commitment towards the same...
agent \( j \) at the same time. Suppose, for instance, that the merchant commits towards the customer to deliver the goods and it also commits towards the same customer to send a receipt, then the merchant would be committed towards the customer to deliver the goods and send the receipt. Formally, \((C_{mer \rightarrow cus} \text{ Deliver}) \land (C_{mer \rightarrow cus} \text{ Receipt}) \rightarrow C_{mer \rightarrow cus} (\text{Deliver} \land \text{Receipt})\). This postulate is incorporated in (Singh, 2008; EL Kholy et al., 2014; Chopra & Singh, 2015).

**P8. [Knowing its conjoin commitment]**

**Formalization:** \((C_{i \rightarrow j} \varphi_1) \land (C_{i \rightarrow j} \varphi_2) \rightarrow K_i(C_{i \rightarrow j}(\varphi_1 \land \varphi_2))\).

**Meaning:** An agent knows about its conjoin commitment.

**Correspondence:** For any frame \( F = (W, \approx_i, \approx_j, \approx_{i \rightarrow j}) \), \( F| = (C_{i \rightarrow j} \varphi_1) \land (C_{i \rightarrow j} \varphi_2) \rightarrow K_i(C_{i \rightarrow j}(\varphi_1 \land \varphi_2)) \) iff \( F \in ES \).

**Proof.** Based on Postulate P7, \( (M, s_1) \models (C_{i \rightarrow j} \varphi_1) \land (C_{i \rightarrow j} \varphi_2) \rightarrow C_{i \rightarrow j}(\varphi_1 \land \varphi_2) \). Thus, it is enough to prove \( (M, s_1) \models C_{i \rightarrow j}(\varphi_1 \land \varphi_2) \rightarrow K_i(C_{i \rightarrow j}(\varphi_1 \land \varphi_2)) \) which can be proved in a way similar to P6.

**Discussion:** The validity of this postulate is captured from the fact that an agent knows about its own commitment (Al-Saqqar et al., 2014). Consequently, in the previous example, the merchant should know that it would be committed towards the same customer to deliver the goods and send the receipt. Formally, \((C_{mer \rightarrow cus} \text{ Deliver}) \land (C_{mer \rightarrow cus} \text{ Receipt}) \rightarrow K_{mer}(C_{mer \rightarrow cus} (\text{Deliver} \land \text{Receipt}))\).

**P9. [Commitment’s chain]**

**Formalization:** \((C_{i \rightarrow j} \varphi) \land (C_{i \rightarrow j} (\varphi \rightarrow \psi)) \rightarrow C_{i \rightarrow j} \psi\).

**Meaning:** Commitments are closed under implication.

**Proof.** Assume that \((M, s_1) \models (C_{i \rightarrow j} \varphi) \land (C_{i \rightarrow j} (\varphi \rightarrow \psi)) \land \neg(C_{i \rightarrow j} \psi)\). From the semantics of commitment, for all \( s_2 \in S \) such that \( s_1 \approx_{i \rightarrow j} s_2 \), we have \((M, s_2) \models (K_i \varphi \land K_j \varphi) \land (K_i (\varphi \rightarrow \psi) \land K_j (\varphi \rightarrow \psi))\). Using the \( K \) axiom of knowledge, \((M, s_2) \models K_i \psi \land K_j \psi\). Since \( s_1 \approx_{i \rightarrow j} s_2 \), then \((M, s_1) \models C_{i \rightarrow j} \psi\) which contradicts our assumption.

**Discussion:** This postulate shows that CTLKC\(^+\) is closed under strict implication. This postulate is integrated in (Singh, 2008; EL Kholy et al., 2014).
P10. **[Knowing its commitment’s chain]**

**Formalization:** \((C_i \rightarrow_j \varphi) \land (C_i \rightarrow_j (\varphi \rightarrow \psi)) \rightarrow K_i(C_i \rightarrow_j \psi)\).

**Meaning:** Knowledge of the commitments is closed under implication.

**Correspondence:** For any frame \(F = (W, \approx_i, \approx_j, \approx_{i \rightarrow_j})\), \(F \models (C_i \rightarrow_j \varphi) \land (C_i \rightarrow_j (\varphi \rightarrow \psi)) \rightarrow K_i(C_i \rightarrow_j \psi)\) iff \(F\) is \(\mathcal{ES}\).

**Proof.** Based on Postulate P9, \((M, s_1) \models (C_i \rightarrow_j \varphi) \land (C_i \rightarrow_j (\varphi \rightarrow \psi)) \rightarrow C_i \rightarrow_j \psi\). Thus, it is enough to prove \((M, s_1) \models C_i \rightarrow_j \psi \rightarrow K_i(C_i \rightarrow_j \psi)\) which was proved in P6.

**Discussion:** This postulate conveys the fact that an agent knows about its implied commitment.

P11. **[Weaken commitment]**

**Formalization:** \(C_i \rightarrow_j (\varphi_1 \land \varphi_2) \rightarrow C_i \rightarrow_j \varphi_1\).

**Meaning:** If \(i\) commits to a conjunction, \(i\) is also commits to each part of the conjunction.

**Proof.** Given \(s_1 \in W\), we must show \((M, s_1) \models C_i \rightarrow_j (\varphi_1 \land \varphi_2) \rightarrow C_i \rightarrow_j \varphi_1\). Assume that \((M, s_1) \models C_i \rightarrow_j (\varphi_1 \land \varphi_2) \land \neg(C_i \rightarrow_j \varphi_1)\). From the semantics of \(C_i \rightarrow_j \varphi\), for all global states \(s_2 \in S\) such that \(s_1 \approx_{i \rightarrow_j} s_2\), we have \((M, s_2) \models K_i(\varphi_1 \land \varphi_2) \land K_j(\varphi_1 \land \varphi_2)\). Thus, \((M, s_2) \models K_i \varphi_1 \land K_j \varphi_1\). Therefore, \((M, s_1) \models C_i \rightarrow_j \varphi_1\) which contradicts our assumption. So, the postulate.

**Discussion:** This postulate says that if agent \(i\) commits to a conjunction, \(i\) is also commits to each part of the conjunction. For example, if the merchant commits to sending both the required goods and the receipt, then the merchant commits to send the goods. Formally, \(C_{mer \rightarrow cus} (Deliver \land Receipt) \rightarrow C_{mer \rightarrow cus} Deliver\). This postulate is incorporated in (Singh, 2008; EL Kholy et al., 2014; Chopra & Singh, 2015).

P12. **[Knowing its weakened commitment]**

**Formalization:** \(C_i \rightarrow_j (\varphi_1 \land \varphi_2) \rightarrow K_i(C_i \rightarrow_j \varphi_1)\).

**Meaning:** An agent knows about each part of its conjuncted commitment.

**Correspondence:** For any frame \(F = (W, \approx_i, \approx_j, \approx_{i \rightarrow_j})\), \(F \models C_i \rightarrow_j (\varphi_1 \land \varphi_2) \rightarrow K_i(C_i \rightarrow_j \varphi_1)\) iff \(F\) is \(\mathcal{ES}\).
Proof. Based on Postulate P11, \((\mathcal{M}, s_1) \models C_{i\rightarrow j}(\varphi_1 \land \varphi_2) \rightarrow C_{i\rightarrow j}\varphi_1\). So, it is enough to prove \((\mathcal{M}, s_1) \models C_{i\rightarrow j}\varphi_1 \rightarrow K_i(C_{i\rightarrow j}\varphi_1)\) which was proved in P6.

Discussion: This postulate shows that if an agent committed to a conjunction, then it would be aware of its own commitment to each part of the conjunction.

P13. [Weaken fulfillment]

Formalization: \(Fu(C_{i\rightarrow j}(\varphi_1 \land \varphi_2)) \rightarrow Fu(C_{i\rightarrow j}\varphi_1)\).

Meaning: If \(i\) fulfills a conjunction, \(i\) is also fulfills each part of the conjunction.

Proof. Assume that \((\mathcal{M}, s_1) \models Fu(C_{i\rightarrow j}(\varphi_1 \land \varphi_2)) \land \neg Fu(C_{i\rightarrow j}\varphi_1)\). From the semantics of \(Fu(C_{i\rightarrow j}\varphi)\), three options are to be considered. According to the first option, there exists \(s_2 \in W\) such that \(s_2 \approx_{i\rightarrow j} s_1\) and \((\mathcal{M}, s_2) \models C_{i\rightarrow j}(\varphi_1 \land \varphi_2)\) and for all \(s \in W\) such that \(s \approx_{i\rightarrow j} s_1\) \((\mathcal{M}, s) \models \neg C_{i\rightarrow j}\varphi_1\). Using P11, we have contradiction when \(s = s_2\). Thus, \((\mathcal{M}, s_1) \models Fu(C_{i\rightarrow j}(\varphi_1 \land \varphi_2)) \land Fu(C_{i\rightarrow j}\varphi_1)\). So, we are done for the first option.

For the second and third options, there exists two states \(s_2, s_3 \in W\) such that \(s_1 \approx_i s_2, s_3 \approx_{i\rightarrow j} s_2\), and \((\mathcal{M}, s_3) \models C_{i\rightarrow j}(\varphi_1 \land \varphi_2)\) and for all \(s \in W\) such that \(s \approx_{i\rightarrow j} s_2\) \((\mathcal{M}, s) \models \neg C_{i\rightarrow j}\varphi_1\). Using P11, we have contradiction when \(s = s_3\). Therefore, \((\mathcal{M}, s_3) \models C_{i\rightarrow j}(\varphi_1 \land \varphi_2) \land C_{i\rightarrow j}\varphi_1\) and so \((\mathcal{M}, s_2) \models Fu(C_{i\rightarrow j}(\varphi_1 \land \varphi_2)) \land Fu(C_{i\rightarrow j}\varphi_1)\). Using Corollary 1, \((\mathcal{M}, s_1) \models Fu(C_{i\rightarrow j}(\varphi_1 \land \varphi_2)) \land Fu(C_{i\rightarrow j}\varphi_1)\) as desired.

Discussion: This postulate conveys the fact that if agent \(i\) fulfills a conjunction, \(i\) is also fulfills each part of the conjunction. For example, if the merchant fulfills its commitment of sending both the required goods and the receipt, then the merchant fulfills sending the goods. Formally, \(Fu(C_{\text{mer-cus}} (\text{Deliver} \land \text{Receipt})) \rightarrow Fu(C_{\text{mer-cus}} \text{Deliver})\).

P14. [Knowing its weakened fulfillment]

Formalization: \(Fu(C_{i\rightarrow j}(\varphi_1 \land \varphi_2)) \rightarrow K_i(Fu(C_{i\rightarrow j}\varphi_1))\).

Meaning: An agent knows about each part of its conjuncted fulfillment.

Correspondence: For any frame \(\mathcal{F} = (W, \approx_i, \approx_j, \approx_{i\rightarrow j})\), \(\mathcal{F} \models Fu(C_{i\rightarrow j}(\varphi_1 \land \varphi_2)) \rightarrow K_i(Fu(C_{i\rightarrow j}\varphi_1))\) iff \(\mathcal{F} \in \mathcal{ES}\).
Proof. Based on Postulate P13, \( Fu(C_{i\rightarrow j}(\varphi_1 \land \varphi_2)) \rightarrow Fu(C_{i\rightarrow j}\varphi_1) \). So, it is enough to prove \((M, s_1) \models Fu(C_{i\rightarrow j}\varphi_1) \rightarrow K_i(Fu(C_{i\rightarrow j}\varphi_1)) \) which was proved in P4.

Discussion: This postulate reflects the fact that if an agent fulfilled a conjunction, then it would be aware of its own fulfillment of each part of the conjunction.

P15. [Strong consistency]

Formalization: \( C_{i\rightarrow j}\varphi \rightarrow \neg C_{i\rightarrow j}\neg\varphi \).

Meaning: When a commitment holds, there is no possibility for committing to the negation of its content.

Correspondence: For any frame \( F = (W, \approx_i, \approx_j, \approx_{i\rightarrow j}) \), \( F \models C_{i\rightarrow j}\varphi \rightarrow \neg C_{i\rightarrow j}\neg\varphi \) iff \( F \) is serial with respect to \( \approx_{i\rightarrow j} \).

Proof. \((\Leftarrow)\) Suppose that \( F = (W, \approx_i, \approx_j, \approx_{i\rightarrow j}) \) is serial with respect to \( \approx_{i\rightarrow j} \), and let \( M = (W, \approx_i, \approx_j, \approx_{i\rightarrow j}, V) \) be any model based on \( F \). Given \( s_1 \in W \), we must show \((M, s_1) \models C_{i\rightarrow j}\varphi \rightarrow \neg C_{i\rightarrow j}\neg\varphi \).

Assume that \((M, s_1) \models (C_{i\rightarrow j}\varphi) \land (C_{i\rightarrow j}\neg\varphi) \). From the semantics of \( C_{i\rightarrow j}\varphi \), for all global states \( s_2 \in S \) such that \( s_1 \approx_{i\rightarrow j} s_2 \), we have \((M, s_2) \models K_i\varphi \land K_i\neg\varphi \). Thus, from the semantics of \( K_i\varphi \), for all \( s_3 \in S \) such that \( s_2 \approx_i s_3 \), we have \((M, s_3) \models \varphi \land \neg\varphi \). So, the contradiction.

\((\Rightarrow)\) Suppose that \( F \) is not serial with respect to \( \approx_{i\rightarrow j} \). We must show \( F \not\models C_{i\rightarrow j}\varphi \rightarrow \neg C_{i\rightarrow j}\neg\varphi \). Since \( F \) is not serial, using an argument by contraposition, then it might be the case that \((M, s_1) \models C_{i\rightarrow j}p \land C_{i\rightarrow j}\neg p \). Therefore, \( F \not\models C_{i\rightarrow j}p \rightarrow \neg C_{i\rightarrow j}\neg p \), as desired.

Discussion: The validity of this postulate is captured from the fact that an agent cannot commit to bring about \( \varphi \) and \( \neg\varphi \) at the same time. This postulate is incorporated in (Singh, 2008; EL Kholy et al., 2014).

P16. [Knowing strong consistency]

Formalization: \( C_{i\rightarrow j}\varphi \rightarrow K_i(\neg C_{i\rightarrow j}\neg\varphi) \).

Meaning: When a commitment holds, then the debtor knows that there is no possibility to commit about the the negation of the commitment content.

Correspondence: For any frame \( F = (W, \approx_i, \approx_j, \approx_{i\rightarrow j}) \), \( F \models C_{i\rightarrow j}\varphi \rightarrow K_i(\neg C_{i\rightarrow j}\neg\varphi) \) iff \( F \) is ES.
Proof. Based on Postulate P15, \((M, s_1) \models C_{i \rightarrow j} \varphi \rightarrow \neg C_{i \rightarrow j} \neg \varphi\). Consequently, it is enough to prove \((M, s_1) \models \neg C_{i \rightarrow j} \neg \varphi \rightarrow K_i(\neg C_{i \rightarrow j} \neg \varphi)\) which is similar to the proof of P6.

Discussion: This postulate is reasonable to be applied in open MASs since it doesn’t make sense for an agent to commit and reason about \(\varphi\) and \(\neg \varphi\) at the same time.

P17. [Nonexistence]

Formalization: \(AG \neg \varphi \rightarrow \neg C_{i \rightarrow j} \varphi\).

Meaning: If the content of the commitment doesn’t hold globally, then the commitment itself doesn’t hold too.

Correspondence: For any frame \(F = (W, \approx_i, \approx_j, \approx_{i \rightarrow j})\), \(F \models AG \neg \varphi \rightarrow \neg C_{i \rightarrow j} \varphi\) iff \(F\) is serial with respect to \(\approx_{i \rightarrow j}\).

Proof. \((\Leftarrow)\) Suppose that \(F = (W, \approx_i, \approx_j, \approx_{i \rightarrow j})\) is serial with respect to \(\approx_{i \rightarrow j}\) and let \(M = (W, \approx_i, \approx_j, \approx_{i \rightarrow j}, V)\) be any model based on \(F\). Given \(s_1 \in W\), we must show \((M, s_1) \models AG \neg \varphi \rightarrow \neg C_{i \rightarrow j} \varphi\). Assume that \((M, s_1) \models AG \neg \varphi \land C_{i \rightarrow j} \varphi\). From the semantics of \(C_{i \rightarrow j} \varphi\), for all global states \(s_2 \in S\) such that \(s_1 \approx_{i \rightarrow j} s_2\), we have \((M, s_2) \models K_i \varphi \land K_j \varphi\). Further, from the semantics of \(K_i \varphi\) and \(AG \varphi\), for all \(s_3 \in S\) such that \(s_2 \approx_i s_3\), we have \((M, s_3) \models \varphi \land \neg \varphi\). So, the contradiction.

\((\Rightarrow)\) We argue by contraposition. Suppose that \(F\) is not serial with respect to \(\approx_{i \rightarrow j}\). We must show \(F \not\models AG \neg \varphi \rightarrow \neg C_{i \rightarrow j} \varphi\). Since \(F\) is not serial, then it might be the case that \((M, s_1) \models AG \neg \varphi \land C_{i \rightarrow j} \varphi\). Therefore, \(F \not\models AG \neg \varphi \rightarrow \neg C_{i \rightarrow j} \varphi\), as desired.

Discussion: This postulate illustrates the fact that, if the content of the commitment (i.e., \(\varphi\)) doesn’t hold in all states, then the commitment itself never holds. This seems reasonable since, from the semantics of the commitment, both agents (i.e., debtor and creditor) become aware of the content in the accessible states which are also reachable. This postulates is incorporated in (EL Kholy et al., 2014).

P18. [Knowing the nonexistence]

Formalization: \(AG \neg \varphi \rightarrow K_i(\neg(C_{i \rightarrow j} \varphi))\).

Meaning: An agent knows that it is not the case that it commits to a content which never holds.

Correspondence: For any frame \(F = (W, \approx_i, \approx_j, \approx_{i \rightarrow j})\), \(F \models AG \neg \varphi \rightarrow K_i(\neg(C_{i \rightarrow j} \varphi))\) iff \(F\) is ES.
Proof. From Postulate P17, \( (\mathcal{M}, s_1) \models AG\neg\varphi \rightarrow \neg C_{i \rightarrow j} \varphi \). So, it is enough to prove \( (\mathcal{M}, s_1) \models \neg C_{i \rightarrow j} \varphi \rightarrow K_{i}(\neg C_{i \rightarrow j} \varphi) \) which has similar proof as P6.

Discussion: This postulate is reasonable to be applied in open MASs since agents should know that they cannot commit about an impossible content.

P19. [Commitment consistency]

Formalization: \( \neg C_{i \rightarrow j} \bot \), where \( \bot \) is read as "constant false proposition".

Meaning: An agent cannot commit to false.

Correspondence: For any frame \( \mathcal{F} = (W, \approx_i, \approx_j, \approx_{i \rightarrow j}) \), \( \mathcal{F} \models \neg C_{i \rightarrow j} \bot \) iff \( \mathcal{F} \) is serial with respect to \( \approx_{i \rightarrow j} \).

Proof. \((\Leftarrow)\) Suppose that \( \mathcal{F} = (W, \approx_i, \approx_j, \approx_{i \rightarrow j}) \) is serial with respect to \( \approx_{i \rightarrow j} \) and let \( \mathcal{M} = (W, \approx_i, \approx_j, \approx_{i \rightarrow j}, \mathcal{V}) \) be any model based on \( \mathcal{F} \). Given \( s_1 \in W \), we must show \( (\mathcal{M}, s_1) \models \neg C_{i \rightarrow j} \bot \). Assume that \( (\mathcal{M}, s_1) \models C_{i \rightarrow j} \bot \). From the semantics of commitment, for all global states \( s_2 \in S \) such that \( s_1 \approx_{i \rightarrow j} s_2 \), we have \( (\mathcal{M}, s_2) \models K_i \bot \land K_j \bot \) which contradicts the consistency axiom of knowledge (i.e., Axiom D). Therefore, \( (\mathcal{M}, s_1) \models \neg C_{i \rightarrow j} \bot \).

\((\Rightarrow)\) Suppose that \( \mathcal{F} \) is not serial with respect to \( \approx_{i \rightarrow j} \). We must show \( \mathcal{F} \not\models \neg C_{i \rightarrow j} \bot \). Since \( \mathcal{F} \) is not serial, then it might be the case that \( (\mathcal{M}, s_1) \models C_{i \rightarrow j} \bot \). Therefore, \( \mathcal{F} \not\models \neg C_{i \rightarrow j} \bot \), as desired.

Discussion: The validity of this postulate is captured from the fact that an agent cannot know false (Fagin et al., 1995). Consequently, similar to knowledge, an agent cannot commit to false. This postulate is integrated in (Singh, 2008; EL Kholy et al., 2014; Chopra & Singh, 2015).

P20. [Knowing committing to false]

Formalization: \( \neg K_i(C_{i \rightarrow j} \bot) \).

Meaning: Committing to false cannot be known.

Correspondence: For any frame \( \mathcal{F} = (W, \approx_i, \approx_j, \approx_{i \rightarrow j}) \), \( \mathcal{F} \models \neg K_i(C_{i \rightarrow j} \bot) \) iff \( \mathcal{F} \) is serial with respect to \( \approx_i \).
Proof. (⇐) Suppose that $\mathcal{F} = (W, \approx_i, \approx_j, \approx_{i \rightarrow j})$ is serial with respect to $\approx_i$. Let $\mathcal{M} = (W, \approx_i, \approx_j, \approx_{i \rightarrow j}, V)$ be any model based on $\mathcal{F}$. Given $s_1 \in W$, we must show $(\mathcal{M}, s_1) \models \neg K_i(C_{i \rightarrow j} \bot)$. Assume that $(\mathcal{M}, s_1) \models K_i(C_{i \rightarrow j} \bot)$. From the semantics of knowledge, for all global states $s_2 \in S$ such that $s_1 \approx_i s_2$, we have $(\mathcal{M}, s_2) \models C_{i \rightarrow j} \bot$, which is a contradiction with Postulate P19. Therefore, $(\mathcal{M}, s_1) \nvdash \neg K_i(C_{i \rightarrow j} \bot)$. 

(⇒) Suppose that $\mathcal{F}$ is not serial with respect to $\approx_i$. We must show $\mathcal{F} \not\models \neg K_i(C_{i \rightarrow j} \bot)$. Since $\mathcal{F}$ is not serial, then it might be the case that $(\mathcal{M}, s_1) \models K_i(C_{i \rightarrow j} \bot)$. Therefore, $\mathcal{F} \not\models \neg K_i(C_{i \rightarrow j} \bot)$, as desired. 

Discussion: This postulate is reasonable to be applied in open MASs since it reflects the fact that an agent cannot know an impossible commitment. Again, the validity of this postulate is captured from the fact that an agent cannot know false (i.e., Axiom D of knowledge).

P21. [Fulfillment consistency]

Formalization: $\neg F u(C_{i \rightarrow j} \bot)$.

Meaning: Committing to false cannot be fulfilled.

Proof. Given $s_1 \in W$, we must show $(\mathcal{M}, s_1) \models \neg F u(C_{i \rightarrow j} \bot)$. Assume that $(\mathcal{M}, s_1) \models F u(C_{i \rightarrow j} \bot)$. According to Postulate P2, $(\mathcal{M}, s_1) \models K_i \bot$ which contradicts the $D$ axiom of knowledge. Thus, $(\mathcal{M}, s_1) \models \neg F u(C_{i \rightarrow j} \bot)$.

Discussion: This postulate says that committing to false cannot be fulfilled. It is reasonable to be applied in open MASs as agents cannot fulfill an impossible commitment.

P22. [Debtors’ knowledge of fulfilling an impossible commitment]

Formalization: $\neg K_i(F u(C_{i \rightarrow j} \bot))$.

Meaning: Fulfillment for committing to false cannot be known by the debtor.

Correspondence: For any frame $\mathcal{F} = (W, \approx_i, \approx_j, \approx_{i \rightarrow j})$, $\mathcal{F} \models \neg K_i(F u(C_{i \rightarrow j} \bot))$ iff $\mathcal{F}$ is serial with respect to $\approx_i$.

Proof. (⇐) Suppose that $\mathcal{F} = (W, \approx_i, \approx_j, \approx_{i \rightarrow j})$ is serial with respect to $\approx_i$. Let $\mathcal{M} = (W, \approx_i, \approx_j, \approx_{i \rightarrow j}, V)$ be any model based on $\mathcal{F}$. Given $s_1 \in W$, we must show $(\mathcal{M}, s_1) \models \neg K_i(F u(C_{i \rightarrow j} \bot))$. Assume
\[(M, s_1) \models K_i(Fu(C_{i \rightarrow j} \perp)).\] From the semantics of knowledge, for all global states \(s_2 \in S\) such that \(s_1 \approx_i s_2\) we have \((M, s_2) \models Fu(C_{i \rightarrow j} \perp)\) which is a contradiction with P21. Thus, \((M, s_1) \models \neg K_i(Fu(C_{i \rightarrow j} \perp)).\)

\((\Rightarrow)\) Suppose that \(F\) is not serial with respect to \(\approx_i\). We must show \(F \not\models \neg K_i(Fu(C_{i \rightarrow j} \perp)).\) We argue by contraposition. Since \(F\) is not serial, then it might be the case that \((M, s_1) \models K_i(Fu(C_{i \rightarrow j} \perp)).\) Therefore, \(F \not\models \neg K_i(Fu(C_{i \rightarrow j} \perp)),\) as desired.

\begin{proof}
\end{proof}

Discussion: This postulate is reasonable to be applied in MASs as debtor cannot know a fulfillment for a nonexisting commitment.

P23. [Creditors’ knowledge of fulfilling an impossible commitment]

Formalization: \(\neg K_j(Fu(C_{i \rightarrow j} \perp)).\)

Meaning: Fulfillment for committing to false cannot be known by the creditor.

Correspondence: For any frame \(F = (W, \approx_i, \approx_j, \approx_{i \rightarrow j}), F \models \neg K_j(Fu(C_{i \rightarrow j} \perp))\) iff \(F\) is serial with respect to \(\approx_j\).

\begin{proof}
The proof is similar to the previous one, but with respect \(j\).
\end{proof}

Discussion: Similar to Postulate P22, the creditor cannot know a fulfillment to a nonexisting commitment.

5.3. Soundness and Completeness

Soundness and completeness of a deduction system illustrate its appropriateness for handling logic (Varga and Vrtersz, 2008). A deduction system is called sound, “whenever its decision problem is solved for a given set of formulas, then this formula set has a special semantic property” (Varga and Vrtersz, 2008). On the other hand, completeness of a deduction system means that “if a set of formulas has the semantic property given by soundness, then the calculus works successfully over that set of formulas” (Varga and Vrtersz, 2008). Soundness can also be analyzed in decision-processes to verify the correctness of business processes and capture their dynamic behaviors (Clempner, 2014).

As mentioned by Singh (2008), the existence of the correspondence between a given set of postulates and their related classes of frames provides the soundness and completeness for the logic under consideration. Thus, the Theorem below results directly from the proofs given in the aforementioned postulates.
Theorem 3. For CTLKC+, the logic generated by any subset of postulates \( \{P_1 - P_{23}\} \) is sound and complete with respect to models that are based on the corresponding classes of frames.

6. Conclusion and Future Work

In this paper, we proved the soundness and completeness of the logic of knowledge and commitments (CTLKC+) using Benthem’s correspondence theory for modal logic. Concretely, we developed a set of postulates to reason about the interactions between knowledge and social commitments in MASs and corresponded them to certain classes of frames. By so doing, we proved that a logic consisting of any subset of those postulates is sound and complete with respect to the models based on those frames. Furthermore, we used the NetBill payment protocol as an application example to illustrate the proposed postulates.

The main insightful and practical implication of this work is that the combined CTLKC+ logic is proven as a consistent, robust and powerful logical tool to model complex but realistic MASs where agents have knowledge and able to manipulate and reason about commitments. This will leverage the use of this logic in practice particularly because it is more expressive than the logics of knowledge and the logics of commitments taken separately from each other. Moreover, the main lesson learned is that correspondence theory is a powerful tool providing a systematic approach to analyze the soundness and completeness of logical languages, which are two complex issues in the field of computational logics.

As future work, we plan to develop dedicated model checking algorithms for CTLKC+ logic and implement them directly on top of the MCMAS symbolic model checker (Lomuscio, Qu, & Raimondi, 2009). Consequently, we will be able to compare the verification results of both techniques (i.e., the reduction (Al-Saqqar et al., 2015) and the direct model checking techniques). Moreover, we aim to investigate the issue of commitment and knowledge uncertainty within a probabilistic version of CTLKC+. Soundness, completeness and complexity analysis of this new probabilistic logic and its model checking are challenging problems yet to be addressed. Another plan for future work is to use CTLKC+ to model and verify web services following the methodology proposed in (Bentahar et al., 2013).
References


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