This work shows the experimental implementation of a chaotic communication system based on two Chua’s oscillators which are synchronized by Hamiltonian forms and observer approach. The chaotic communication scheme is realized by using the commercially available positive-type second generation current conveyor (CCII+), which is included into the AD844 device. As a result, experimental measurements are provided to demonstrate the suitability of the CCII+ to implement chaotic communication systems.

Keywords: Chua’s oscillator; circuit realization; Hamiltonian forms approach; current conveyor; secure communication system; chaos.

1. Introduction

An undoubtedly relevant application of synchronization of chaotic oscillators [Cruz-Hernández, 2001; Khan & Singh, 2008; Kilinc et al., 2008; Nijmeijer & Mareels, 1997; Tsay et al., 2005] is in the field of secure/private communications; see e.g. [Alvarez & Li, 2006; Cruz-Hernández et al., 2005; Cruz-Hernández & Romero-Haros, 2008; Matras & Basso, 2008; Trejo-Guerra et al., 2008]. Although many kinds of chaotic oscillators have been introduced, in electronics, Chua’s circuit has been widely used because it allows the development of new designs and applications [Barboza & Chua, 2008; Bilotta et al., 2007; Caflagna & Grassi, 2004; Caponetto et al., 2005; Chua, 1994; Demirkol et al., 2008; Kilic, 2003; Tlelo-Cuautle & Muñoz-Pacheco, 2007; Yu et al., 2007]. Basically, it generates the double-scroll attractor [Cruz & Chua, 1993; Shil’nikov, 1993; Tlelo-Cuautle et al., 2006], and up to now many kinds of implementations have been reported, some at the integrated circuit level, see e.g. [Cruz & Chua, 1993; Tlelo-Cuautle et al., 2006]; by using field programmable analog array [Caponetto et al., 2005], and by using current-feedback operational amplifiers (CFOAs) [Elwakil & Kennedy, 2000; Kilic, 2004; Sánchez-López et al., 2008; Senani & Gupta, 1998].

In Chua’s chaotic system, circuit designers proposed new topologies to implement the Chua’s
diode and inductor elements [Kilic, 2003]. Besides, since the CFOA\(^1\) provides higher bandwidth compared to conventional operational amplifiers, it is a good candidate to enhance Chua’s system [Elwakil & Kennedy, 2000; Kilic, 2004; Sánchez-López et al., 2008; Senani & Gupta, 1998]. However, since the CFOA consists of a positive-type second generation current conveyor (CCII+) in cascade connection with a voltage follower [Tlelo-Cuautle et al., 2006], then this work is oriented to show the usefulness of the CCII+ to realize a chaotic communication system based on two synchronized Chua’s oscillators. Henceforth, Chua’s diode is realized by using two CCII+s as shown in [Senani & Gupta, 1998], while the simulated inductance is realized by using four CCII+s.

The design of the CCII+ at the transistor level of abstraction can be revised in [Fakhfakh et al., 2003], as well as the simulated inductance which is described in Sec. 4. This process is verified experimentally by implementing a master-slave communication system.

The rest of the paper is organized as follows: in Sec. 2, Chua’s oscillator is described by their state space form. These equations are arranged to apply Hamiltonian approach [Sirca-Ramirez & Cruz-Hernández, 2001] to design the observer in Sec. 3. The chaotic communication system by using only CCII+s is shown in Sec. 4, while the synchronization and signal transmission results are given in Sec. 5. Finally, Sec. 6 summarizes some important remarks.

2. Chua’s Oscillator

As a difference from the so-called Chua’s circuit, Chua’s oscillator only takes an extra linear resistor in series with the inductor [Chua et al., 1993], as shown in Fig. 1. In this work, the resistance \(R_L\) will be considered as a parasitic effect belonging to the simulated inductance which is described in Sec. 4. By taking the nodal currents, the following system of equations arise

\[
\begin{align*}
C_1 \frac{dv_{C1}}{dt} &= G(v_{C2} - v_{C1}) - g(v_{C1}) \\
C_2 \frac{dv_{C2}}{dt} &= G(v_{C1} - v_{C2}) + i_L \\
L \frac{di_L}{dt} &= -v_{C2} - i_L R_L
\end{align*}
\]

where \(G = 1/R\) and the nonlinear characteristic of Chua’s diode is given by

\[
g(v_{C1}) = G_{sc}v_{C1} - \frac{1}{2}(G_2 - G_1)
\times (|v_{C1} + E| - |v_{C1} - E|)
\]

The normalized set of equations is given by the state space form

\[
\begin{align*}
\dot{x}_1 &= \alpha(x_2 - x_1 - f(x_1)) \\
\dot{x}_2 &= x_1 - x_2 + x_3 \\
\dot{x}_3 &= -\beta x_2 - \gamma x_3
\end{align*}
\]

where the nonlinear function is described as

\[
f(x_1) = bx_1 + \frac{1}{2}(a - b)(|x_1 + 1| - |x_1 - 1|)
\]

With solutions given by \(x_1, x_2\) and \(x_3\); the selection of \(x_1 = v_{C1}/E, x_2 = v_{C2}/E,\) and \(x_3 = i_L R/E\) implies that the parameters have also to be defined as \(\alpha = C_1/C_2, \beta = R^2 C_2/L\) and \(\gamma = R L C_2/L\). Thus, the main elements of the system are related to the bifurcation parameters \(\alpha, \beta\) and \(\gamma\). These parameters allow the chaotic regime due to the influence on the eigenvalues of the system. A set of known parameters is

\[
\alpha = 9, \quad \beta = \frac{100}{7}, \quad a = -\frac{8}{7}, \quad b = -\frac{5}{7}
\]

Parameter \(\gamma\) can be chosen \(\gamma < 3\). The function \(f(x_1)\) is described by the parameters \(a = RG_1\) and \(b = RG_2\) which allocate its slopes and the normalized break point \(E = 1\), related to the signal amplitude and it is arbitrarily selected to enlarge/contract the attractor.

\(\text{CFOA Datasheet available in: http://www.analog.com/static/imported-files/data_sheets/AD844.pdf}\)
3. Synchronization by Hamiltonian Forms Approach

According to [Cruz-Hernández et al., 2005], Chua’s system can be synchronized by Hamiltonian forms approach. First, the system is expressed in a Hamiltonian form with destabilizing vector field $F$, as described by (4), with integers $n \geq m$.

$$
\dot{x} = J(y) \frac{\partial H}{\partial x} + (I + S) \frac{\partial H}{\partial x} + F(y), \quad x \in \mathbb{R}^n,
$$

$$
y = \frac{\partial H}{\partial x}, \quad y \in \mathbb{R}^m
$$

(4)

Here, the energy conservative part is related to the skew symmetric matrix $J$, while the non-conservative part is, in general, represented by the skew symmetric matrix $S$. $I$ is a constant skew symmetric matrix; $H(x)$ denotes a smooth energy function associated to the system and $y$, the linear output mapping. Thus, the normalized Chua’s oscillator (2) in Hamiltonian form is given by (5), with the gradient vector given by (6).

$$
\begin{pmatrix}
x_1 \\
x_2 \\
x_3
\end{pmatrix} =
\begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & \beta \\
0 & -\beta & 0
\end{pmatrix}
\frac{\partial H}{\partial x}
+ \begin{pmatrix}
-\alpha^2 & \alpha & 0 \\
\alpha & -1 & 0 \\
0 & 0 & -\beta \gamma
\end{pmatrix}
\frac{\partial H}{\partial x} + \begin{pmatrix}
-\alpha f(x_1) \\
0 \\
0
\end{pmatrix}
$$

(5)

$$
\frac{\partial H}{\partial x} =
\begin{pmatrix}
\frac{1}{\alpha} & 0 & 0 \\
0 & \frac{1}{\beta} & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
x_3
\end{pmatrix}
$$

(6)

An observer for (4) is described by (7), where $K$ represents the observer’s gain.

$$
\begin{pmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{pmatrix} =
\begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & \beta \\
0 & -\beta & 0
\end{pmatrix}
\frac{\partial H}{\partial x}
+ \begin{pmatrix}
-\alpha^2 & \alpha & 0 \\
\alpha & -1 & 0 \\
0 & 0 & -\beta \gamma
\end{pmatrix}
\frac{\partial H}{\partial x} + \begin{pmatrix}
-\alpha f(y) \\
0 \\
0
\end{pmatrix}
$$

$$
+ \begin{pmatrix}
k_1 \\
k_2 \\
k_3
\end{pmatrix} e_y
$$

(8)

Matrix $M$ is calculated to accomplish

$$
2 \left( S - \frac{1}{2}(KC + CTK^T) \right)
$$

$$
= \begin{pmatrix}
-2\alpha(\alpha + k_1) & \alpha(2 - k_2) & -\alpha k_3 \\
\alpha(2 - k_2) & -2 & 0 \\
-\alpha k_3 & 0 & -2\beta \gamma
\end{pmatrix}
$$

(9)

By means of the Sylvester’s Theorem [Ogata, 1995], and by solving the required conditions for the selection of a non-negative observer’s gain $K$, one achieves:

$$
k_1 > 0
$$

$$
0 \leq k_2 \leq 4
$$

$$
0 \leq k_3 < \frac{\sqrt{2} k_1}{\alpha}
$$

(10)

If we assume for simplicity $k_2 = k_3 = 0$, it is clear that for all $k_1 > 0$, $M$ is then negative definite matrix. However, in the experiment, the coupling stage was realized by setting $k_3 = 0$, while $k_1$ and $k_2$ are positive constants as shown in the next section.

4. Circuit Construction

In this section, the mixed mode characteristic of the CCH+ is exploited to implement the Chua’s diode and the simulated inductor in straight form.
An important issue is that this realization allows integrated circuit design. The construction of the chaotic circuit is shown in Fig. 2, where the simulated inductance is implemented by four CCII+s (L1 to L4), two resistors (RL1 and RL2) and one capacitor (CL).

The CCII+ is included into the commercially available CFOA AD844. Its main parasitic effects are associated to the parasitic resistance in terminal X (RX), and the finite resistances presented at terminals Y and Z. From Fig. 2, by applying the symbolic method introduced by Tlelo-Cuautle et al. [2009], the equivalent analytic inductance obtained by this arrangement is

\[ Z(s) = \frac{sCL + G_Y}{sG_1CL + G_2 + G_1G_2} \]  

The term \( G_Y \) represents the conductance at terminal Y, while \( G_1 = 1/(RL1 + 2RX) \) and \( G_2 = 1/(RL2 + 2RX) \), and \( CL \) is an external capacitor. In this manner, in the ideal case, when \( G_Y = 0 \) and \( RX = 0 \), (11) becomes \( Z(s) = s \cdot CL \cdot RL1 \cdot RL2 \).

Equation (11) embeds the resistance denoted by \( RL \) in Fig. 1, which is a parasitic effect to the CCII+-based simulated inductance. An important thing is that \( RL \) simplifies the synchronization procedure by turning negative all roots of \( M \).

5. Experimental Results

5.1. Synchronization results

Figures 4 and 5 detail synchronization results obtained with the proposed scheme, both in time and in phase planes, respectively.

5.2. Information transmission

Once the circuit synchrony has been observed, the transmission process can be carried out by several strategies. The main ones are chaotic masking, shift keying and chaotic modulation. If it is preferred to work the devices in the linear region in this part, one can keep using the CCII+ to implement the chaotic masking scheme shown in Fig. 6.

The signal \( S_1 \) represents the information to be encrypted as signal \( S_2 \), and finally recovered as signal \( S_3 \). Due to the mixed nature of the CCII+, one can consider the possibility of using either voltage or current mode signals for the transmission. Figure 7 shows the required connections in each
Fig. 3. Complete transmitter/receiver circuits using only current conveyors.
Fig. 4. Signals (a) channel 1: $x_1$, Y-scale: 5V/div; channel 2: $\hat{x}_1$, Y-scale: 5V/div, (b) channel 1: $x_2$, Y-scale: 5V/div; channel 2: $\hat{x}_2$, Y-scale: 5V/div, synchronized in time.

Fig. 5. Signals (a) $x_1$ and $\hat{x}_1$, X-scale and Y-scale: 2V/div, (b) $x_2$ and $\hat{x}_2$, X-scale and Y-scale: 0.5V/div, synchronized in phase plane.

Fig. 6. Chaotic masking, block diagram encryption.

The constant $D$ represents a reduction factor of signal $S_1$ compared to $S_2$.

When considering the high impedance in terminals Y and Z of the CCII+ respect to X [Fakhfakh et al., 2007], $R_X$ should be taken into account because it generates tracking errors known as voltage-gain ($A_v$) between Y and X, and current-gain ($A_i$) between X and Z. In the ideal case, $A_v = A_i = 1$, however, in the real case these tracking errors affect the acquisition of the signal $S_3$. 
Chaotic Communication System Using Chua’s Oscillators Realized with CCII+s

According to

\[ S_3 = A_v^2 A_i^2 S_1 + A_v A_i D(A_v A_i x_i - \hat{x}_i) \]  

for the voltage mode circuit, and

\[ S_3 = A_v^2 S_1 + A_v A_i \left( \frac{\hat{x}_i - A_v x_i}{R_X} \right) \]  

for the current mode one. It is clear that the current approach is not just more accurate, it also allows the determination of the exact value for \( R_X \).

Once this consideration has been made, the complete design with the discussed current mode approach is shown in Fig. 3. RS and RA are chosen to adjust the chaotic signal magnitude, while \( R_{in} \) and \( R_{out} \) are used to manipulate the external signal as voltage. State \( x_2 \) has been taken as the encryption signal \( x_i \). Figure 8 shows the recovered signals for several frequencies of \( S_1 \), as well as \( S_2 \) sequences.

Fig. 7. Current conveyor based chaotic masking scheme for (a) voltage and (b) current mode.

Fig. 8. (a), (c), (e) and (g) Comparison between the transmitted (channel 1) and received (channel 2) signals, Y-scale channel 1 and channel 2: 1 V/div; and (b), (d), (f) and (h) transmitted (channel 1) and encrypted (channel 2) signals, Y-scale channel 1: 1 V/div; Y-scale channel 2: 20 mV/div; for: (a) and (b) 100 Hz, (c) and (d) 1 KHz, (e) and (f) 10 KHz, and (g) and (h) 50 KHz.
Fig. 8. (Continued)
6. Conclusions

An experimental implementation of Chua’s oscillator has been presented using only CCII+s. The circuit performance has been reviewed experimentally using a commercial CCI+ included into the AD844 from Analog Devices.

Parasitic effects of the implementation are analyzed according to the general data provided by the manufacturer to explain the most susceptible points of the design despite assistance. Furthermore, the synchronization of two Chua’s oscillators is shown by applying the Hamiltonian approach, in order to implement a chaotic communication system.

On the other hand, since chaotic masking is one of the potential forms to solve the transmission problem of encrypted information, then the versatility of the CCI+ suggests different approaches to provide some solutions in this area. As a result, a chaotic communication system realized with only CCII+s has been proposed and verified experimentally.

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