A Game Theory Approach for Inter-Cell Interference Management in OFDM Networks

Ali Y. Al-Zahrani and F. Richard Yu
Department of Systems and Computer Engineering, Carleton University, Ottawa, ON, Canada
Email: aazahrain@connect.carleton.ca and richard_yu@carleton.ca

Abstract—Inter-cell interference management is one of the most important issues in next generation wireless networks. In this paper, a transmit power adaptation method using a non cooperative game theory approach is developed in such a way to reduce the inter-cell interference in the whole network. The gaming problem is formulated by allowing that a subchannel could be shared by multiple neighboring cells (i.e., universal frequency reuse). The data rate is enhanced by finding the optimum transmit power for each co-channel user using game theory-based scheme. The performance of the proposed scheme is analyzed and compared with existing scheme. Simulation and analytical results are presented to show the effectiveness of the proposed scheme.

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a promising candidate for high data rate mobile communication due to its exceptional physical layer characteristics. For OFDM cellular networks, the frequency reuse factor is of high importance to enhance the spectral efficiency of the whole system. However, if the frequency resources are reused aggressively, inter-cell interference is going to be a serious problem that could distress the whole system.

Inter-cell interference (or co-channel interference) in its essence is a kind of contention for access to the media. For such a conflict, game theory is considered a potential solution. In other words, user equipments (UEs), who are in an OFDM wireless network located in adjacent cells and operating on the same subchannel, are in fact affecting each other through inter-cell interference. Such interaction can be modeled as a game whose players are those UEs which are competing for network resources (i.e., bandwidth and energy). Obviously, any action taken by a user affects the performance of other users in the network. Accordingly, game theory is the natural method for studying such interaction. Game theory approach has been extensively studied in CDMA-based networks to control the intra-cell interference. However, there are few works that considered game theory to control inter-cell interference in OFDM-based networks [1]–[4].

In [2]–[4], decentralized scheduling schemes are proposed for inter-cell interference control as alternatives for the centralized types which are too complex. In these mentioned papers, the competing players are the schedulers in each base station or the co-channel users in adjacent cells. Moreover, while the games’ objectives of these papers are different, these works considered the case where the action set available to the players consists of frequencies and/or time resources only.

On the other hand, in our proposed scheme, ICI is reduced by controlling the transmit power through a noncooperative game in which the transmit power is considered to be the action set available to the players (users).

In [1], the game components are as same as those in our proposed scheme except that [1] added one more dimension in the action set that is the rate choice on each subchannel. However, authors in [1] considered only a simple case of 2 users, each in different cell which are competing on 2 subchannels. Alternatively, our proposed scheme considered multiple users in multiple cells.

There are several motivations for choosing the non cooperative game theory approach. First, due to the tremendous complexity associated with global optimization allocation approaches, it looks reasonable to delegate part of the allocation process to a selfishly smart users using a non cooperative game with a network-oriented rules. Second, the trend of the future networks is to get rid of central controllers and migrate to a more distributed systems which could be highly supported by game theory approaches. Such decentralized strategy aided with game-based power control could allow for dynamic soft frequency reuse which may leads to a plug and play operation and universal frequency reuse. Third, game theory approaches does not concern about user-rate maximization, but rather it tries to maximize the rate of the whole network while satisfying the minimum requirement of every user. Accordingly, game theory approaches try to make the level of service evenly equal over all users wherever they are, and hence it is implicitly fair.

The rest of this paper is organized as follows. Section II provides a necessary and brief background about game theory. System model and assumptions are introduced in Section II. Detailed description of the noncooperative game is presented in section III. Section IV discusses the simulation results and then it is followed by the conclusion in Section V.

II. GAME THEORY: AN OVERVIEW

A game is described by a set of rational players, the actions associated with the players and the utilities (or payoffs) for the players: \( G = (K, \{A_k\}, u_k) \).

- \( K = \{1, 2, ..., K\} \) is the set of rational players.
- \( A_k = \{a_{k1}, a_{k2}, ..., a_{k_v}\} \) is the set of actions available to player \( k \), where \( v_k \) is the number of available actions for that player.
- \( \mathcal{AP} = A_1 \times A_2 \times ... \times A_K = \{(a_1, a_2, ..., a_K)\} \) is
the set of the actions profiles, that is all the possible combinations of the players’ actions, where \( a_k \in A_k \).

- \( u_k : A_1 \times A_2 \times ... \times A_K \rightarrow \mathbb{R} \) is the utility function for each player \( k \). It maps an actions profile to a numerical real number. If \( a_{k1}, a_{k2} \in A_k \), player \( k \) prefers \( a_{k1} \) over \( a_{k2} \) if and only if \( u_k(a_{k1}) \geq u_k(a_{k2}) \).

In a noncooperative game, each player seeks to choose its action which maximizes its own utility, mathematically:

\[
\text{maximize } u_k, \quad \forall k = 1, ..., K.
\]  

A NE is an actions profile \((a^*_1, a^*_2, ..., a^*_K)\), such that no player can unilaterally improve its own utility, i.e.,

\[
u_k(a^*_k, a^*_{-k}) \geq u_k(a_k, a^*_{-k}) \quad \forall a_k \in A_k \text{ and } k = 1, ..., K, \]

where \( a^*_{-k} = (a^*_1, ..., a^*_{k-1}, a^*_{k+1}, ..., a^*_K) \).

At NE no player has any incentive to change its strategy. Accordingly, NE is a stable outcome of the game \( G \). One efficient way to obtain a Nash equilibrium is to use the best responses of the players. Consider a player \( k \). For any given actions of the players other than \( k \) (i.e., \( a_{-k} \)), the best response \( B_k \) of player \( k \) is the action \( a_k \in A_k \) that maximizes its utility, i.e.,

\[
B_k(a_{-k}) = \{a_k \in A_k: u_k(a_k, a_{-k}) \geq u_k(a_{k'}, a_{-k}) \quad \forall a_k' \in A_k\}
\]

In a cellular network, all the bandwidth is available to every base station. It is assumed that every user is aware of its channel strength as well as the amount of co-channel interference \( I_k \). The base stations are asynchronized in terms of scheduling. In other words, if a base station \( j \) is in a scheduling process for a certain subchannel, the other base stations could be in operation mode so that the amount of interference coming to \( j \) cell is almost constant. Each cell consists of user equipments and their assigned base station. The total number of perfectly orthogonal OFDM subchannels is \( M \). Moreover, the total number of users within one cell is \( K \) and the total number of co-channel users is \( J \). Since, OFDM is the system’s transmission scheme, the channel is considered flat during resource block duration. Considering the uplink, the signal to interference plus noise ration (SINR) on the \( m \) th subchannel of the \( k \) th user’s signal at the base station, can be expressed as:

\[
\gamma_k^m = \frac{p_h^m h_{k,k}^m}{\sum_{j \neq k} p_h^m h_{j,k}^m + \sigma_k^m} = \frac{p_h^m h_{k,k}^m}{I_k^m + \sigma_k^m}
\]

where \( p_h^m \) and \( p_h^m \) are the transmit power on the \( m \) th subchannel, from user \( k \) and the interfering user at cell \( j \), respectively. \( h_{k,k}^m \) and \( h_{j,k}^m \) are the desired signal channel and the interferer channel, respectively. \( \sigma_k^m \) is the thermal noise power at base station of interest \( k \) and on \( m \) th subchannel. \( I_k^m \) is the total interference experienced by base station of interest \( k \) on subchannel \( m \).

III. NONCOOPERATIVE GAME FORMULATION

Let \( G = (\mathcal{J}, \{\mathcal{P}_j\}, \{u_j\}) \) represent a game where:

- \( \mathcal{J} = \{1, 2, ..., k, ..., J\} \) is the set of user equipments in adjacent cells who are using the same subchannel.
- \( \mathcal{P}_j = \{p_j : p_j \in [0, p_{max}]\} \) is a continues set of transmit power which represents the space of user’s actions. \( p_{max} > 0 \), is the maximum allowable transmit power per subchannel. The selected action (transmit power) has to be smart enough to maximize the user’s utility.
- \( u_j \): is the \( j \) th user’s utility.

Since the choice of the utility function has a great impact on the nature of the game and on the user’s behaviors, the following subsection is dedicated to introduce and analyze the proposed utility function.

A. UTILITY FUNCTION

In the context of wireless communication, utility function is a mathematical expression that quantifies the level of satisfaction a user gets by using the system resources.

The factors that have an effect on the quality of multimedia services include delay and clarity. The main causes of delay are the low data rate and the frequent retransmission which are basically because of low level of SINR. Moreover, weak SINR increases the bit error rate (BER) which degrades the level of clarity, specially in voice services. As a result, weak received signal and interference are the underlying sources of service degradation. One way to deal with such situation is by fixing an application-based BER while enhancing the throughput, which should be variable and adaptive.

When maximizing the spectral efficiency is the main objective, it is preferable to define the user’s utility as a concave increasing function of the user’s SINR [5]–[8]. To this end, the first function comes into mind is the logarithmic Shannon capacity function or a function proportional to it. However, for a fixed interference \( I \) and fixed desired channel power \( h_{j,k} \), the best way for a user to maximize such utility function in a non-cooperative game is transmitting with the highest possible power. Such behavior will increase the level of interference in the network and hence reduce the system’s spectral efficiency rather than improving it. As a result, such a function is not a complete utility function. To complete it, a pricing term (or cost) should be incorporated to it so that it prevents the users from always transmitting at full power. Thus, If SINR is fixed, the utility function should be a decreasing function with respect to the transmit power.

Accordingly, for subchannel \( m \), the following user’s utility function is devised for the on-hand game-based power control:

\[
u_j(p_j, \gamma_j) = \ln(1 + \frac{\gamma_j^m}{\beta_j}) - \alpha_j p_j^m, p_j, \gamma_j \geq 0
\]

where \( \beta_j \) is an application-dependent positive factor that takes into account the required BER and also it represents a mathematical approximation factor for adaptive transmission, \( \beta = \frac{-\ln(5BER)}{1.3} \) [9]. If the required BER for all users are
equal, then there is no need for $\beta$ since it become a common factor among all users. $\alpha_j$ is a user’s location-dependent pricing factor which makes the price for radiating power dependent on the user’s location. If the user is at cell edge, it should be allowed to transmit relatively higher power to compensate its channel weakness and overcome the higher interference, but this reasonable strategy is going to incur such a user higher loss in its utility value. To be fair, it should be assigned a relatively low value of $\alpha_j$. However, for the central user, since it has the privilege of strong channel and weak interference, it should be firmly restricted from radiating too much power to protect the whole system from unwanted higher interference. To restrict such a user, its $\alpha_j$ should be a relatively high value. As a result, the price (second term in the utility function) is not uniformly equal over the users, in fact as the users go further from the base station, as the price of every watt they radiate goes down. In general, the value of $\alpha_j$ should be proportional with channel power $h_{j,j}$ and inverse proportional with the amount of interference $I_j$, i.e., $\alpha_j \propto h_{j,j}$ and $\alpha_j \propto \frac{1}{I_j}$.

According to the designed utility function in (5), it is important to note the following properties, assuming the game is played over the subchannel $m$:

1) For a fixed transmit power $p_j$,
   a) $\frac{\partial u_j(p_j,\gamma_j)}{\partial p_j} = \frac{1}{p_j^{\alpha_j}} > 0$, i.e., utility is an increasing function of SINR.
   b) $\frac{\partial^2 u_j(p_j,\gamma_j)}{\partial \gamma_j^2} = -\frac{1}{(\beta_j + \gamma_j^2)^2} < 0$, i.e., the rate of the increasing utility is decreasing with respect to SINR. In other words, as SINR gets higher, the utility function gets slower.

2) For a fixed SINR $\gamma_j$,
   a) $\frac{\partial u_j(p_j,\gamma_j)}{\partial p_j} = -2\alpha_j p_j^{\alpha_j-1} < 0$, i.e., utility is a decreasing function of the transmit power.
   b) $\frac{\partial^2 u_j(p_j,\gamma_j)}{\partial p_j^2} = -2\alpha_j < 0$, i.e., the rate of the decreasing utility is increasing with respect to the transmit power. In other words, the utility function decline faster, as the user increases its transmit power.

3) If $h_{j,j}$ and $I_j$ are fixed, then $u_j(p_j,\mathbf{p}_{-j})$ is a concave function of transmit power $p_j$. To prove this property, let us assume that $\ln(1 + \frac{p_j^{\alpha_j} h_{j,j}^m}{\beta_j (\gamma_j^2 + \sigma_j^2)}) = \Psi_j(p_j,\mathbf{p}_{-j})$ and $-\alpha_j p_j^{\alpha_j} = \Omega_j(p_j)$. Now, since $\Psi_j(p_j,\mathbf{p}_{-j})$ and $\Omega_j(p_j)$ are concave functions of $p_j$, then their sum $u_j(p_j,\mathbf{p}_{-j}) = \Psi_j(p_j,\mathbf{p}_{-j}) + \Omega_j(p_j)$ must be also a concave function of $p_j$.

Properties 1.b and 2.b are due to the fact that utility function is a concave function of SINR and transmit power, respectively. Why concavity?

Before we answer this question, it is important to introduce the concept of marginal utility. Marginal utility is defined as the additional satisfaction, or additional amount of utility, gained from each extra unit of consumption (i.e., marginal utility = derivative of the utility w.r.t. consumption argument).

There are two ways for a user to increase its utility; first, by improving its SINR through transmitting more power. Second way is by reducing the cost (price) which is done by transmitting less power (i.e., getting more utility by being friendly with the network). For our utility function defined in (5), the marginal utility of SINR is $\psi_j = \Psi_j'(p_j,\gamma_j) = \frac{\partial u_j(p_j,\gamma_j)}{\partial \gamma_j}$, while the marginal utility of holding the power is $\omega_j = \Omega_j'(p_j) = \frac{\partial u_j(p_j,\gamma_j)}{\partial p_j}$. From property 1.b, the marginal utility of SINR decreases as SINR increases. Moreover, property 2.b tells us the marginal utility of holding power decreases as we hold more power, but if more power is radiated, the price is going to increase significantly.

In this case, since the user is looking to maximize its utility, the concavity property is going to encourage him to stop radiating more power when it reaches the maximum utility. Interestingly, After reaching the maximum utility, the utility is going to decrease if the user increases its power simply because the marginal utility of SINR is going to diminish while the additional price is going to increase dramatically causing the utility function to deteriorate as it is depicted in Fig. 1. In other words, while increasing the power, the user will reach a point where marginal utility of SINR is less than marginal utility through cost reduction, forcing the user to stop pumping more power.

B. Game Equilibrium

With the system model described in previous section, it is preferred to find the optimal transmit power for each user $j$ in the adjacent cells who are using subchannel $m$. The $p_j$ should be most satisfying for user $j$ as well as minimizing for interference in the network. This could be found through the following formulated optimization problem:

$$\text{maximize } u_j(p_j,\mathbf{p}_{-j}). \quad (6)$$

Property (3) indicates that the utility function $u_j(p_j,\mathbf{p}_{-j})$ is a concave function of the user’s transmit power $p_j$. Then
there exists an equilibrium as follow:

\[ u_j(p_j, p_{-j}) = \ln(1 + \gamma_j) - \alpha_j p_j^2, \text{ where } \gamma_j = \frac{p_j^m h_j^m}{I_j + \sigma_j^2}. \quad (7) \]

To find the power that maximizes the utility, take the derivative for both sides with respect to the transmit power \( p_j \), then equalize it to zero.

\[ \frac{\partial u_j}{\partial p_j} = \frac{1}{\beta_j + \gamma_j} h_{j,j} - 2 \alpha_j p_j = 0, \quad \text{for } j = 1, 2, \ldots J. \quad (8) \]

equivalently;

\[ \frac{\partial u_j}{\partial p_j} = \frac{h_{j,j}}{\beta_j (\sum_{i \neq j} p_i h_{i,j} + \sigma^2) + h_{j,j} p_j} - 2 \alpha_j p_j = 0, \quad (9) \]

where \( i \in J \), is an interferer for user \( j \).

Since there are \( J \) users, then there are \( J \) equations with \( J \) unknown variables (i.e., \( p_j; j = 1, 2, \ldots J \)). However, To solve this system, all parameters (i.e., \( h_{j,j}, \beta_j, h_{i,j}, \alpha_j \) for all \( j \)) of the above system are required to be sent to a central entity for computation which could cause a huge burden on the network. Instead a distributed algorithm has been suggested in the next subsection.

C. Distributed Power Control

Let us rearrange Eq. (9):

\[ \frac{h_{j,j}}{\beta_j (\sum_{i \neq j} p_i h_{i,j} + \sigma^2) + h_{j,j} p_j} = 2 \alpha_j p_j. \quad (10) \]

Now, for \( j = 1, 2, \ldots, k, \ldots J \), obviously the left hand side (LHS) of equation (10) is a monotonically decreasing function with respect to the transmit power \( p_j \) while the right hand side (RHS) is a monotonically increasing function of \( p_j \). Therefore, there is a unique intersection which is the equilibrium.

Moreover, Equ. (8) can be rewritten as:

\[ \frac{1}{\beta_j + \gamma_j} h_{j,j} (\sum_{i \neq j} p_i h_{i,j} + \sigma^2) + h_{j,j} p_j = 2 \alpha_j p_j. \quad (11) \]

By assuming \( \frac{h_{j,j}}{\sigma^2} = c f_j^j(p_{-j}) \), which is a known constant factor due to the assumed knowledge of \( h_{j,j} \) and \( I_j \), Eq. (11) can be written in terms of marginal utilities as follows:

\[ \psi_j \cdot c f_j = -\omega_j. \quad (12) \]

It can be inferred from Eq. (12) that the maximum utility happens when the marginal price (or the negative of marginal utility of cost reduction) equals the marginal utility of SINR times a constant factor. This suggests that in general, when the marginal utility of price reduction is less than the product of marginal utility of SINR and \( c f_j \), then increasing the power is a good option. On the other hand, if the marginal utility times \( c f_j \) is less than marginal price, user \( j \) should decrease its radiated power. As a result, the following algorithm is proposed in [8] for such game-based power control:

1) Set the following variables: initial transmit power vector \( p^0 \), initial positive step size \( \delta^0_j \) for all \( j \in J \), number of iteration \( t = 0 \) and the accuracy parameter \( \epsilon \).

2) for every \( j \in J \), iterate: \( p_j^{t+1} = p_j^t + \delta^t_j \cdot [c f_j^j \omega_j + \psi_j^j] \).

- if the sign of \([c f_j^j \omega_j^t + \psi_j^t] \) is same as the sign of \([c f_j^j \omega_j^t + \psi_j^t] \), then double the step size of user \( j \), i.e., \( \delta_j^{t+1} = 2 \delta_j^t \), otherwise divide it by 2. This feature is for accelerating the convergence.

3) if \( |[p_j^{t+1} - p_j^t]| > \epsilon \), increment \( t \) and go to step 2. Otherwise stop.

IV. Simulation Results and Discussions

In this section, the effectiveness of game-based power control for interference management is shown through computer simulation. Two networks having different power transmitting schemes have been simulated. While the first network is equipped with the game-based power control, the other one is just transmitting with equal constant power; \( p_{\text{max}} = \frac{P_{\text{total}}}{M} \).

In addition to the system model and the associated assumptions mentioned in before, the simulation parameters are displayed in Table I.

Figs. 2 and 3 show the performance results which have been observed for a user density of 25 user per a cell. As it is depicted, the proposed game-based power control scheme copes with the inter-cell interference and hence enhances the
aggregate throughput as well as the cell edge user throughput. While both systems exploits multiuser diversity as well as employing adaptive modulation and coding technique, the gain in the proposed scheme is solely due to interference reduction of about 20 dB, as it is shown in Fig. 4.

Due to multiuser diversity, as the number of users increases in the network, the aggregate throughputs of both systems increase as well (See Fig. 5). However, it is obvious that the proposed scheme always outperforms the traditional constant-power scheme. Furthermore, it is also noticeable that as the load increases as the gap between the two curves increases too which proves the robustness of the proposed game-based scheme.

V. CONCLUSIONS AND FUTURE WORK

A novel interference reduction scheme using game theory approach is presented in this paper. The utility function considered here makes sure that the level of service gotten by a user exactly worths the price it pays. Based on simulation results, the game-based power control scheme outperforms the reference traditional constant-power scheme not only in terms of the higher aggregate throughput, but also in achieving significant enhancement for the throughput and fairness of cell edge user. This proposed scheme is based an economic model which study the power control for cellular system in such a way to minimize the interference in the network. According to this model, it might be worthy in future to consider controlling the number of users who are allowed to share the same subchannel.

REFERENCES