MULTIPATH CHANNEL TRACKING IN OFDM SYSTEMS

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ABSTRACT

High-quality channel estimation and tracking is central for coherent detection in wireless communication systems. Typical tracking algorithms assume the number of multipath components and delays to be known and constant, while their amplitudes may vary in time. In this work we focus on the more realistic assumption that also the number of channel taps is unknown and time-varying. The more complex estimation problem arising from this assumption is solved using Random-Set Theory, which allows one to regard the multipath-channel response as a single set-valued variable. Using a particle-filter implementation, we derive an estimator that simultaneously tracks the amplitudes and the number of channel taps for orthogonal frequency division multiplexed (OFDM) systems.

I. INTRODUCTION

In wireless communication, mobility causes time-varying multipath propagation, while coherent detection requires high-quality channel estimation and tracking at the receiver. In fact, large channel estimation errors may have a serious effect on the received data bit-error-rate (BER), and may cause error-rate floor [1]. Conversely, channel estimation can be avoided by using differential detection, at a cost, however, of a loss in signal-to-noise ratio (SNR). The assumption that the number of multipath components and their delays is known and constant, while amplitudes are time-varying, yields relatively simple algorithms for tracking the channel response [2, 3].

In this work, we deal with multipath channel estimation with an unknown and time-varying number of paths. Here we use Random-Set Theory (RST), a mathematical tool that has recently found applications in multi-sensor multi-target tracking [4], multi-speaker tracking [5], and multiuser detection [6, 7]. In general, RST is a probability theory defined on collections of elements where not only each element is random, but the number of elements is also random. With this theory, Bayesian filtering equations can be derived which allow to determine the evolution with time of the channel estimator [8].

The problem of tracking a time-varying channel with an unknown number of multipath components has been treated in the past by developing a sequential estimator which separates an acquisition mode from a tracking mode [9]. The acquisition mode first estimates the number of multipath components via the minimum description length (MDL) criterion, then estimates, using an Estimation of Signal Parameter via Rotational Invariance (ESPRIT) algorithm, the multipath components delays. In the tracking mode, the multipath time delays are first tracked by an interpath interference cancellator (IPIC) delay locked loop (DLL); next, assuming correct delay information, a minimum mean-square-error (MMSE) estimator evaluates the component gains. The performances of this algorithm, in terms of mean-square-error and of probability of detection of the path, are satisfactory even at low SNR values. Its only downside is that it requires the number of paths to remain constant, and an acquisition mode to precede the estimation. In our work, we remove both requirements.

The balance of this paper is organized as follows. Section II. states the problem. Section III. provides the Bayesian-filter recursions needed to solve the tracking problem. Section IV. describes channel tracking in orthogonal frequency division multiplexing (OFDM) systems, and shows some numerical results. Finally, Section V. summarizes our findings.

II. STATEMENT OF THE PROBLEM

We consider an OFDM system employing $K$ orthogonal sub-carriers: $N$ subcarriers, occupying the central part of the spectrum, are used for transmission, while the $K - N$ sub-carriers located at the boundaries are set to zero in order to avoid out-of-band interference. The data stream, after conversion from serial to parallel, is partitioned into blocks of length $N_{in}f$, while $N_{pt} = N - N_{in}f$ pilot symbols are uniformly interleaved with data to enable channel estimation: the resulting modulation scheme is represented in Fig. 1. Assuming, with no loss of generality, that the contributions from the transmit and receive filters can be compensated and denoting by $i_m$, $m = 1, \ldots, N_{pt}$, the $m$th pilot subcarrier index and by $p$ the signaling interval, the observed signal, to be processed for channel estimation, can be given in vector form as [9]

$$y_p = D_p F_p h_p + z_p$$

where $y_p = [y_{p,1}, \ldots, y_{p,N_{pt}}]^T$, $y_{p,ik}$ represents the $k$th pilot subcarrier observation sample at time $p$, $D_p = \text{diag}(d_{p,1}, \ldots, d_{p,N_{pt}})$, with $d_{p,ik}$ the training data on the $k$th

Figure 1: Symbols pattern in OFDM systems.
pilot subcarrier at time $p$, $z_p = [z_{p,1}, \ldots, z_{p,N_{pt}}]^T$, $z_{p,ik}$ representing the zero-mean complex Gaussian additive noise having variance $\sigma_z^2$, $h_p = [h_1(pT_s), \ldots, h_{L(pT_s)}(pT_s)]^T$, with $h_{\ell}(pT_s)$ the complex gain of the $\ell$th path at time $p$, and

$$\{F_p\}_{m, \ell} = e^{-j2\pi m \tau_{p}(pT_s)}$$

for $m = 1, \ldots, N_{pt}$ and $\ell = 1, \ldots, L(pT_s)$ the number of active paths and $\tau_{p}(T_s)$ the delay of the $\ell$th path during the $p$th interval. $T$ represents the sampling interval and $T_s$ the time duration of an OFDM symbol. Notice that $T_s = NT + T_g$, with $T_g$ the guard time. Clearly, if the multipath delay spread is smaller than the guard time, the $n$th subcarrier at time $p$ experiences a frequency-domain flat fading with coefficient $H_{p,n}$ expressed as

$$H_{p,n} = \sum_{\ell=1}^{L(pT_s)} h_{\ell}(pT_s) e^{-j2\pi n \frac{\tau_{p}(pT_s)}{T_s}}$$

(3)

The problem under consideration can now be formulated as follows: Given the observations (1), and a model for the channel response evolution, determine a causal estimator for $h_p$, relying upon $\{y_{1:p}\}$, the observations available up to interval $p$, incorporating all of the prior information on the channel behavior.

III. CHANNEL TRACKING

A traditional approach to solve the problem stated at the end of previous section assumes that the vector to be estimated has constant length $L_{max}$, the maximum number of paths, and implements Bayes recursions to obtain estimates of the vector sequence $[h_1(pT_s), \ldots, h_{L_{max}}(pT_s)]^T$. Disappearance of a path yields a zero in the corresponding vector entry. Kalman Filtering (KF) can be thus applied, possibly in its extended form if the tap weights do not evolve according to a linear Gaussian model [3].

Our new approach relies upon an estimate, done in a single step, of the active paths, which incorporates all the available information concerning the channel evolution: RST provides the natural framework to solve this problem, which appears conceptually close to other problems as outlined in [5, 6, 7].

A. An RST model for the multipath channel

Denote by $\mathcal{H}_{p}^{(k)}$ the following singleton-or-empty random set [4, 8]:

$$\mathcal{H}_{p}^{(k)} = \begin{cases} \{\emptyset\} & \text{if path } k \text{ is not present} \\ \{h_p^{(k)}\} & \text{if path } k \text{ is present} \end{cases}$$

(4)

For the sake of simplicity we have assumed a Tapped-Delay-Line (TDL) channel model [10], wherein the path delays are equispaced. More general models with non-equispaced delays can be accommodated with the same theory by augmenting the single-path state-space as $h_p^{(k)} = [k, a_p^{(k)}, \tau_p^{(k)}]^T$ where $\tau_p^{(k)}$ is the delay of the $k$th path during the $p$th signaling interval.

In our notation, $a_p^{(k)}$ is the complex $k$th multipath gain at time $p$. The channel realizations at epoch $p$ are thus completely described by the set

$$\mathcal{H}_p = \bigcup_{k=1}^{L_{max}} \mathcal{H}_p^{(k)}$$

(5)

which is a random set on the hybrid space $\{1, \ldots, L_{max}\} \times \mathbb{C}$ (see [4, 8]). For future reference, we introduce the random sets $\pi(\mathcal{H}_p)$ and $\pi'(\mathcal{H}_p)$, denoting the projection of $\mathcal{H}_p$ onto $\{1, \ldots, L_{max}\}$ and onto $\mathbb{C}$, respectively:

$$\pi(\mathcal{H}_p) = \bigcup_{k: \pi_k^{(k)} \neq \emptyset} \{k\}$$

(6)

$$\pi'(\mathcal{H}_p) = \bigcup_{k: \pi_k^{(k)} \neq \emptyset} \{a_p^{(k)}\}$$

(7)

If $S_p$ denotes the set of paths surviving from epoch $p-1$ into epoch $p$, and $B_p$ the set of newly born paths, we thus have

$$\mathcal{H}_p = S_p \cup B_p$$

(8)

The constraints [6, 7]

$$\pi(S_p) \subseteq \pi'(\mathcal{H}_p) \subseteq \pi'(\mathcal{H}_{p-1})$$

(10)

reflect the facts that no component being active at time $p-1$ can migrate to the set of new paths, and that the paths surviving at epoch $p$ are a subset of those active at epoch $p-1$. For the sake of simplicity, we invoke the assumption of [5] that at most one new component is allowed to be born at each epoch³, whereby

$$B_p = \begin{cases} \{[\ell, a_p^{(\ell)}]^T\} & \text{with prob. } P_{birth} \\ \emptyset & \text{with prob. } 1 - P_{birth} \end{cases}$$

(11)

with $\ell \in \{1, \ldots, L_{max}\} \setminus \pi(\mathcal{H}_{p-1})$, and $P_{birth}$ the probability that a new path arises. As a consequence, we obtain the conditional density

$$f_{B_p|\mathcal{H}_{p-1}}(B_p \mid \mathcal{H}_{p-1}) = \begin{cases} P_{birth} f_{a_p^{(\ell)}(a_p^{(\ell)})} & \text{if } B_p = \{[\ell, a_p^{(\ell)}]^T\} \\ 1 - P_{birth} & \text{if } B_p = \emptyset \\ 0 & \text{if } |B_p| > 1 \end{cases}$$

(12)

where $\ell \in \{1, \ldots, L_{max}\} \setminus \pi(\mathcal{H}_{p-1})$, and $f_{a_p^{(\ell)}(a_p^{(\ell)})}$ is the probability density function of the $\ell$th path gain at epoch $p$.

Similarly, we have

$$S_p = \bigcup_k S_p^{(k)}$$

(13)

with

$$S_p^{(k)} = \begin{cases} \emptyset & \text{with probability } P_{death} \\ \{a_p^{(k)}\} & \text{with probability } 1 - P_{death} \end{cases}$$

(14)

where $P_{death}$ is the probability that an active path disappears. Assuming that distinct paths survive or die independent of each other, we have the following properties:

³We hasten to say that removal of this assumption, which simplifies the tracking problem, does not invalidate the theory presented here.
• The conditional probability density function (pdf) of the random set $S_p$, given $\mathcal{H}_{p-1}$ can be derived from generalized convolution of the pdf’s of the random sets $S_p^{(k)}$ [5, 6]:

$$f_{S_p|\mathcal{H}_{p-1}}(S_p | \mathcal{H}_{p-1}) = P_{\text{death}}^{(k)}|S_p| \prod_{f \in \pi(S_p)} f_{a_f|\mathcal{H}_{p-1}}(a_f^{(k)} | a_{f-1}^{(k)})$$

(15)

with $S_p \subseteq \mathcal{H}_{p-1}$, and $f_{a_f|\mathcal{H}_{p-1}}(a_f^{(k)} | a_{f-1}^{(k)})$ the transition density describing the evolution of the gains of the surviving paths.

• The random set sequences $S_p$ and $B_p$ are conditionally independent given $\mathcal{H}_{p-1}$;

• $(\mathcal{H}_{p})_{p=1}^{\infty}$ forms a Markov sequence.

As a consequence, the transition density $f_{\mathcal{H}_p|\mathcal{H}_{p-1}}(\mathcal{H}_p | \mathcal{H}_{p-1})$ can in turn be determined through the generalized convolution formula, which, when specialized to the current scenario, yields [6]:

$$f_{\mathcal{H}_p|\mathcal{H}_{p-1}}(\mathcal{H}_p | \mathcal{H}_{p-1}) = \int f_{\mathcal{H}_p}(\mathcal{H}_p \cap \mathcal{H}_{p-1}) f_{\mathcal{H}_p}(\mathcal{H}_p \setminus (\mathcal{H}_p \cap \mathcal{H}_{p-1})) |\mathcal{H}_{p-1}|$$

(16)

In the following, for notational simplicity, we omit the subscript of the pdf’s, when no confusion arises.

The basic step to achieve causal estimates of the random set sequence $(\mathcal{H}_{p})_{p=1}^{\infty}$, based upon the observations $y_{1:p}$ is the implementation of Bayesian recursions in the form [4]

$$f(\mathcal{H}_p|y_{1:p-1}) = \int f(\mathcal{H}_p|\mathcal{H}_{p-1}) f(\mathcal{H}_{p-1}|y_{1:p-1}) \delta\mathcal{H}_{p-1}$$

(17)

$$f(\mathcal{H}_p|y_{1:p}) \propto f(y_{1:p} | \mathcal{H}_p) f(\mathcal{H}_p | y_{1:p-1})$$

(18)

where the operation involved in (17) is a set integration in the sense specified in [8]. The notation $\delta\mathcal{H}_p$ for the differential emphasizes this.

Since closed-form expressions for integrals (17)-(18) do not seem to be achievable, we resort to “particle filtering,” or Sequential Monte Carlo (SMC) methods to approximate Bayesian recursions [11]. The RFS SMC filter is described as follows.

The a posteriori pdf is approximated by the a set of particles as

$$f(\mathcal{H}_p | y_{1:p}) \approx \sum_{i=1}^{M} w_p^{(i)} m_{\mathcal{H}_p}(\mathcal{H}_p^{(i)})$$

(19)

where $m_{\mathcal{H}_p}(\mathcal{Y})$ is the "0-1" measure, defined as follows

$$\int_{C} m_{\mathcal{H}_p}(\mathcal{Y}) d\mathcal{Y} = \begin{cases} 1, & \text{if } \mathcal{Y} \subseteq C \\ 0, & \text{otherwise} \end{cases}$$

(20)

In (19), $\mathcal{H}_p^{(i)}$ is the $i$th set “particle,” $w_p^{(i)}$ is its “weight,” and $M$ is the total number of particles. Asymptotic convergence properties of the SMC RFS filter has been proved in [12], showing that, for sufficient large $M$, the mean-square approximation error of the SMC RFS filter is inversely proportional to $M^\alpha$, for some constant $0 < \alpha \leq 1$, while the implementation complexity is approximately linear with $M$.

Once the a posteriori density $f(\mathcal{H}_p | y_{1:p})$ is obtained, an estimate of $\mathcal{H}_p$ can be obtained in several ways, as outlined in [8]. A pair of Bayesian estimators can indeed be defined, known as GMAP-I (or “Marginal Multitarget Estimator”) and GMAP-II (or “Joint Multitarget Estimator”). GMAP-I is a two-stage estimator, wherein the set cardinality is estimated first. Defining

$$f(n_p | y_{1:p}) \Delta \int_{|\mathcal{H}_p|=n_p} f(\mathcal{H}_p | y_{1:p}) \delta\mathcal{H}_p$$

(21)

we obtain

GMAP-I:

$$\tilde{n}_p = \arg \max_{|\mathcal{H}_p|} f(n_p | y_{1:p}), \quad \mathcal{H}_p = \arg \max_{|\mathcal{H}_p|=\tilde{n}_p} f(\mathcal{H}_p | y_{1:p}),$$

(22)

GMAP-II performs the estimation in a single step:

$$\tilde{\mathcal{H}}_p = \arg \max_{|\mathcal{H}_p|} f(\mathcal{H}_p | y_{1:p}) c_{|\mathcal{H}_p|}^{|\mathcal{H}_p|}$$

(23)

where $c$ is a small constant determined by the cost function that this estimator minimizes [8].

Here we define a third estimation rule, which amounts to first estimating the identities of the paths at epoch $p$, and then estimating only the weights of the active paths as the expected a posteriori value, while setting to zero the weights corresponding to inactive paths. Specifically, we define the GMAP-III estimator as

GMAP-III:

$$\tilde{\mathcal{H}}_p = \arg \max_{|\mathcal{H}_p|} f(\mathcal{H}_p | y_{1:p}), \quad \mathcal{H}_p = \arg \max_{|\mathcal{H}_p|=\tilde{\mathcal{H}}_p} f(\mathcal{H}_p | y_{1:p}),$$

(24)

where

$$f(\mathcal{H}_p | y_{1:p}) = \int_{\mathcal{H}_p} f(\mathcal{H}_p | y_{1:p}) \delta\mathcal{H}_p$$

(25)

Clearly, in GMAP-III, we first estimates the discrete parameter (similar to GMAP-I), then we rearrange the admissible particles to form $f(\mathcal{H}_p | y_{1:p})$ and finally we compute the standard EAP estimation.

IV. NUMERICAL RESULTS

We consider a DFT size of $K = 64$, with $N = 53, N_{pt} = 4$, and a frequency spacing $D_f = 16$ between two pilot subcarriers. Pilot subcarriers are thus located at

$$i_m = \left( m - \frac{N_{pt} + 1}{2} \right) D_f, \quad m = 1, \ldots, N_{pt}$$

(26)

The average energy per pilot symbol, $\sigma_e^2$, is uniform, and a 4-QAM modulation scheme is assumed. As to the channel, we assume a uniform multipath delay profile, multipath spread smaller than the guard time, and uncorrelated path gains. The overall channel energy is normalized to one. Consider first the transmission of $Q = 100$ OFDM symbols through a channel.
with $L_{\text{max}} = 3$, $P_{\text{birth}} = 0.05$, and $P_{\text{death}} = 0.05$. The path gains are assumed to follow a Gauss-Markov model as

$$f(a_{|n|}^{(k)}) = \mathcal{N}(a_{|n|}^{(k)}, 0, \sigma_{n}^{2})$$

(27)

$$f(a_{|p-1|}^{(k)}) = \mathcal{N}(a_{|p-1|}^{(k)}, \lambda \sigma_{p-1}^{2}, (1 - \lambda^{2})\sigma_{p}^{2})$$

(28)

with $\sigma_{n}^{2}$ the average energy of one path, and $\lambda = 0.999$. This choice of parameters is made in order to compare the performance of our estimation procedures with those of a Kalman filter operating on a fixed number $L_{\text{max}}$ of paths, as advocated in [3]. A bootstrap filter with $M=10,000$ particles has been employed in the simulation of GMAP-I, GMAP-II and GMAP-III. Fig. 2 shows the frequency-domain mean-square error (FMSE)

$$\text{FMSE} = \frac{1}{N_{\text{inf}}Q} \sum_{p} \sum_{n} \left| H_{p,n} - \hat{H}_{p,n} \right|^{2}$$

(29)

for GMAP-I, GMAP-II, GMAP-III and Kalman filter (KF), where $\hat{H}_{p,n}$ corresponds to the frequency domain channel estimates on the $n$th subcarrier during the $p$th interval which can be obtained by random-set estimate. The advantage of the approach we advocate here is apparent. This behavior may be justified upon perusal of Figs. 3-4, which show the estimated real and imaginary components of the active paths as a GMAP-III is employed, and contrasting these with Figs. 5-6, referring to KF estimation. In both cases we have chosen a signal-to-noise ratio SNR $\Delta \sigma_{n}^{2}/\sigma_{p}^{2} = 20$ dB. These figures show that a KF operating on $L_{\text{max}}$ paths suffers from a transient effect from disappearing/new paths, while GMAP-III, which relies upon a joint estimate of path identities and gain values, appears more suited to a time-varying scenario. The transient effect becomes heavier and heavier as the SNR decrease. This behavior is justified by Figs. 7-8, which show the estimated real and imaginary components of the active paths using GMAP-III, while Figs. 9-10 refer to KF estimation. In both cases we have chosen a signal-to-noise ratio SNR $\Delta \sigma_{n}^{2}/\sigma_{p}^{2} = 10$ dB.

We point out some differences between constant and time-varying number of multipath components scenarios. Indeed, in [13], a channel estimator is considered for constant number of channel paths which attempt the tracking of five multipath components while the actual number of path is only three and it is shown that such a mismatch ends up in a slight degrades of performance. In time-varying multipath components such a behavior is not expected being necessary a joint estimation of the channel order and channel gain.

V. CONCLUSIONS

We have examined the estimation of a multipath channel with an unknown, and time-varying, number of paths. Using Random-Set Theory (RST), Bayesian filtering equations were derived which allow to determine the evolution with time of the channel estimator. The estimators advocated here outperform previously known systems.

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REFERENCES


Figure 3: GMAP-III estimation for $L=3$, SNR $= 20$ dB. Estimation of the real component.

Figure 4: GMAP-III estimation for $L=3$, SNR $= 20$ dB. Estimation of the imaginary component.

Figure 5: Kalman filter estimation for $L=3$, SNR $= 20$ dB. Estimation of the real component.

Figure 6: Kalman filter estimation for $L=3$, SNR $= 20$ dB. Estimation of the imaginary component.

Figure 7: GMAP-III estimation for $L=3$, SNR $= 10$ dB. Estimation of the real component.

Figure 8: GMAP-III estimation for $L=3$, SNR $= 10$ dB. Estimation of the imaginary component.

Figure 9: Kalman filter estimation for $L=3$, SNR $= 10$ dB. Estimation of the real component.

Figure 10: Kalman filter estimation for $L=3$, SNR $= 10$ dB. Estimation of the imaginary component.