Multifractal analysis of axial maps applied to the study of urban morphology

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1. Introduction

Urban spaces are commonly defined by street layout, which is an important element of urban morphology as determined by the distribution of buildings on a certain site. Street patterns have been classified as irregular, radial-concentric, rectangular and grid (Dickinson, 1961); star, satellite, linear, rectangular grid, other grid, baroque network and lacework (Lynch, 1981); regular, concentric and irregular (DTLR & CABE, 2001); and by over 100 additional descriptors for streets outlines, including radial, grid, tree and linear (Marshall, 2005). City morphology is classified according to the way streets are arranged; over time, several urban morphologies in a major community are influenced by various planning regulations and socio-economic conditions.

Urban morphology provides information about the structural characteristics of a city; it provides insight into the structural origins of and impacts of historical change on the chronological processes concerning construction and reconstruction of a city.

An understanding of urban morphology facilitates the projection of future growth in municipal site. Therefore, different scientific approaches have focused on the development of statistical models, such as the Geographical Information System (GIS)-based approach (Allen & Lu, 2003; Xiao et al., 2006), that can be used to estimate future urban growth and obtain information about city sustainability (Czerkauer-Yamu & Frankhauser, 2010; Van Diepen & Henk, 2003). By utilising graphic tools to quantify and define characteristics of urban morphology, Hillier and Hanson (1984) reproduce and simplify the spatial composition of a city by determining the minimum number of axial lines required to cover all areas of an urban lattice; this approach graphically and accurately represents street networks through axial maps. This procedure has been successfully applied to city modelling (Jiang & Claramunt, 2002), urban design (Jeong & Ban, 2011), spatial distribution of urban pollution (Croxford, Penn, & Hillier, 1996), prediction of human movement (Jiang, 2007) and road network analysis (Duan & Wang, 2009; Hu, Jiang, Wang, & Wu, 2009).

Urban morphology has also been described by several authors as exhibiting a fractal nature (Batty, 2008; Batty & Longley, 1987, 1994; Benguigui, Czamanski, Marinov, & Portugalu, 2000; De Keersmaecker, Frankhauser, & Thomas, 2003; Feng & Chen, 2010; Frankhauser, 1998). Based on this notion, the morphology of a city can be described by a single fractal dimension. Milne (1990) argues that not all landscapes are fractal due to their contemporary patterns because several past processes have individually influenced their complexity. Therefore, a single fractal dimension may not always sufficiently describe an urban area due to the presence of heterogeneous fractal properties, as Encarnação, Gaudiano, Santos, Tenedório, and Pacheco (2012) suggests. The multifractal analysis proposed in this research solves this dilemma by obtaining generalised fractal dimensions, or Rényi spectra, to describe data. The research is a meaningful contribution to the study of urban patterns because the multifractal approach transforms irregular data into a
compact form and amplifies small differences between variables (Folorunso, Puente, Rolston, & Pinzon, 1994). In addition, a multifractal framework exhibits a significant advantage over traditional methods through the integration of variability and scaling analyses (Kravchenko, Boast, & Bullock, 1999; Lee, 2002; Stanley & Meakin, 1988). Multifractal analyses have been successfully applied to the study of river network morphology with significant results (De Bartolo, Gaudio, & Gabriele, 2004; De Bartolo, Grabiele, & Gaudio, 2000; De Bartolo, Veltri, & Primavera, 2006; Rinaldo, Rodriguez-Iturbe, Rigon, Ijjasz-Vasquez, & Bras, 1993). De Bartolo et al. (2004) has applied the Sandbox method to overcome limitations detected in the most frequently used multifractal fixed-sized algorithm, the Box-Counting algorithm (Mach, Mas, & Sagues, 1995), to describe river networks. River networks and streets have common properties such as hierarchy and critical self-organisation (Chen, 2009). Based on this context, this research proposes to describe urban patterns by using the multifractal Sandbox method. Apparently, it is the first study that characterises different urban areas to obtain a set of fractal dimensions. Two neighbourhoods in the city of Cordoba (Andalusia, southern Spain) are analysed. The street networks are defined by the Space Syntax algorithm (Hillier & Hanson, 1984). To complete the multifractal analysis, a lacunarity study is conducted. Lacunarity describes the texture appearance of an image, on different scales, based on the distribution of gaps or holes. It is applied to real data sets that may have a fractal or multifractal nature (Plotnick, Gardner, Hargrove, Prestegaard, & Perlmutter, 1996). This analysis is frequently utilised to develop urban spatial configurations (Alves Junior & Barros Filho, 2005; Amorim, Barros Filho, & Cruz, 2009; Myint & Lam, 2005) and segregation; Wu & Sui, 2001).

The paper is organised as follows: Section 2 introduces the Space Syntax algorithm and the multifractal theory that is required to model and describe urban patterns. Section 3 details the study areas and discusses the main results of a multifractal analysis. Section 4 includes conclusions and recommendations for further research.

2. Combination of axial maps and multifractal analysis to describe urban morphology

2.1. Axial maps

Space Syntax is a set of tools that describes spatial configuration through connectivity lines that cover all areas of a plane. This set of lines comprises an axial map (Peponis, Wineman, Bafna, Rashid, & Kim, 1998; Turner, Penn, & Hillier, 2005), which is one of the primary tools of Space Syntax. According to Turner (2006), an axial map is an abstraction of the space between buildings, which is depicted by straight lines and drawn according to a formal algorithm. The lines represent edges and intersections of lines represent junctions or connections between edges. Based on the algorithm, the axial map is the minimal set of axial lines that are linked in such a way that they completely cover the space.

Several authors have developed different computer programs for the construction of axial maps based on the initial proposal of Hillier and Hanson (1984). Some programs have been implemented in GIS, such as Axman, which was created by Nick Dalton at University College in London (Major, Penn, & Hillier, 1997) and Axwoman, which was developed by Jiang, Claramunt, and Batty (1999). Both are used to draw axial lines with a computer and analyse axial maps of urban and interior spaces; the main difference between the programs is that Axman is a Mac-based application and Axwoman is a Windows-based application. Other software have been designed to generate axial maps automatically. Turner et al. (2005) introduced a universal-platform software, called Depthmap, to perform a set of spatial network analyses designed to explain social processes within a built environment. AxialGen, which was developed by Jiang and Liu (2009), is a research prototype for automatically generating axial maps to demonstrate that axial lines constitute a true skeleton of an urban space. Although it is a good approximation of the axial map proposed by Hillier and Hanson (1984), this prototype has not been sufficiently tested. This research, which follows the method proposed by Turner et al. (2005), extracts axial maps that successfully demonstrate and implement the algorithm proposed by Hillier and Hanson (1984) by obtaining accurate results with Depthmap. To translate formal language into algorithmic implementation, the definition of axial maps given by Hillier and Hanson (1984) was clarified and rewritten as "An axial map is the minimal set of lines such that the set taken together fully surveils the system and every axial line that may connect two otherwise unconnected lines is included". To create the axial map, Turner et al. (2005) establish two conditions: (i) the reduction from the all-lines map to a unique minimal axial graph of the system and (ii) the ability to surveil the entire system through axial lines and the preservation of topological rings.

Any possible axial lines are calculated from a map in which all streets and blocks (polygons) are displayed. Every axial line is defined by joining two intervisible vertices. There are three different possibilities (Fig. 1): (i) both of the vertices are convex, (ii) one vertex is convex and one is concave or (iii) both vertices are concave. The reduction to a single minimal axial-line map is based on the rule that if any line connects to a line, its neighbours do not join the two lines; the single line is retained or removed. When two lines have the same connection, the longest line is chosen and the other one is removed. A second condition needs to be applied to obtain the preservation of topological rings and surveillance of the entire system. To surveil the system, the algorithm chooses those lines from which every point of the system is visible. Therefore, this new condition is added to the last criterion. To complete the topological rings, the algorithm executes a triangulation around a polygon edge so that it is visible from the three axial lines that are around the geometry (Fig. 2).

This research describes urban morphology as studied through axial maps and obtained with Depthmap software for two areas of Cordoba in Andalusia, which is in southern Spain (Fig. 3). This city is situated at 37°50'44" latitude and 04°50'23" longitude; its average elevation is 123 m above sea level. Cordoba, which is located in the Guadalquivir River Valley, has an area of 1245 km² and a population of 329,723 inhabitants (data provided by the Statistic National Institute). Cordoba was founded by the Romans.
in the first half of the 2nd century BC because of its privileged geographical location. The city has been walled since its founding. Its fortification was expanded when the Arabs conquered Cordoba. With the arrival of Christians to the city, the wall was enlarged further. Due to population growth, Cordoba began to expand beyond its walls in 1905. Today, the city centre is situated in the area that was previously walled.

Two neighbourhoods of Cordoba are studied, through a combination of axial maps, to understand their urban morphology (Fig. 4) and multifractal analysis. These areas were chosen because they exhibit similar densities of axial lines (Table 1).

2.2. Multifractal analysis

2.2.1. Sandbox method

The Sandbox method introduced by Tél, Fülöp, and Vicsek (1989) and developed by Vicsek (1990) and Vicsek, Family, and Meakin (1990) allows the complete reconstruction of the multifractal spectrum for both positive and negative moment orders, \( q \). It overcomes limitations of the Box-Counting algorithm, whose main disadvantage is the incorrect determination of the fractal dimensions for negative moment orders (De Bartolo et al., 2004).

A Sandbox algorithm involves randomly selecting \( N \) points that belong to the street network and then counting, for each point \( i \), the number of pixels \( M_i(R) \) that belong to the street network inside a region of a given radius \( R \) (a circle), which is centred at a specified point. The advantage of this method is that the circles are centred on the structure such that there are no boxes with few elements inside (pixels), thereby avoiding border effects (Fig. 5).

By choosing arbitrary points as centres, the average value of the mass and their \( q \)th moments over randomly distributed centres can be computed as \( \langle |M(R)|^q \rangle \), with \( q \) as the probability moment order. Thus,

\[
\sum_i \left( \frac{M_i}{M_0} \right)^{q-1} \frac{M_i}{M_0} \propto \left( \frac{R}{L} \right)^{q D_q}
\]

where \( M_0 \) stands for the total mass of the cluster or lattice mass, and \( L \) is the lattice size equal to 1 after normalisation. Based on Falconer (1990), this normalisation does not modify the measure because it is a geometrically invariant transformation. Considering the ratio \( M_i/M_0 \) as a probability distribution on an approximating fractal, the following averaged expression can be derived as:

\[
\left\langle \frac{M_i}{M_0} \right\rangle^{q-1} \propto \left( \frac{R}{L} \right)^{q D_q}
\]

According to Eq. (2), the selection of the centres must be uniform over the approximating fractal. The generalised fractal dimension \( D_q \) of moment order \( q \) is defined as (Tél et al., 1989):

\[
D_q(R/L) = \frac{1}{q-1} \lim_{R \to 0} \frac{\ln(|M(R)/M_0|^{q-1})}{\ln(R/L)}, \quad \text{for } q \neq 1
\]

De Bartolo et al. (2004) determined the solution for \( D_q \) when \( q = 1 \) in the Taylor’s expansion around
squares linear regression as the slope of the scaling curves of the covering regions, as given by the expression connecting the moments of probability to the radius length of monofractally distributed measures, values when the heterogeneity of the distribution increases. For

\[ D_0 = \lim_{R \to 0} \frac{\ln[M(R)/M_0]}{\ln(R/L)} \]

The generalised dimensions can be obtained through least squares linear regression as the slope of the scaling curves \( \ln[M(R)/M_0] \) versus \( \ln(R/L) \) for \( q \neq 1 \), and \( \ln[M(R)/M_0] \) versus \( \ln(R/L) \) for \( q = 1 \), with \( \ln(R/L)_{\text{upper}} \) and \( \ln(R/L)_{\text{lower}} \) as the inner and outer cut-off lengths, respectively. \( D_0 \) is the fractal dimension, also known as the capacity dimension, of the set over which the measure is performed; \( D_1 \) is the information dimension and \( D_2 \) is the correlation fractal dimension (Grassberger, 1983; Grassberger & Procaccia, 1983). \( D_0 \) is a decreasing function with respect to \( q \) for a multifractally distributed measure (Caniego, Martin, & San Jose, 2003).

The relationship between the spectrum of the generalised fractal dimensions \( D_q \) and the multi-fractal spectrum \( f(\alpha) \) with \( \alpha \) as the Lipschitz–Hölder exponent, is given through the sequence of mass exponents \( \tau_q \) (Hentschel & Procaccia, 1983), which is a function connecting the moments of probability to the radius length of the covering regions, as given by the expression

\[ \tau_q = (q - 1)D_q \]

The mass exponent sequences are interpolated with fifth-order polynomials to obtain the multifractal spectrum \( f(\alpha) \) by means of the Legendre transform, which is defined by the relations (Halsey, Jensen, Kadanoff, Procaccia, & Shraiman, 1986):

\[ \alpha_q = -d\tau_q/dq \]
\[ f(\alpha_q) = q\alpha_q + \tau_q \]

The Lipschitz–Hölder, or singularity exponent \( \alpha_q \), quantifies the strength of the measure singularities. \( f(\alpha) \) is an inverted parabola for multifractally distributed measures, with a wider range of \( \alpha \) values when the heterogeneity of the distribution increases. For monofractally distributed measures, \( \alpha \) is equivalent for all regions of the same size and the multifractal spectrum consists of a single point (Kravchenko et al., 1999). The highest value of the multifractal spectrum \( f(\alpha_q) \) corresponds to the fractal dimension \( D_0 \). Both provide information about the degree of filled space.

### 2.2.2. Lacunarity

The lacunarity concept was introduced by Mandelbrot (1982) to differentiate texture patterns that may have the same fractal dimension but are different in appearance. It can be used with both binary and quantitative data in one, two and three dimensions. It is a measure of the gaps or holes in a space distribution. Thus, lacunarity measures the spatial heterogeneity of pixels in an image. If an image has a large amount of gaps, it has a high lacunarity; in contrast, if an image has a translational invariance, it has a low lacunarity. Several algorithms have been proposed to measure this property (Gefen, Meir, & Aharony, 1983; Lin & Yang, 1986; Mandelbrot, 1982).

Regarding this issue, the current research is based on the use of the “Gliding Box” algorithm proposed by Allain and Cloitre (1991) and popularised by Plotonick, Gardner, and O’Neill (1993). This method consists of a square box of size \( r^2 \) that slides over a space of total size \( M \), the mass \( S \) is counted inside of the box at each point in the sliding process. Beginning with the first pixel located in the first column and row of images, the box slices along each pixel of the image. In this research, the images are binary, thus, the mass corresponds to the number of pixels that fill the space, whose value is 1. The process is repeated for each new box size until the box size equals the image size. The frequency distribution of the box masses is \( n(S, r) \). This frequency distribution is converted into a probability distribution \( Q(S, r) \) by dividing each frequency value by the total number of boxes for each size \( N(r) \). Then, the first and second moment of the distribution are defined as:

\[ Z(1) = \sum SQ(S, r) \]
\[ Z(2) = \sum S^2Q(S, r) \]

The lacunarity is defined by the relationship between these moments as:

### Table 1

<table>
<thead>
<tr>
<th>Axial lines</th>
<th>Axial lines length</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of lines</td>
<td>Area (km²)</td>
</tr>
<tr>
<td>Area 1</td>
<td>169</td>
</tr>
<tr>
<td>Area 2</td>
<td>56</td>
</tr>
</tbody>
</table>

Fig. 4. Areas selected for this work.
The first moment can be described by the mean $E(S)$ and the second moment can be described by the variance $\text{Var}(S)$ of the masses as follows:

$$Z(1) = E(S)$$
$$Z(2) = \text{Var}(S) + E^2(S)$$

Therefore, the lacunarity for each box size can be defined by

$$A(r) = \frac{Z(2)}{Z^2(1)}$$

This approach uses a measure based on a multiscale analysis that is dependent on scale. In this research, lacunarity is calculated in an area of $256 \times 256$ pixels located in the image centre to avoid border effects (Fig. 6). For each area, the lacunarity is calculated for box sizes ranging from $r = 1$ to $r = 256$ in multiples of 2.

3.1. Description of study areas

Two neighbourhoods in the city of Cordoba, Spain, are studied. Area 1 presents an irregular morphology as illustrated by Fig. 7, which shows the corresponding axial map. This irregularity is a consequence of its unique history (i.e., Islamic heritage combined with centralised administrative influences). Today, this area exhibits its typical Mudejar urban morphology, which is characterised by rectilinear main avenues and tortuous secondary streets. This neighbourhood reflects Roman, Arab and Christian settlements and is located in the city centre; it is the main business area for the community. Area 2 is a residential neighbourhood; it consists of houses that were built in the 1950s and are based on a regular pattern (Fig. 7) that corresponds to modern planning and socio-economic requirements.
Table 1 shows the number of axial lines, density of axial lines per area and statistical parameters of the axial line length. These data are extracted from the axial maps that were generated for both areas. Although Area 1 has a slightly higher density of axial lines, it displays shorter axial lines than Area 2. Table 2 shows the statistical analysis of the block size for each study area. The average block size has been normalised over the surface of each zone to obtain a suitable comparison between the study areas. This
3.2. Multifractal analysis results

The Sandbox method of axial-map analysis is applied using the algorithm proposed by Tél et al. (1989), Vicsek (1990) and Vicsek et al. (1990). To calculate the mass M(R) or number of network points in a circle of a given radius R, the algorithm begins with a circle of a maximum radius of 0.25 and a minimum radius of 0.0025 for all axial maps. The minimum radius is chosen so that two pixels or two street network points must be inside the circle.

Fig. 8 shows the scaling curves for \( q \in [-5, 5] \) at 0.25 increments. This study is limited to this range of \( q \) values to avoid instability in the multifractal parameters, because higher and lower moment orders may magnify the influence of outliers in the measurements, as stated by Zeleke and Si (2005). These curves are fitted linearly to obtain the fractal dimension \( D_q \). To obtain the best fit for \( R^2 \), the linear regression is cut between a lower limit \( (R/L)_{\text{lower}} \) and an upper limit \( (R/L)_{\text{upper}} \) for \( q = 0 \) (Table 3). The sequences of mass exponent \( \tau_q \) are detailed in Fig. 9 for each area. As shown, the \( \tau_q \) curve for Area 1 exhibits multifractal behaviour, due to the different slopes for negative and positive moment orders. However, for Area 2, \( \tau_q \) is almost a straight line, which confirms a quasi-fractal nature. In all cases, the generalised fractal dimension \( D_0 \) is a decreasing function with \( D_0 > D_1 > D_2 \), as verified in Table 3, which confirms the multifractal nature (Fig. 10). \( D_0 \) shows a strong dependence on the values of \( q \) for Area 1. However, for Area 2, \( D_0 \) tends to be constant, which denotes a much closer monofractal or simply fractal behaviour \( (D_0 \approx D_1 \approx D_2) \) and a more uniform street distribution than in Area 1. This finding influences the shape and length of the multifractal spectra shown in Fig. 11. Points that belong to the spectrum in Area 2 tend to be grouped compared with the dissemination of points exhibited in Area 1, which denotes the existence of a quasi-monofractal nature (one fractal dimension is sufficient to describe this zone). Table 3 illustrates that both areas have similar space-filling values \( D_0 \). They almost have equivalent fractal dimensions although they display distinct morphologies. Area 2 (quasi-fractal case) has a lower number of axial lines than Area 1 (multifractal case) because it fills the space in a more efficient way.

To address two city areas with different morphologies and similar \( D_0 \), lacunarity was calculated to distinguish texture patterns for the two fractal networks. Lacunarity curves are displayed in Fig. 12 and indicate a minimum value when the variance is 0 and a maximum value of one when the box size is 1. Lacunarity curves are straight and decrease almost linearly. This result corresponds to variability in the geometric distribution. Area 1, which exhibits a greater dispersion of its elements, presents the highest lacunarity value and, therefore, a heterogeneous street distribution in every box size. At the beginning, the curve of Area 2 is straight for low box-size values, which indicates heterogeneous lacunarity values. However, for a box size equal to \( 32 \times 32 \), the lacunarity is more homogeneous. Lacunarity can be related to factors that influence urban morphology (e.g., block size). Area 1 shows the highest coefficient of variation of block size (Table 2) and the highest lacunarity values. Opposite results were found for Area 2.

4. Discussion

Previous works have considered city morphology to be fractal (Batty, 2008; Batty & Longley, 1994; Benguigui et al., 2000; De Keersmaecker et al., 2003; Feng & Chen, 2010; Frankhauser, 1998). The capacity dimension \( D_0 \) of urban form can be observed as a ratio of filling a city space (Chen & Lin, 2009), which provides

### Table 2
Block statistical analysis.

<table>
<thead>
<tr>
<th>Numbers of blocks</th>
<th>Average (km²/km²)</th>
<th>Maximum</th>
<th>Minimum</th>
<th>CV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area 1</td>
<td>105</td>
<td>7.20E–03</td>
<td>2.70E–02</td>
<td>2.75E–04</td>
</tr>
<tr>
<td>Area 2</td>
<td>69</td>
<td>7.80E–03</td>
<td>6.33E–03</td>
<td>9.55E–04</td>
</tr>
</tbody>
</table>

### Table 3
Multifractal parameters obtained from applying the Sandbox method.

<table>
<thead>
<tr>
<th></th>
<th>( D_0 )</th>
<th>( D_1 )</th>
<th>( D_2 )</th>
<th>( R/L_{\text{lower}} )</th>
<th>( R/L_{\text{upper}} )</th>
<th>( R_{\text{upper}}^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area 1</td>
<td>1.7654</td>
<td>1.7371</td>
<td>1.7168</td>
<td>–4.8</td>
<td>–2.2</td>
<td>0.9995</td>
</tr>
<tr>
<td>Area 2</td>
<td>1.7575</td>
<td>1.7562</td>
<td>1.7543</td>
<td>–4.5</td>
<td>–2.5</td>
<td>0.9990</td>
</tr>
</tbody>
</table>
a new approach for performing urban spatial analysis. Dimension $D_0$ is connected to the frequent measurements of a city shape and vice versa, within a certain range of spatial scales (Chen, 2011).

Based on the results of this research, the prior notion that the morphology of a city can only be considered as fractal in nature is valid for city zones with regular morphology, such as Area 2 in which $D_0 \approx D_1 \approx D_2$. In this case, the fractal dimensions have corresponding values (as a monofractal case), as illustrated in Table 3. However, when a city structure exhibits more complexity (Area 1), a multifractal nature emerges with different fractal dimensions, such as $D_0 > D_1 > D_2$, that provide more information about the street network distribution.

Table 3 shows that dimension $D_0$ has similar values for both study areas, which implies similar space fillings. However, their morphologies are different because they can be derived from fractal dimensions $D_1$ and $D_2$. According to Davis, Marshak, Wiscombe, and Cahalan (1994), $D_1$ provides a measure of the degree of heterogeneity in the spatial distribution of a variable. In addition, the information dimension characterises the distribution and intensities of singularities (high values of street network density) with respect to the mean. If $D_1$ is lower, the distribution of singularities in the street network density will be sparse. To the contrary, if $D_1$ increases, these singularities will have lower values that exhibit a more uniform distribution.

This finding is in agreement with the $D_1$ values listed in Table 3 for Areas 1 and 2. Area 1 yielded a higher $D_1$ because it has more blocks with several sizes and irregular shapes (Fig. 7a) than Area 2, which has a smaller number of blocks that are similar in size and resemble rectangles (Fig. 7b). This finding is consistent with
the CV values shown in Table 2 (CV_Area 1 > CV_Area 2). When $D_0 = D_1$, a monofractal case exists. This description might be valid for Area 2; however, it is not valid for Area 1 ($D_0 > D_1$), which exhibits a multifractal nature.

Correlation dimension $D_2$ describes the uniformity of the street network density among several selected zones (circles of radius $R$). $D_2$ is related to the probability of finding pixels that belong to the street network within a given distance when beginning on a pixel that belongs to this object. $D_2$ is confirmed to be higher for Area 2 (Table 3) and more uniform than Area 1, as shown by Fig. 7. As in the case of $D_1$, this finding is a result of relevant differences in the number, size and shape of blocks that determine the analysed morphologies of the neighbourhood. Area 2 can be considered quasi-fractal because $D_0 \approx D_2$, whereas Area 1 has a multifractal nature ($D_0 > D_2$), as previously stated.

According to these results, the Rényi dimension spectrum can be used to describe urban morphology instead of a single-valued fractal dimension. This situation is determined by different urban morphology generative processes (city growth according to several planning/socio-economic regulations) over time, which results in regular and irregular areas.

5. Conclusions

With the goal of performing a multifractal analysis of urban morphology, street layouts are extracted using a Space Syntax algorithm for two urban neighbourhoods, thereby obtaining axial maps of each area to accurately represent their spatial configurations. The Sandbox multifractal method and lacunarity measurements are applied to describe street networks. Multifractal analysis is efficient for characterising the morphology of street networks with the advantage of parameters that are independent over a range of scales. According to the generalised dimension spectra obtained in this research, it is convenient to consider the existence of the multifractal nature of some urban zones, especially when addressing cities with frequent irregular morphology; these areas cannot be described by a single fractal dimension as previous research states. Instead, infinite fractal dimensions for these zones should be theoretically considered. In practice, capacity, information and correlation fractal dimensions are sufficient parameters for characterising irregular morphologies.

This research is based on a case study of two neighbourhoods in the same city. Although the regular and irregular urban patterns studied here are frequently observed in other cities, the replication of this analysis would establish the generality of the conclusions reached in the case study of Cordoba. The differences found for the fractal dimensions obtained for the neighbourhoods in this research are influenced by urban patterns derived from planning regulations and diverse socio-economic situations, such as the spatial and temporal evolution of land values (i.e., Hu, Cheng, & Wang, 2012). Thus, with the objective of aiding urban planning and management, further analyses should be performed to relate specific fractal dimensions to different planning and socio-economic regimes. The study of relationships between fractal dimensions and market context is suggested because the city street network is determined by business principles in many cases.

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