Low Complexity Distributed Model for the Compensation of Direct Conversion Transmitter’s Imperfections

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Abstract—In modern communication systems, nonlinearity in power amplifiers (PAs) and in-phase and quadrature-phase (I/Q) imperfections in the transmitter are of enormous concern. With the increase in the importance for highly energy efficient and low complexity models, there is a need to develop low complexity digital predistortion (DPD) methods. In this paper, we present a novel memory polynomial based distributed two block model to alleviate these impairments. Various performance metrics are used to evaluate the design performance and complexity of proposed model as compared to the state of the art predistorter model. Simulation and measurement results indicate the ability of the proposed model to meet the desired design purpose with reduced complexity in terms of number of coefficients, dispersion coefficient, condition number and number of floating point operations required for computing various steps in the inverse modeling algorithm. This is achieved while maintaining reasonable performances in terms of NMSE and ACEPR. The major attribute of the model is the reduction in complexity of the system. The number of complex valued coefficients and the number of floating point operations (FLOPs) are both reduced by 17%–56%, matrix conditioning is improved by 12–33 dB and the dispersion coefficient is reduced by 16–42 dB as compared to the previously proposed joint modulator and power compensation technique.

Index Terms—ACEPR, digital predistortion (DPD), in-phase and quadrature-phase (I/Q) imbalance, memory polynomial, NMSE, PA, spectral regrowth.

I. INTRODUCTION

T

HE quality of direct conversion transmitter output suffers in band distortion and out of band interference with adjacent channel due to the nonlinearity of PA [1], gain/phase imbalances in modulator and DC leakages from local oscillator [2]. These problems have been elaborated in literature and methods have been proposed to mitigate these imperfections [3]–[28]. Accumulated effect of these imperfections is even more pronounced for highly envelope varying signals with advanced access techniques such as wideband code division multiple access (WCDMA) or modulation techniques such as orthogonal frequency division multiplexing (OFDM). When passed through a PA these signals suffer from intermodulation distortion. Digital predistortion (DPD) based methods for the compensation of PA nonlinearity can be found in literature [3]–[10]. However, these methods do not perform satisfactorily in the presence of modulator imbalances. Hence digital predistortion methods that are able to compensate for I/Q imbalances along with PA nonlinearity are of great essence. In recent literature, a Volterra based model presented in [11] compensates for I/Q imperfections and has reasonable modeling performance but the number of coefficients is very large which increases the complexity and it does not address PA nonlinearity issue. Similarly real valued focused time delay neural network presented in [12] also has a high complexity. Although modulator can be tuned to remove such imperfections using conventional offline hardware tuning and characterization approaches, it requires extra hardware circuitry and discontinuity in transmission [13]–[14]. Therefore digital predistortion techniques capable of compensating for PA as well as modulator imperfections without stopping transmission are attractive in practical sense [17], [20]–[22]. In this regard, an efficient and low complexity parallel Hammerstein (PH) method is presented in [20], which employs a parallel structure to split the predistorter into non-conjugate and conjugate branches to compensate for both the PA nonlinearity and frequency dependent I/Q mismatch. However, in doing so the authors have shown that the number of complex valued parameters increases as compared to a series structure. Two block models have been reported in literature [9], [10], [13], [24] with the aim of reducing the complexity of the DPD models. The methods proposed in [9] and [10] consider the effect of PA nonlinearity by considering three blocks and two blocks models respectively, while the method proposed in [13] considers a two block model, where the first block compensates for the PA nonlinearity and the second block mitigates I/Q imbalance, but the authors have not mentioned any specific model to mitigate the effect of PA nonlinearity i.e. no specific DPD model is proposed and deals specifically with the I/Q compensator assuming that the PA is linearized. However, as reported in [20], these techniques require extra hardware because of the separate processing of the two blocks. Hence, the novelty in the proposed work is to adapt these blocks jointly using single step estimation requiring only one measurement for device characterization. The method proposed in [24] provides a two block model for a dual band power amplifier, where the first block uses a truncated Volterra series to compensate for the modulator imbalances and non-linear memory effect of the dual band PA, while the second block is a static two dimensional Look Up Table (LUT) based model. A dual input based compound structure has been proposed in [25] where the authors have compared their performance and complexity with the dual input nonlinear modeling approach presented in [26]. It has been shown by the authors that although their complexity is better than the dual input model [26], it is still quite high. Hence an important aspect of the proposed method is to reduce the complexity of the system. The proposed model employs a memory polynomial first block which aims to mitigate the effects of dynamic nonlinearity of the power amplifier, and a mildly nonlinear second block with memory capable of modulator’s nonlinear imperfection compensation. Simplicity, ease of implementation and low complexity are important attributes in any

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characteristics. So if we consider the effect of memory, the above equation becomes

\[ y_{PA}(n) = \sum_{j} \sum_{m} a_{m,j} x(n - m) |x(n - m)|^{j} \]  

(2)

\( m \) is the memory index. Another problem in the transmitter occurs due to the gain and phase mismatch introduced by the mixer between the in phase and quadrature components which occurs during the upconversion of the baseband signal to the carrier frequency. Consider an input signal to the transmitter of the form

\[ x(t) = A(t) \cos(w_{c}t + \phi(t)) \]  

(3)

Here \( A(t) \) and \( \phi(t) \) represent time varying amplitude and phase respectively and \( w_{c} \) is the carrier frequency. In terms of in phase and quadrature components it can be written as

\[ x(t) = I(t) \cos(w_{c}t) - Q(t) \sin(w_{c}t) \]  

(4)

where \( I(t) \) and \( Q(t) \) are the in phase and quadrature phase components respectively. In a typical transmitter, these \( I \) and \( Q \) signals pass through a digital-to-analog converter. After being converted to analog signals they are passed through a low pass filter to remove the alias products. A local oscillator produces a 90 degree phase difference between the two branches which are then combined. However, all these components are not ideal; hence the output signal adopts the form given below [12]

\[ y_{I/Q}(t) = (I(t) + d_{i}) \cos(w_{c}t) - \Delta \alpha(Q(t) + d_{q}) \sin(w_{c}t + \Delta \phi) \]  

(5)

where \( d_{i} \) and \( d_{q} \) are the in phase and quadrature DC offsets, and \( \Delta \alpha \) and \( \Delta \phi \) are the gain and phase imbalances, respectively. \( I/Q \) imbalance in transmitters produce Mirror Frequency Interference (MFI) resulting in adjacent channel interference [14]. Also these imbalances and local oscillator leakages tend to produce extra intermodulation distortions at the output, thus degrading the performance of the transmitter. [14] presents an \( I/Q \) imbalance model as follows

\[ y_{I/Q}(t) = w_{1}(t) \ast x(t) + w_{2}(t) \ast x^{*}(t) \]  

(6)

where \( x(t) \) is the input to the transmitter and \( y_{I/Q}(t) \) is the imbalanced output and

\[ w_{1}(n) = \frac{h_{I}(n) + h_{Q}(n) \Delta \alpha e^{\Delta \phi}}{2} \]  

(7)

and

\[ w_{2}(n) = \frac{h_{I}(n) - h_{Q}(n) \Delta \alpha e^{\Delta \phi}}{2} \]  

(8)

Here \( \Delta \alpha \) represents the gain imbalance and \( \Delta \phi \) represents the phase imbalance. Eq. (6) in frequency domain can be written as

\[ Y_{I/Q}(w) = W_{1}(w)X(w) + W_{2}(w)X^{*}(w) \]  

(9)

The conjugate part results in mirror frequency imaging. Similar to PA model, in polynomial form eq. (6) can be written as [24]

\[ y_{I/Q}(n) = \sum_{m} b_{m} x(n - m) + \sum_{m} c_{m} x^{*}(n - m) \]  

(10)

Above equation shows the model for linear \( I/Q \) imperfections. However, as mentioned in [11], [15], [26], the \( LO \) phase offsets and the reconstruction filters in the direct conversion transmitters give rise to nonlinear imbalance. Hence a true \( I/Q \) imperfection compensator should consider these nonlinear effects. The proposed model considers the effects of nonlinear imperfection as shown in the next section.
III. PROPOSED MODEL FOR THE MITIGATION OF POWER AMPLIFIER NONLINEARITY AND I/Q IMBALANCE

A. Memory Polynomial Block With A Mildly Nonlinear Dynamic Block

Fig. 3 shows the schematic for the proposed model. The digital predistortion is carried out using the indirect learning architecture [29]. A memory polynomial with nonlinearity order $N$ and memory depth $M$ can be written as

$$y_1(n) = \sum_{m=0}^{M} \sum_{j=0}^{N} a_{m,j} x(n-m) |x(n-m)|^j$$ \hspace{0.5cm} (11)

$y_1(n)$ represents the complex output samples of the memory polynomial model for given input samples $[x(n) x(n-1) \ldots x(n-M)]$ and $a_{m,j}$ represents the polynomial coefficients. This output serves as the input to the second block where another set of coefficients aim to mitigate $I/Q$ imbalance. The output of this block is written as

$$y_2(n) = \sum_{m'=0}^{M} \sum_{k=0}^{K} (b_{m'}^* y_1(n-m') |y_1(n-m')|^k$$

$$+ \epsilon_{m'}^* y_1(n-m') |y_1(n-m')|^k) + LO$$ \hspace{0.5cm} (12)

$LO$ term represents the local oscillator leakage. In matrix notation, (11) and (12) can be written as

$$Y_1 = Xa$$ \hspace{0.5cm} (13)

$$Y_2 = Vb$$ \hspace{0.5cm} (14)

$a$ is the vector of coefficients generated by the memory polynomial block to account for the PA nonlinearity and $b$ is the vector of coefficients generated by the second block to account for the nonlinear $I/Q$. Here $X$ is the matrix of input data and considers both the nonlinearity and memory in the system and is given by

$$P$$ denotes the $P^{th}$ data sample. The output of the first block serves to generate a matrix $V$ which takes the complex and complex conjugate combinations of $Y_1$ and can be written as

$$V = [v_1 \ v_1^* \ 1]$$ \hspace{0.5cm} (16)

This formation of matrix $V$ i.e. appending the conjugate part of the data with the real data forms the basis behind $I/Q$ imbalance mitigation. Here $v_1^*$ is the conjugate of the above matrix $v_1$. The final output $Y_2$ is the predistorted signal which is fed to the PA to obtain the linearized signal. The linear least square approach can be used to extract these coefficients given by the following expressions

$$\hat{a} = (X^H X)^{-1} X^H Y$$ \hspace{0.5cm} (18)

$$\hat{b} = (V^H V)^{-1} V^H Y$$ \hspace{0.5cm} (19)

(18) and (19) are calculated using Moore-Penrose pseudo inverse to extract the parameters. Also, it is important to note that one needs to have information only about the PA input and output characteristics for extracting the coefficients which consequently generate $Y_1$ and $Y_2$. The number of complex valued coefficients for the proposed model is $(N+1) \times (M+1) + (K+1) \times (M+1) + 2+1$. Inverting the matrix is a complexity problem and if the dimensions of the matrix are large, the complexity of the system increases enormously.

B. Justification for the Proposed Model

As mentioned earlier literature shows that distributed block models reduce the complexity of the system. However, it is not the only advantage of distributed systems. Distributed models also provide a
and (12), we get the following form (see appendix)

In the order of their appearance. Table A.1 in the appendix shows here we see that by choosing different values of $j$, the model presented in [13] mitigates imperfections in the reverse order as they appear. In literature both kind of models can be found. The proposed model mitigates the imperfections in the reverse order more spectral rich nonlinear model as compared to a joint memory

more spectral rich nonlinearity as compared to the joint models. For sake of simplicity, assume both the blocks are static. By using eqs. (11) and (12), we get the following form (see appendix)

$$y_2 = \left\{ \sum_j a_j x^j \times \sum_k b_j^k x^k A_j^k \right\} + \left\{ \sum_j a_j^* x^j \times \sum_k c_j^k x^k A_j^k \right\} + \text{bias}$$

(20)

Here we see that by choosing different values of $j$ and $k$, we get a more spectral rich nonlinear model as compared to a joint memory polynomial model such as the one provided in [20]. In addition to this, the proposed model mitigates the imperfections in the reverse order as they appear. In literature both kind of models can be found. The model presented in [13] mitigates imperfections in the reverse order of their appearance, while the model proposed in [24] alleviates them in the order of their appearance. Table A.1 in the appendix shows that in terms of performance, it does not make a difference. The output suffers from various distortions and the aim is to find the set of coefficients which can alleviate these imperfections.

IV. PERFORMANCE AND COMPLEXITY ANALYSIS

A. Measurement Setup

The experimental validation of the proposed model is done using a class AB LDMOS (Laterally Diffused Metal Oxide Semiconductor) power amplifier with a drain voltage of 28 volts and gate voltage of 9 volts designed for a maximum power of 3 dBm.

Various signals listed in Table I are fed to this amplifier to obtain the distorted nonlinear output signal. A gain imbalance of 1 dB and phase imbalance of 3 degrees was induced in all these signals. Also, an offset of 1% in the in-phase component and 1.5% in the quadrature-phase component was introduced to emulate the effect of LO leakage. These signals are then provided to the proposed models for their validation as effective predistorters capable of mitigating the PA nonlinearity and I/Q imbalance. As a special case, an asymmetric WCDMA 1101 signal was generated using Agilent ADS having a PAPR of 12 dB, which when fed to the PA produces an in-band distortion as shown in the spectrum in Fig. 9.

<table>
<thead>
<tr>
<th>Signal</th>
<th>PAPR (dB) (approx.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>WCDMA 101</td>
<td>10.6</td>
</tr>
<tr>
<td>WCDMA 111</td>
<td>10.5</td>
</tr>
<tr>
<td>WCDMA 1111</td>
<td>11</td>
</tr>
<tr>
<td>WCDMA 1101</td>
<td>12</td>
</tr>
</tbody>
</table>

Table I

PEAK TO AVERAGE POWER RATIOS OF THE SIGNALS USED

B. NMSE and ACEPR

First the proposed digital predistorter design is evaluated for its performances for inverse modeling. The figures of merit used are normalized mean square error (NMSE) and adjacent channel error power ratio (ACEPR). NMSE determines the error in the model estimation and the in-band modeling performance and can be given by [3]

$$\text{NMSE (dB)} = 10 \log_{10} \left( \frac{\sum_{n=1}^{N} |y_{\text{measured}}(n) - y_{\text{estimated}}(n)|^2}{\sum_{n=1}^{N} |y_{\text{measured}}(n)|^2} \right)$$

(21)

Here $y_{\text{measured}}(n)$ and $y_{\text{estimated}}(n)$ are the measured and the estimated outputs respectively and $n$ denotes the samples. ACEPR measures the out of band performance of the model and is given by [3], [9].

$$\text{ACEPR} = \frac{1}{2} \left( \int_{\text{lower adj. channel}} |E(f)|^2 df + \int_{\text{upper adj. channel}} |E(f)|^2 df \right) \int_{\text{channel}} |y_{\text{measured}}|^2 df$$

(22)

$E(f)$ is the discrete Fourier transform of the error between the measured signal and the estimated signal. For a fair comparison, all the models evaluated in Tables II and III were trained with the same number of data samples i.e. 10,000 samples and tested for 25,000 data samples. Figs. 4–8 elaborate the in band and out of band modeling performance of the proposed model for the above mentioned input data. In Fig. 4, we see that if only the first block is employed, the model does not attain reasonable NMSE indicating that it is not able to produce an effective inverse model for the measured output signal. However, as we introduce the second block and add nonlinearity and memory to it, the NMSE goes as low as –34.67 dB which is similar to the state of the art PH model as indicated in Table II. Similar observations are shown in Fig. 5. Figs. 6–8 show the error spectrums of the proposed model for WCDMA 101, WCDMA 111 and WCDMA 1111 signals respectively. It can be seen that the model is able to achieve a reasonably low error which result in low ACEPR.

However, without I/Q and dc offset compensation, the model has very high in-band error. Fig. 9 shows the modeling performance for WCDMA 1101 signal. It can be seen that without I/Q and dc offset compensation, the in band distortion cannot be mitigated, however, the proposed model seems to work quite satisfactorily and able to cancel the in band distortion.
C. Complexity and Numerical Stability Issues for Models

At this stage, a question of complexity might arise in the mind of the readers. The figures of merits used in this work for complexity evaluation and numerical stability are dispersion coefficients, condition number, number of coefficients, and floating point operations (FLOPs) required for implementing the model.

1) Condition Number and Dispersion Coefficients: While extracting model parameters the Moore-Penrose pseudo inverse is used to calculate the inverse of the matrix (in our case $X$ and $V$). This pseudo inverse, is however, sensitive to the changes in the matrix conditioning [9]. Dispersion coefficient, on the other hand, determines the spread of coefficients over the domain. Tables II and III show the complexity of the model and compare it to the proposed Parallel Hammerstein (PH) model [20]. In order to fully appreciate the advantage of the proposed model over the existing techniques, one has to consider the above mentioned metrics. It can be seen in Table II that after the coefficients are generated, the original matrix construction, the number of FLOPs increase as a square of the number of columns, hence by reducing the number of coefficients the running time of the system can be reduced.

2) FLOPs for Calculating Matrix Inverse: Another important figure of merit for complexity evaluations is the number of floating point operations (FLOPs) required to compute the model. The FLOPs calculations are done at various stages of computation.

For matrix inversion, Moore-Penrose pseudo inverse is calculated using singular value decomposition (SVD), which according to [31] is a two phase/step process. The first step is reducing the matrix (e.g. $X_{u,v}$) into bi-diagonal matrices, the running time for which is $\mathcal{O}(n^3)$ FLOPs (where the $O$ notation is the measure of the running time of an algorithm). In the second step the bi-diagonal matrix is then diagonal-ized, the computation time for which (for machine epsilon $\varepsilon_{\text{machine}}$) is $\mathcal{O}(\log(\log(\varepsilon_{\text{machine}})))$ FLOPs. Hence the running time of SVD algorithm is $\mathcal{O}(n^2)$ FLOPs. In our case, matrices $X$ and $V$ require inversion during coefficient extraction. For matrix $X$, $u$ equals $L$ (number of samples) and $v$ is 15 (number of coefficients).

Hence the running time of SVD for matrix $X$ is $\mathcal{O}(225 \times L)$. During matrix construction, the number of rows depends on the data points chosen and the number of columns depends upon the nonlinearity order and memory depth of the respective block. As indicated above, the number of FLOPs increase as a square of the number of columns, hence by reducing the number of columns (as attained by the proposed model and shown in Table III) the running time is reduced significantly.

3) FLOPs for Matrix-Coefficients Multiplication: Eqs. (13) and (14) show that after the coefficients are generated, the original matrices are multiplied by the coefficient vector. For rectangular matrices multiplication [33] e.g. $X_{u \times v} \times a_{v \times w}$, the running time is bounded by $\mathcal{O}(uvw)$. Since $a$ is the vector of coefficients, $w$ equals 1. So the running time for the multiplication is $\mathcal{O}(15 \times L)$. Again we see that by reducing the number of coefficients the running time of the system can be reduced.

4) FLOPs for Calculating the Outputs of the Blocks ($y_1(n)$ and $y_2(n)$): The method employed in [34] is used to calculate the number of FLOPs (floating point operations) required to compute the respective polynomial models of both the blocks. For e.g. for the memory polynomial series, for memory index ’$m$’ and nonlinearity order 2, can be written as $a_{0,m}x(n-m)+a_{1,m}x(n-m)x(n)+a_{2,m}(n-m)x(n-m)]^2$. Here, since both are complex integers, 6 FLOPs
TABLE II
COMPLEXITY ANALYSIS AND STABILITY OF THE PROPOSED

<table>
<thead>
<tr>
<th>Input Signal</th>
<th>Proposed Model (N=4, M=2, K=2, M*=2)</th>
<th>PH Model [20] (memory=4, non-linearity=5/3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dispersion coefficient (dB)</td>
<td>Condition Number (dB)</td>
</tr>
<tr>
<td>WCDMA 101</td>
<td>a (eq. 3) 48.3</td>
<td>b (eq. 4) 31.1</td>
</tr>
<tr>
<td>WCDMA 111</td>
<td>49.2</td>
<td>32.3</td>
</tr>
<tr>
<td>WCDMA 1111</td>
<td>51.1</td>
<td>25.3</td>
</tr>
</tbody>
</table>

TABLE III
COMPLEXITY COMPARISON

<table>
<thead>
<tr>
<th>Metric</th>
<th>Proposed Model</th>
<th>PH Model [20]</th>
<th>% reduction Proposed Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficients</td>
<td>34</td>
<td>41</td>
<td>17</td>
</tr>
<tr>
<td>FLOPs</td>
<td>318</td>
<td>385</td>
<td>17.4</td>
</tr>
<tr>
<td>Running time for SVD in terms of FLOPs for L data samples</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proposed Model</td>
<td>PH Model [20]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>X</td>
<td>O(225×L)</td>
<td>O(361×L)</td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>O(361×L)</td>
<td>O(1681×L)</td>
<td></td>
</tr>
<tr>
<td>Running time for matrix-coefficients multiplication for L data samples</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Proposed Model</td>
<td>PH Model [20]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Xa (eq. 3)</td>
<td>O(15×L)</td>
<td>O(19×L)</td>
<td></td>
</tr>
<tr>
<td>Vb (eq. 4)</td>
<td>O(19×L)</td>
<td>O(41×L)</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 10. Power spectral density of PA output, proposed model (23 coefficients) and PH model [20] (53 coefficients) for WCDMA 1101 signal.

ACPR is an important figure of merit to calculate energy leakage into the adjacent channel and is given by [20]

\[ \text{ACPR}_{(dB)} = -10 \log_{10} \left( \frac{\int_{\text{adj. channel}} S(f)}{\int_{\text{in band}} S(f)} \right) \]  \hspace{1cm} (23)

Where \( S(f) \) denoted the power spectral density, adj. channel and in band denote the adjacent channel and desired channel. It can be seen that without any predistortion the output signal suffer from high distortion resulting in a poor ACPR. However by applying the proposed model the ACPR improves significantly. Hence, the model shows reasonable ACPR with reduced complexity which essentially meets the desired purpose of the proposed method i.e. keeping similar performance while reducing the coefficients, condition number and dispersion coefficients significantly as compared to the PH model. The PSDs of the proposed model and the PH model show that the proposed model is able to linearize the signal better than the PH model. A complexity reduction of around 56 percent in terms of number of FLOPs and number of coefficients is achieved.

V. CONCLUSION

In this paper, we explored a novel memory polynomial based distributed model for the mitigation of PA nonlinearity and I/Q mismatch. Various figures of merit and experimental results show that the model is able to reduce the complexity significantly, while maintaining reasonable performance. Number of coefficients and associated complexity is much lower as compared to PH model, without losing any in-band and out-of-band performance.

APPENDIX

The outputs of the two blocks \( y_1(n) \) and \( y_2(n) \) are given by eqs. (11) and (12) respectively. For the sake of simplicity eq. (11) can be written as

\[ y_1 = \sum_j a_j x^j \]  \hspace{1cm} (A.1)

The output of the first block provides the input to the second block, hence the output of second block can be written as

\[ y_2 = \sum_k b'_k \left\{ \sum_j a_j x^j \left| \sum_j a_j x^j \right| \right\} \]

\[ + \sum_k c'_k \left\{ \sum_j a_j^* x^j \left| \sum_j a_j x^j \right| \right\} \]  \hspace{1cm} (A.2)

For the sake of simplicity, we do not consider memory into the system and we drop the notation for the samples i.e. \( y_2(n) \) is just written as \( y_2 \). Let

\[ A_j = \left| \sum_j a_j x^j \right| \]  \hspace{1cm} (A.3)
Then the final equation becomes

\[
\gamma_2 = \left\{ \sum_j a_j x_j |x_j| + \sum_k b_k |x_k| a_f \right\} + \left\{ \sum_j a_j^2 |x_j|^2 + \sum_k c_k |x_k| a_f \right\} + \text{bias} \quad (A.4)
\]

This completes the proof.

Secondly, if we change the position of the blocks i.e. compensate for the modulator imperfections first, followed by the PA nonlinearity then the performance remains the same, as shown in the table below.

<table>
<thead>
<tr>
<th>Signal</th>
<th>Proposed model</th>
<th>Reverse of the proposed model</th>
</tr>
</thead>
<tbody>
<tr>
<td>WCDMA 1111</td>
<td>-34.75</td>
<td>-34.57</td>
</tr>
</tbody>
</table>

### REFERENCES


