Disruption management in flight gate scheduling

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This paper introduces models of robustness in flight gate assignments at airports. We briefly repeat the general flight gate assignment problem and disruptions occurring in airline scheduling. Recovery strategies and robust scheduling are surveyed as the main methods in disruption management. We present a non-robust flight gate assignment model and incorporate two approaches of robustness.

Keywords and Phrases: flight gate assignment, robust scheduling, disruption management.

1 Introduction

Due to the rapid growth of air transport traffic, how to efficiently allocate gates at airports to incoming or outgoing flights has become one of the most important problems which managers of airlines and airports have to concern about. The problem, which can be modeled as a scheduling problem, is often referred to as flight gate scheduling.

The main purpose of flight gate scheduling is to find an assignment of flights, i.e. the aircraft serving a flight, to aircraft stands, as well as start and completion times for processing an aircraft at the position it has been assigned to. Flight-to-gate assignments not only affect an airport’s operating efficiency but also its level

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Disruption management in flight gate scheduling

of service. In practice, a good solution to the problem can prevent congestion and inconvenience at the airport.

A number of analytical models have been developed that help airport authorities more effectively assigning daily flights to gates. For example, Braaksma (1977), Babic, Teodorovic and Tosc (1984), Mangoubi and Mathaisel (1985), Vanderstraeten and Bergeron (1988), Bihr (1990), Zhang, Cesaroni and Miller (1994), Cheng (1997), Yan and Chang (1998), Haghani and Chen (1998), Bolat (2000a,b), and Yan and Huo (2001). For a more detailed introduction we refer to Dorndorf et al. (2005).

Usually, objectives of flight gate scheduling are:

1. the number of expensive aircraft towing procedures (that otherwise decrease the available time for some ground service operations on the ramp as well as in the terminal) has to be reduced;
2. the total walking distance for passengers has to be minimized;
3. the deviation of the current schedule from a reference schedule has to be minimized to increase schedule attractiveness and passenger comfort;
4. the number of un-gated (open) aircraft activities has to be minimized;
5. preferences of certain aircrafts for particular gates have to be maximized.

Three classes of constraints are usually considered:

1. one gate can process only one aircraft at the same time;
2. service requirements and space restrictions with respect to adjacent gates must be fulfilled;
3. minimum ground time and minimum time between subsequent aircraft have to be assured.

Although those deterministic models could work well in the planning stage and provide authorities the so-called optimal schedules, they can hardly generate schedules which are really optimal in the operation stage. Disruptions which interrupt the operation of airlines are always in the way. In fact, there are many factors that will affect gate assignments during real operations, including: (1) flight or gate breakdown, (2) flight earliness or tardiness, (3) emergency flights, (4) severe weather conditions, (5) errors made by staff and many others. Moreover, it often happens that a slight disruption occurred in the morning would cause a severe breakdown in the evening due to the ‘knock-on effect’.

Unfortunately, most researches only focus on the performance improving of static gate scheduling. Very little concern has been addressed to the way to cope with disruptions in flight gate scheduling.

This paper surveys a large amount of models and techniques developed to deal with disruptions in airline industry, either reactive after disruptions or taking proactive steps to consider them when schedules are made. Moreover, we concentrate on the application of robustness in flight gate scheduling. Two new methods are proposed to generate robust flight gate schedules which are expected to be flexible to slight disruptions.

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The remainder of the paper is organized as follows: In Section 2 we detail the disruptions considered in the flight gate scheduling and in Section 3 we present several typical techniques of disruption management. In Section 4, methods of robust scheduling in airline industry are surveyed. In Section 5 a deterministic gate scheduling model is introduced, which is followed by two new robust flight gate scheduling methods based on it. We conclude in Section 6.

2 Disruption management

Airlines spend an enormous amount of resources and efforts in generating optimal flight schedules to maximize their profits. However, these schedules are often interrupted by various internal and external reasons, e.g. bad weather, equipments breakdown, and crew unavailability. These disruptions make deviations from the initial plan inevitable. If not managed properly and timely, such deviations will severely affect the airlines performance in terms of revenue, operational efficiency, and customer satisfaction.

2.1 The objective of disruption management

Disruptions are inherently associated with uncertainties or unexpected events. However, it is not necessary or possible to act on all unexpected events. Some unexpected events, such as minor delays, do not require changes of plans and cause very limited inconvenience. Other events may be so serious that it may not be possible to do anything about them. So only in case it is necessary to do something about the event, we will define it as a disruption. It is a situation where one or more activities in one or more of the key resource areas (e.g. crew or aircraft) have deviated from the resource plan. Subsequent activities in the affected lines of work either cannot start on time or can start on time, but only after controller intervention.

So the airlines can either predict the possible disruptions when planning to take proactive decisions or react to the disruptions after they occur to take recovery decisions. Accordingly, we use the phrase disruption management (DM) here to describe the efforts to avoid the disruptions beforehand and the efforts to solve the disruptions afterwards, which are corresponding to the robust planning process and recovery process respectively. The definition here is different from the traditional ones that only focus on efforts in the recovery process.

As for the objectives of disruption management, they fall into two aspects.

Maximize the real profits. A disruption unavoidably brings changes to the revenue and some costs including excess crew costs and costs of compensation to passengers, etc., so it is a necessary objective to maximize the profits in the changed environments.

Return to the original plan as soon as possible. Disruption management focuses on attempting to preserve, as much as possible, the original operational plan

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Disruption management in flight gate scheduling

(even if this means non-optimal solutions to the original objective function). The rationale behind this approach is that in many cases there are significant implicit costs that are involved in breaking away from the original plan. These implicit costs are difficult to measure (or even estimate) and therefore they are not represented in the original objective function. Furthermore, any deviation of the new plan from the original might cause passenger dissatisfaction and bad effects on the future business.

Therefore, disruption management should be both: maximizing profits while staying close to the initial plan.

2.2 The disruption management process

Airlines require a large number of resources in the daily operation but the principal resources for an airline company are crew and aircraft. Together with the passengers they constitute the three main aspects which must be planned and monitored to obtain operational efficiency. In most cases the airlines draw out a plan several months before operation. As the time of the operation approaches, the plan is adjusted to changes, which is denoted as the tracking process. The traditional methods in these processes are to get the optimized plan, while in robust planning slack should be built into the plan where it is possible to be particularly vulnerable to disruptions.

While on the day of operation, crew, aircraft and passengers interact closely with each other and any change will possibly have a direct impact on them. It is the control process in which the operation is monitored during execution. When the observed situation deviates from the situation planned for, it must be decided whether an intervention is necessary or whether it is a disruption in other words.

In case of an intervention the recovery process is triggered. The first step is to identify the possible relevant actions and to evaluate these. The evaluation will involve evaluations from various perspectives to ensure that the option must be feasible for crew as well as aircraft and should preferably minimize passenger inconvenience. The process of identifying and evaluating the options is an iterative one and in principle it will continue until the option acceptable for all perspectives is found. Based on the agreed option, it can then be decided when to execute the final decision for committing the resources (earlier or later than necessary should be avoided). The proper time is dependent on the option considered; once a decision has been taken it must be implemented and the monitoring must continue. The control process will continue until the flight is completed or canceled. The whole process is shown in Figure 1 (see also KOHL et al. 2004).

The measures taken in a robust planning and recovery process are essential factors in disruption management, which are denoted as robust scheduling and recovery strategy. The following sections review the two methods.
3 The recovery problem

The airlines nowadays use some recovery strategies in their operations such as an assignment of standby crew, swap, and delay of a flight, etc. In research, according to the three factors in airline operation that are aircrafts, crew, and passengers, the recovery strategies fall into three categories. Among the three kinds of strategies, the aircraft recovery strategy receives the most significant attention partly because the aircraft is the easiest resource with the lowest complexity in rules and also the scarcest resource which is fixed first. Crew recovery problems and passenger recovery problems will not be considered here, see, for instance, the passenger flow model of Lettovsky (1997). As for the integrated strategy, it is still a challenge and no real-life results are reported yet. Therefore, in this paper we focus the review on aircraft recovery, which gives some suggestions to flight gate recovery strategy problems.

In recovery problems a paramount important element is decision costs. In mathematical modeling, costs reflect how preferable each decision possibility is. Preferable here means a better objective function value i.e., the objective function will be a measure of how preferable the revised flight schedule is. The most common approach to quantifying decision costs is to estimate the real costs associated with each decision.

3.1 Aircraft recovery problems

For a given airline, the flight schedule includes all flights between any two destinations, the original departure and arrival times, the expected flight durations and the tail assignments. Given an original flight schedule and one or more disruptions, the dedicated Aircraft Recovery Problem (ARP) consists of delaying a flight, canceling a flight as well as swapping an aircraft to a flight assignment. The term swap
denotes that two flights, designated to be undertaken by two specific aircrafts, are interchanged between these aircrafts.

3.1.1 Decision cost

Estimates of the real costs associated with each of these include the following.

Delay costs:
- passengers leaving for other flights undertaken by different airlines;
- compensations to passengers;
- crew planning issues;
- ill-will from the passengers.

Swap costs:
- redirecting passengers to new gates;
- ill-will of the passengers as a consequence of changing gates (i.e. a longer walking distance to the gate);
- redirecting luggage, supplies, crew, etc. to aircraft different from those planned.

Cancellation costs. The costs of cancellations are similar to those of delays, but will often be larger, i.e. there are often more costly ramifications associated with a cancellation than with a delay.

The above-mentioned costs are only some of the more important ones to consider when calculating the actual operating costs of an airline. Most of them will be difficult, if not impossible to estimate. It may be futile to base an objective function on real operating costs. Instead, most models focus on quantifying the basic principles that flight controllers adhere to.

3.1.2 Review on ARP

Mathematical models. The aircraft recovery problem is often considered as a network flow model with side constraints. In most models disruptions, cancellations, delaying, and swaps are permitted but no maintenance, crew restriction, and passenger connections. Swaps in these models are limited to the same aircraft type but some authors allow for swaps between different types, e.g. Argüello, Bard and Yu (1997) present a method for rescheduling the aircraft routings using swaps between different fleet types.

Objectives. The objectives for ARP models are either to minimize the disturbances of the original schedule or to maximize the profits. The first kind of objectives is achieved by directly dealing with the issues of delays and swaps. For instance, Teodorovic and Stojkovic (1990) set up a model with two objectives, which aims to maximize the number of flights flown and minimize the total passenger delays respectively.

While most objectives fall into the second class with types of minimizing the costs or maximizing the profits. Jarrah et al. (1993) develop a delay model and
a cancellation model aiming to minimize the costs associated with delays and cancellations. The models of Argüello et al. (1997) are built to minimize the costs including measures for passenger inconvenience and lost flight revenue. In turn, Yan and Yang (1996), Yan and Tu (1997) develop four models to minimize the cost of schedule repair. The objective of the model by Yan and Young (1996) is to maximize the profit that is the result of the airline revenue subtracted by the cost of cancellation and delay.

**Solutions.** As the size of realistic problems is too large for traditional methods, heuristics are incorporated into the algorithm to make it feasible for realistic ARP. Three methods are proposed by Andersson (2001): a Lagrangian relaxation, Dantzig–Wolfe decomposition for small-sized problems, and tabu search for large problems.

The crew recovery problem consists of delaying a crew, placing a crew on standby (cancellation), using a standby crew, swapping a crew to flight assignments. The related decision costs also reflect certain basic flight control principles, however, the principles for crew members are not very well defined because there are too many principles in the airlines and their ranking is difficult to ascertain.

### 3.2 The flight gate recovery problem

An aircraft is assigned to a flight gate to complete some activities, such as the arrival of the aircraft, the departure or parking at the gate, as flight gates are scarce and expensive resources, it is very important to use the available gates in the best possible way. The aircraft gate reassignment problem occurs when more than one aircraft requires the same gate at the same time. In this case, the airline must revise the gate assignment to minimize the disruption effects.

Not much research has been done in the area of flight gate recovery. Gu and Chung (1999) describe a genetic algorithm approach for solving the gate reassignment problem. By using a global search technique on quantified information, this genetic algorithm approach can efficiently find minimum extra delayed time solutions that are as effective as or more effective than solutions generated by experienced gate managers. An integral minimum cost flow model is introduced by Bard et al. (2001). This model aims at reconstructing airline schedules in response to delays by transforming the routing problem into a time-based network in which the overall time horizon is divided in discrete periods. The transformation is polynomial with respect to the number of airports and flights. An optimum of the model corresponds to the optimal solution of the original problem under some slight conditions.

### 4 Robust scheduling in airline

Several recovery methods have been introduced above. These methods can help us to go back to the original schedules or to reschedule resources after disruptions on an immediate basis. This reactive strategy is reflected in airline companies where
millions of dollars are spent in recovery costs and systems to help airlines better react to disruptions (McDowell 1997). Disruptions are expensive and lead to loss of time, money, and customer goodwill.

The costs of recovery processes are immense, let alone the costs of disruptions themselves. Rather than reacting, something has to be done in advance to prevent disruptions. In other words, there is a need to take proactive steps and work out robust schedules which are flexible to upcoming disruptions.

Since the late 1990s, lots of papers have been written on the airline robust scheduling problem. We can roughly classify those with respect to the resources considered. Aircrafts are the most frequently considered resources in the airline industry. How to schedule all the aircrafts efficiently remains a tough problem, let alone the efforts of making the aircraft schedules or fleet schedules more robust.

The deterministic problem is usually modeled as proposed by Barnhart (1997) in which minimizing the cost is the objective and three groups of constraints ensure that each flight must be assigned to exactly one routing, keep the flow balance at all the airports and control the total number of aircrafts in use with respect to the number of aircrafts available.

The difference between methods of aircraft robust scheduling lies in the measurement of robustness. Ageeva (2000) proposed an approach for incorporating robustness into airline scheduling. Through the approach, a robust schedule could be obtained which is able to provide ways for more aircraft routes to intersect at different points, so that corresponding aircrafts can be easily switched to other flights if needed. The so-called overlaps in every schedule are then counted as the degree of robustness. At last, the most robust schedule is selected among all the optimal alternatives as the ultimate solution.

Rosenberger, Johnson and Nemhauser (2001) apply the hub connectivity, i.e. the number of legs in a route starting and finishing at different hubs with only stops at spokes in between, as the measurement of robustness, while Bian et al. (2003) consider the amount of aircrafts on ground as the measurement of robustness of airline schedules.

Lim and Wang (2005) understand arrival and departure times of aircraft as an outcome of a random variable. In their model they aim for reducing an estimation of the expected value of gate conflicts. They solve the problem using a hybrid two-stage meta-heuristic. The first stage is a tabu search for finding an initial solution while the second stage uses a local search for improving the solution.

Crew is another kind of important resource for airline industry. Billion dollar crew costs are second after fuel costs, thus airlines always seek to efficiently use the crew resources (Graves et al., 1993, Anbil, Tanga and Johnson 1992). The crew scheduling problem is usually modeled as a set partition problem (Schaefer et al., 2004)

\[
\begin{align*}
\text{min} & \quad cx \\
\text{s.t.} & \quad Ax = 1 \\
& \quad x \in \{0, 1\}
\end{align*}
\]
Quite a lot of papers could be found on techniques for solving the set partition problem (see Anbil et al., 1992, Barnhart et al., 1998, Chan and Yano 1992, Chu, Gelman and Johnson 1997, Desrosiers et al., 1991, Gershkoff 1989, Graves et al., 1993, Hoffman and Padberg 1993, Klabjan, Johnson and Nemhauser 2001, Marsten, Muller and Killion 1979, for example). However, the optimal schedules obtained can hardly perform well in real world where disruptions could occur anytime. To find out the crew schedules which could be ‘optimal in practice’, some modifications to the original model are needed. Schaefer et al. (2000) proposed a stochastic extension to the deterministic crew scheduling problem. They modify the objective function by introducing operational costs which take the upcoming disruptions into account. Two different methods are proposed to estimate the operational costs. One is calculating expected costs through simulation while the other one is incorporating a penalty function into the objective function. They then solve the modified model using the parallel primal–dual simplex algorithm.

Ehrgott and Ryan (2002) develop a bicriteria optimization model to generate Pareto optimal crew schedules. The objectives are the cost and robustness of a schedule. Similar to the effort of estimating the operational costs, an objective function is developed to penalize schedules which are not robust. They then minimize this objective function while at the same time maintaining a cost-effective solution.

Yen and Birge (2000) also consider the problem of crew scheduling under uncertainty. They formulate the problem as a two-stage integer stochastic program, where the first stage is the crew scheduling problem and the second stage involves penalties for delays.

Note all the three methods mentioned above involve penalties for possible disruptions. Actually, incorporating a penalty function to the objective is a commonly used method in robust scheduling. However, to the best of our knowledge, little or no effort has been expended on applying penalties for the gate assignment problem (GAP).

As already mentioned, the deterministic GAP can be modeled as a quadratic assignment problem or a linear integer program. Up to now, the most popular idea of incorporating robustness into gate scheduling is applying buffer times. For example, Hassounah and Steuart (1993) show that planned buffer times could improve schedule punctuality. Yan and Chang (1998) and Yan and Huo (2001) use in their static gate assignment problems a fixed buffer time between two continuous flights assigned to the same gate to absorb the stochastic flight delays. Yan and Chang (1998) develop a multicommodity network flow model. Moreover, they use Lagrangian relaxation with sub-gradient optimization and some heuristics to solve the GAP. Yan and Huo (2001) formulate a dual objective 0–1 integer programming model for the aircraft position allocation. The first objective tries to minimize passenger walking time while the second objective aims at minimizing passenger waiting times. The authors argue that, e.g. during peak hours, an aircraft might have to wait for an available gate, and hence passengers have to wait on the aircraft until a gate is available. A simulation framework is proposed by Yan, Shieh and Chen (2002),
which is not only able to analyze the effects of stochastic flight delays on static gate assignments, but can also evaluate flexible buffer times and real-time gate assignment rules.

Recently, some authors try to take into account the dynamic character of the GAP. A delayed departure may delay the arrival of another aircraft scheduled to the same gate, or require the flight to be reassigned. When gate idle times are distributed uniformly among the gates, the probability that the delayed departure will still be earlier than the arrival of the next flight is maximized. One of the first attempts to realize an approach aiming at robust schedules is due to Bolat and As-Saifan (1996) (Bolat 1999, 2000a and 2000b). The author proposes to utilize gates as uniformly as possible to provide schedule robustness to small changes of input data. Furthermore, mathematical models and (optimal and heuristic) procedures are proposed to provide solutions with minimum dispersion of idle time periods for the GAP.

5 New models

As already mentioned in the section before, incorporating a penalty function into the objective is a commonly used method to solve the robust scheduling problem in airline industry. In this section, we propose two models for gate scheduling.

Before we go to the models, an introduction to the deterministic gate scheduling model which is used as a basis to construct new models is needed. This model has been proposed by Dorndorf (2002). The gate assignment problem is considered as a resource-constrained project scheduling problem with generalized precedence constraints, as presented in Dorndorf, Pesch and Phan Huy (2000b). The idea is to assign available airport flight gates to three possible aircraft activities (arrival; optional intermediate parking activity, the length of which depends on the ground time; departure) and to schedule the start and completion times of the activities at the positions.

Compared with previous models there are several new contributions. Firstly, the three activities are modeled separately and, hence, can potentially be assigned to different positions. The aircraft can be moved to another assigned position using tow tractors, a procedure which is called towing. Secondly, in contrast to the standard objective function commonly used (which minimizes passenger walking distance), a complex objective function which is a combination of several partial objectives is introduced.

Three objectives are considered most important after intensive discussions with airport managers:

- maximization of total flight-gate preferences;
- minimization of the number of towing activities; and
- minimization of the deviation of the new gate assignment from a so-called reference schedule.
It should be noted that the total objective only depends on the gate assignment and does not depend on the start and completion times of aircraft processing activities at the assigned positions. Moreover, the overall objective function takes into account both passenger comfort and convenience for airport services.

Each aircraft activity (arrival, departure, or parking) $i$ can be described by its start time $S_i$ and by its completion time $C_i$. It is evident that the start time for an arrival activity and the completion time of a departure activity are fixed and given a priori according to some timetable. All other start and completion times are decision variables of the model.

Let $V$ denote the set of all activities as a unification of the sets of arrival, parking and departure activities, that is, $V = V^a \cup V^p \cup V^d$. Each activity $i$ has a minimum required processing time $p^\text{min}_i$. Activity $i$ can be assigned to different flight gates from the associated gate set $M_i$ of possible gates which is a subset of the set of all gates $M$. $M_i$ is the selected gate for activity $i$. To cope with the situation where the constraints do not allow assigning all aircraft to real gates, a fictitious gate $M^0$ with unlimited capacity is introduced. Every set $M_i$ contains this dummy gate, and an assignment to the dummy gate will be penalized in the objective function.

If two linked activities are assigned to different flight gates, then they require a towing procedure to be moved from one position to another one. Two activities are linked if they are subsequently served by the same aircraft (e.g. arrival–parking or parking–departure). Towing takes some fixed processing time $d^\text{tow}_{iM_i jM_j}$. Let $\varepsilon^\text{tow}$ represent the set of linked activities. It follows that the completion and start times of two linked activities $i$ and $j$ should satisfy the equality $C_i + d^\text{tow}_{iM_i jM_j} = S_j$ to provide continuous processing.

Gates are disjunctive resources that can only process one aircraft at a time (the only exception is, of course, dummy gate $M^0$). Between the processing of two activities $i$ and $j$ at the same gate a fixed set-up time $d^\text{setup}_{iM_i jM_j} \in \mathbb{N}_0$ must pass. The set-up time can reflect the time required to push back the first aircraft from the gate and for moving the second aircraft to the gate as well as the duration required for setting up gate equipment. So, the basic disjunctive constraints that forbid simultaneous assignment of two aircraft to the same gate have to be added to the model. Additionally, these constraints must cover so-called ‘shadowing’ restrictions $(i, M_i, j, M_j)$ between gates $M_i$ and $M_j$ that can be interpreted as follows: if gate $M_i \in M_i$ is assigned to activity $i$, then activity $j$ must not be processed simultaneously at gate $M_j \in M_j$. The set of all shadowing restrictions is denoted with $\varepsilon^\text{shadow}$. Table 1 summarizes the notation used in this model.

For illustration, we use examples in the style of Fig 2 which shows the assignment for gate $M^1$. From the figure we can find that gate $M^1$ would be allocated to the arrival activity 1 from 12:00 a.m. to 13:30 p.m., to departure activity 2 from 13:30 p.m. to 14:00 p.m. and to parking activity 3 from 16:00 p.m. to 18:00 p.m. The hollow bar from 15:00 p.m. to 16:00 p.m. shows gate 1 is free in the interval.

The model is summarized as follows:

Find a schedule $(S, C, M)$ which assures the following constraints:
Table 1. Summary of notation.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e_{\text{shadow}}$</td>
<td>Set of all shadowing restrictions</td>
</tr>
<tr>
<td>$e_{\text{low}}$</td>
<td>Set of linked activities</td>
</tr>
<tr>
<td>$d_{\text{set-up}}^{M_i,M_j}$</td>
<td>Minimum set-up time between two activities $i$ and $j$ at the same gate</td>
</tr>
<tr>
<td>$d_{\text{tow}}^{M_i,M_j}$</td>
<td>Minimum time of towing between two linked activities</td>
</tr>
<tr>
<td>$i$</td>
<td>Activity</td>
</tr>
<tr>
<td>$r$</td>
<td>Index of robustness</td>
</tr>
<tr>
<td>$p_i^{\text{min}}$</td>
<td>Minimum processing time of activity $i$</td>
</tr>
<tr>
<td>$u_{ij}[0,1]$</td>
<td>Preference value associated with every activity-gate combination</td>
</tr>
<tr>
<td>$w_i[0,1]$</td>
<td>Priority weight associated with every activity</td>
</tr>
<tr>
<td>$C_i$</td>
<td>Completion time of activity $i$</td>
</tr>
<tr>
<td>$S_i$</td>
<td>Starting time of activity $i$</td>
</tr>
<tr>
<td>$t_i^a$</td>
<td>Fixed arrival time of activity $i$</td>
</tr>
<tr>
<td>$t_i^d$</td>
<td>Fixed departure time of activity $i$</td>
</tr>
<tr>
<td>$M_i$</td>
<td>Gate that activity $i$ is assigned to</td>
</tr>
<tr>
<td>$M'_i$</td>
<td>Reference gate of activity $i$</td>
</tr>
<tr>
<td>$M_i$</td>
<td>Set of gates associated to activity $i$</td>
</tr>
<tr>
<td>$\mathcal{M}$</td>
<td>Set of all gates</td>
</tr>
<tr>
<td>$V$</td>
<td>Set of all activities</td>
</tr>
<tr>
<td>$V^a$</td>
<td>Set of arrival activities</td>
</tr>
<tr>
<td>$V^p$</td>
<td>Set of parking activities</td>
</tr>
<tr>
<td>$V^d$</td>
<td>Set of departure activities</td>
</tr>
</tbody>
</table>

Fig. 2. Assignment for gate $M^1$.

**Minimal processing time**

$$S_i + p_{i}^{\text{min}} \leq C_i \quad \forall i \in V$$

**Continuous processing**

$$C_i + d_{M_i,M_j}^{\text{low}} = S_j \quad \forall (i,j) \in e_{\text{low}}$$

**Disjunctive activities and set-up times**

For any activities $i,j \in V$ such that either $M_i = M_j \neq M^0$ or $\exists (i,M_i,j,M_j) \in e_{\text{shadow}}$ one of the following condition must be fulfilled

$$C_i + d_{M_i,M_j}^{\text{setup}} \leq S_j, \quad \text{activity } i \text{ must precede } j$$

or

$$C_j + d_{M_i,M_j}^{\text{setup}} \leq S_i, \quad \text{activity } j \text{ must precede } i$$

**Start and completion time**

$$S_i = t_i^a \quad \forall i \in V^a$$

$$C_i = t_i^d \quad \forall i \in V^d$$

$$S_i, C_i \in \mathbb{N}_0 \quad \forall i \in V$$

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Gate selection

\[ M_i \in M_i \quad \forall i \in V. \]

The objective function is a linear combination of several goals:

1. the maximization of the total assignment preference score;
2. the minimization of the number of required towing operations; and
3. the minimization of the deviation from a given reference gate schedule.

Using goal weights \( \alpha_i \), which are non-negative real numbers, the objective function \( z(M) \) is constructed as follows:

\[
z(M) := \min \alpha_1 z_1 + \alpha_2 z_2 + \alpha_3 z_3
\]

where

\[
z_1 := -\sum_{i \in V} w_i u_i M_i,
\]

\[
z_2 := \left| (i, j) \in \text{low} : M_i \neq M_j \right|,
\]

\[
z_3 := \sum_{i \in V : M_i \neq M_i^\prime} w_i.
\]

Typically, a \( u_{ij} \in [0, 1] \) is a preference value associated with every activity–gate combination, \( w_i \in [0, 1] \) is a priority weight associated with every activity and \( M_i^\prime \) denotes the reference gate of activity \( i \). It is obvious that the choice of appropriate preference weights and priorities as well as the ordering of the partial goals by importance using parameters \( \alpha_1, \alpha_2, \alpha_3 \) may have a substantial impact on the optimal gate schedule.

The basic optimization algorithm is a truncated branch and bound procedure (see Dorndorf et al., 2000b). The algorithm proceeds by assigning gates to the activities and by resolving resource conflicts that might appear. In comparison with a standard branch and bound procedure, it has several distinctive features. First of all, it uses two different types of branching:

1. branching over flight gates by assigning the best gate to some unscheduled activity according to some rule and accepting or forbidding this assignment afterwards,
2. branching over disjunctive constraints by resolving resource conflicts and defining which activity from the set of already scheduled ones (an activity is considered to be scheduled if it has a gate assignment) must be the predecessor.

The second feature of the proposed method is that it uses constraint propagation techniques (Dorndorf, Pesch and Phan Huy 2000a). This means that at each node of the binary search tree induced by the branching scheme constraint propagation techniques are applied to reduce the search space until a fixed point has been computed.

For dealing with large instances arising in practice (which have a huge number of aircraft activities and airport gates), the branch and bound procedure was upgraded.
Disruption management in flight gate scheduling

by combining it with additional problem decomposition (variable partitioning) techniques. Additionally, large neighborhood search techniques have been implemented. Computational experiments with large real-life data as well as with manually constructed small examples demonstrate the effectiveness of the proposed technique especially in comparison with the results of a modern rule-based decision support system.

5.1 Model 1

An approach we propose to incorporate robustness into flight gate scheduling is to provide fault tolerant recovery paths which ensure the schedule to be flexible to slight disruptions. In other words, a flight gate schedule is said to be robust if it provides enough flexibility that parts of the schedule can be recovered in the event of irregularities.

A commonly used recovery path in the airline industry is resources switching, which means switching resources between certain activities in the case of disruption to prevent the disruption from propagating to the following activities. For example, in the event that an aircraft fails to depart on time, resource switching allows the arriving aircraft, which is originally assigned to the same gate as the delayed flight, to be reassigned to another gate with little impact on other activities.

Some robustness-related concepts need to be introduced first: a gate $M^n(n \neq 0)$ is available in $[t_1, t_2]$ if no activity is assigned to it in the time interval.

Figure 3 shows a part of the gate schedule in an airport. One can observe that gate $M^1$ is available in the time interval $[15,16]$ and $[18,19]$. A gate $M^n(n \neq 0)$ is virtually available in $[t_1, t_2]$ if only a sequence of activities $i \in V_p$ is assigned to it in the time interval.

Again, in Figure 3 we can see that gate $M^1$ is virtually available in $[16,18]$ because only a parking activity 3 is processed in this interval. Similarly, $M^1$ is virtually available in $[20,22]$.

A schedule is considered robust at activity $i \in V^a \cup V^d$ if there exists at least one available or virtually available gate $M^n \in \mathcal{M}_i \setminus \{M^0, M_j\}$ in the time interval $[S_i, C_i]$. From Figure 4, which is another illustration of a flight gate schedule, we can find the time interval between the start and completion time of activity 1 ($[12,13.5]$) to be totally covered by the time interval of parking activity 6 ($[12,14]$) which is assigned to gate $M^2$. Hence, the schedule is robust at activity 1. Similarly, we can also find some other activities in the schedule where the schedule is robust, such as 4, 8, and 9.

<table>
<thead>
<tr>
<th>$M^1$</th>
<th>Activity 1 (arrival)</th>
<th>Activity 2 (departure)</th>
<th>Activity 3 (parking)</th>
<th>Activity 4 (arrival)</th>
<th>Activity 5 (parking)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td></td>
<td>18</td>
<td>19</td>
<td>20</td>
<td>21</td>
</tr>
</tbody>
</table>

Fig. 3. A part of a flight gate schedule.

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A schedule is considered absolutely robust if for every activity $i \in V^a \cup V^d$, there exists an available or virtually available gate $M^a \in M_i \backslash \{M^0, M_i\}$ in the time interval $[S_i, C_i]$.

Generally speaking, the robustness of a flight gate schedule could be measured through counting such activities in the schedule. In other words, we can increase the robustness of a flight gate schedule by providing more available or virtually available gates to all the arrival and departure activities, so that activities could be reassigned to those gates if needed.

It is quite reasonable that an available gate would provide more possibilities for an activity being ultimately assigned to the preferable gate. As for the virtually available gate, we assume that parking activities are associated with lower priority weight $w_i$ than arrival and departure activities, therefore could be interrupted (i.e. be towed away to other gates or apron) without any major impact on the outcome. It will be understandable under this assumption that virtually available gates are also favorable to increase the robustness of a flight gate schedule.

More specifically, our goal is to minimize $|A| - |R|$, where the set of arrival and departure activities is $A$ and $R$ is the set of activities at which a schedule is robust.

A precise way to measure robustness is to compute the percentage of robust activities in the schedule, i.e.

$$r := \frac{\sum_{i \in R} w_i}{\sum_{i \in A} w_i} \in [0, 1]$$

which we will refer to as the index of robustness. Obviously, the higher $r$, the more robust a schedule is.

Returning to Figure 4, the set of arrival and departure activities consists of activities 1, 2, 4, 7, 8 and 9. Among them, activities 1, 4, 8 and 9 are considered as robust activities. Table 2 shows the priority weight associated with every activity. Therefore, the index of robustness is

$$r = \frac{0.8 + 0.8 + 0.8 + 0.8}{0.8 + 0.7 + 0.8 + 0.7 + 0.8 + 0.8} \approx 0.7$$

As already mentioned above, our goal is to increase the robustness of flight gate schedules by providing more available or virtually available gates to arrival and departure activities. To realize it, we can incorporate the measure of robustness into the original flight gate scheduling model.
Table 2. Priority weight associated with every activity.

<table>
<thead>
<tr>
<th>i</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_i )</td>
<td>0.8</td>
<td>0.7</td>
<td>0.2</td>
<td>0.8</td>
<td>0.1</td>
<td>0.2</td>
<td>0.7</td>
<td>0.8</td>
<td>0.8</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Our way to account for robustness while generating a flight gate schedule is by incorporating the index of robustness \( r \) into the objective function used in the original model. \( r \) can be introduced into the objective function \( z(M) \) described above with a certain weight \( \alpha_4 \) indicating its importance. Hence, the objective function of the modified model is constructed as follows:

\[
z(M) := \min z_1 z_2 + z_3 z_3 + \alpha_4 z_4,
\]

where

\[
z_4 := \frac{\sum_{i \in R} w_i}{\sum_{i \in A} w_i}.
\]

and \( z_1 \) is the total assignment preference score, \( z_2 \) and \( z_3 \) denote the number of required towing operations and the deviation from a given reference gate schedule respectively.

It is interesting to point out that we can still apply a truncated branch and bound procedure (Dorndorf et al., 2000b) to search for the optimal schedule. Moreover, by modifying the objective function, we are more likely to find the robust flight gate schedule.

5.2 Model 2

Uncertainty often occurs in flight gate scheduling. Apart from stochastic approaches, which are often too extensive, fuzzy approaches are adequate to model uncertainty. First, we present a general way of introducing fuzzy logistics into scheduling problems as it is described by Hapke, Jaszkiewicz and Słowiński (1998). Furthermore, we describe simple modifications of the above model which integrate a robustness measure using fuzzy sets.

A general introduction into fuzzy sets can be found by Zimmermann (1987). Fuzzy robust approaches have been presented in the literature by Wang (2002) and Anglani et al. (2003). If the flight gate scheduling problem is modelled as a combinatorial optimization problem the adoption of fuzzy logic for achieving robustness is helpful. Hapke, Jaszkiewicz and Słowiński (2000) present a Pareto simulated annealing algorithm for solving the fuzzy multiobjective combinatorial optimization problem.

In classical set theory, the membership of elements in relation to a set is assessed in binary terms according to a crisp condition – an element either belongs or does not belong to the set. By contrast, fuzzy set theory permits the gradual assessment of the membership of elements in relation to a set. This is described with the aid of a membership function \( \gamma_A : X \rightarrow [0, 1] \), with \( A \subseteq X \). \( \gamma_A \) assigns to every element
Corresponding are at first monotonic increasing and from some point monotonic decreasing. The of membership. Let us assume that membership functions considered in this paper are at first monotonic increasing and from some point monotonic decreasing. The corresponding fuzzy set to the membership function \( \gamma_A \) of \( A \subset X \) is defined by

\[ A := \{(x, \gamma_A(x))|x \in X\}. \]

Hapke et al. (1998) present a solution procedure for a multiobjective, multimode project scheduling problem under multicategory resource constraints. They assume that all the time parameters of activities are uncertain and modelled by fuzzy numbers. Therefore, the objectives with time cost criteria (e.g. project makespan, and maximum lateness) are also uncertain; so, their values are fuzzy numbers as well. In the above model this would imply that the start times \( S_i = t_i^\prime \) of arrival activities and the completion times \( C_i = t_i \) of departure activities are modeled as fuzzy numbers. In this case \( \gamma_S(x) = 1 \) means that \( x \) is the scheduled time for \( S_i \) which is most likely to appear. \( \gamma_S(x) = 0.9 \), for example, indicates that \( S_i = x \) appears with high probability. Having uncertain start and completion times implies that the restrictions and the objectives become to some extend uncertain. Especially, a comparison rule to decide whether \( C_i > S_j \), \( C_i = S_j \) or \( C_i < S_j \) is needed. If such a comparison rule decides \( C_i < S_j \) we are furthermore interested to what extent \( C_i < S_j \) holds. This would give us a measure of robustness for an assignment of \( i \) and \( j \) to the same gate. Hapke et al. (1998) present two comparison rules.

The weak comparison rule (WCR) can be described as follows: two fuzzy sets \( A := \{(x, \gamma_A(x))|x \in X\} \) and \( B := \{(x, \gamma_B(x))|x \in X\} \) are to be compared. Let \( A^* \) and \( B^* \) be possible realizations of \( A \) and \( B \). The degree to which \( A \geq B \) is calculated as the sum of areas in which \( A^* \geq B^* \) is indicated minus the sum of areas in which \( A^* \leq B^* \) is indicated. To describe these areas, we take advantage of the variable \( x' := \max\{\min\{\gamma_A(x), \gamma_B(x)\}\}\).

In other words, \( x' \) describes the intersection of \( \gamma_A \) and \( \gamma_B \) at which one membership function has, and one has not yet reached its maximum. The aforesaid areas are the ones which are encased by the membership functions \( \gamma_A \) and \( \gamma_B \). The areas left of \( x' \) indicate \( A^* \geq B^* \) if \( \gamma_B > \gamma_A \) and they indicate \( A^* \leq B^* \) if \( \gamma_B < \gamma_A \). The areas right of \( x' \) indicate \( A^* \geq B^* \) if \( \gamma_B < \gamma_A \) and they indicate \( A^* \leq B^* \) if \( \gamma_B > \gamma_A \). See for example Figure 5. Area I is bounded at the bottom by \( \gamma_A \) and at the top by \( \gamma_B \). So at this place the realization possibility of \( B \) is higher. Area I is left of \( x' \) so it describes the realization possibility for a low \( x \). Therefore \( A^* \geq B^* \) can be indicated at this place. Same is true for area IV while II, III and V indicate \( B^* \geq A^* \). Because in this case, the areas II, III and V are bigger than I and IV, \( B > A \) can be concluded. Furthermore, we can measure the degree to what \( B > A \) by the surface area of II, III and V minus those of I and IV.

The WCR allows to decide for every pair \((A, B)\) of fuzzy sets whether \( A > B, A \sim B \) or \( A < B \). It is especially suited to compare fuzzy sets of the objective function. However, it is not suitable for scheduling restrictions, as it might lead to infeasible
Disruption management in flight gate scheduling

Fig. 5. The comparison of two fuzzy numbers.

schedules. In Figure 5, \( B > A \), but as the areas I and IV show, in some cases \( A^* \geq B^* \). This leads to the necessity of a second comparison rule.

According to the strong comparison rule (SCR) \( A \) dominates \( B \) if \( A > B \) according to the WCR and there are no areas where \( B^* \geq A^* \) is possible. The application of this comparison rule for the restrictions leads to a feasible schedule for every realization of fuzzy time parameters.

Hapke et al. (1998) solve the uncertain multiobjective, multimode project scheduling problem in two stages: first, they use Pareto simulated annealing to find an approximate representation of the efficient solutions. In contrast to the crisp case, in which Pareto efficient solutions are searched, they regard WCR-non-dominated solutions as efficient. Secondly, the WCR-non-dominated solutions are searched to find a schedule that yields the best compromise between conflicting fuzzy objectives. This is done by an interactive procedure called LBS-discrete (light beam search) by Jaszkiewicz and Słowiński (1997) adapted to solve decision problems with fuzzy criteria.

In the following, we will describe how a fuzzy approach can be integrated into our model. The arrival times \( t^a_i, i \in V^a \) and the departure times \( t^d_i, i \in V^d \) will no longer be regarded as fixed times but as fuzzy numbers. For example, \( \gamma_{t^a_i}(x) \) describes the possibility that \( t^a_i = x \). \( S_i \) and \( C_i \) therefore turn out to be fuzzy numbers too. Instead of forcing \( S_i + p^\text{min}_i \leq C_i \) the minimal processing time restrictions have to be modified so that \( C_i \) has to dominate \( S_i + p^\text{min}_i \) according to the strong comparison rule. Same holds for the disjunctive activities restrictions in which \( S_j \) has to dominate \( C_i + d^\text{set-up}_{M_i/M_j} \) or \( S_i \) has to dominate \( C_j + d^\text{set-up}_{M_i/M_j} \) according to the strong comparison rule respectively. The degree to what the dominance is fulfilled, which can be calculate by the weak comparison rule, is an indication for robustness. The average degree of dominance could therefore be chosen as a further objective. Note that the present objectives are not dependent on the starting and completion times. They can be calculated without uncertainty.

A more simple idea of using fuzzy sets for our model is the following in which we regard \( t^a_i \) and \( t^d_i \) as fixed values. Disruptions are often more critical if constraints
are tight; e.g. especially for the minimal processing time constraints and for the precedence constraints in the above model. For instance, if the processing time of a schedule is close to the minimum processing time $p_{\text{min}}^i$, this schedule may not be able to absorb delays without undertaking some recovery actions, which indicates that this schedule does not have much robustness. We may therefore measure robustness by means of the slack of these constraints. The question of how much slack is sufficient for a satisfactory robustness will be answered using fuzzy sets.

First of all, let us introduce three different kinds of slack variables $sl^k (k = 1, 2, 3)$:

1. For every processing time constraint
   \[ S_i + p_{\text{min}}^i \leq C_i \quad \forall i \in V \]
   the corresponding slack is defined by
   \[ sl_1^i := C_i - S_i - p_{\text{min}}^i \quad \forall i \in V. \]

2. For any precedence constraint, i.e. for any activities $i, j \in V$ such that either $M_i = M_j \neq M^0$ or $\exists (i, M_i, j, M_j) \in \varepsilon_{\text{shadow}}$ the corresponding slack is defined by
   \[ sl_2^{(i, j)} := \max \left\{ S_j - C_i - d_{\text{set-up}}^{i, j, M_i, M_j}, S_i - C_j - d_{\text{set-up}}^{i, j, M_i, M_j} \right\} \quad \forall (i, j) \in \{(i, j) \in V \times V | M_i = M_j \neq M^0 \lor (i, M_i, j, M_j) \in \varepsilon_{\text{shadow}}\}. \]

3. Furthermore, if many activities are assigned to a certain gate in a schedule, which means disruptions of one activity may be more liable to propagate to the following activities which are also assigned to that gate, little robustness is contained in this kind of schedules. Therefore, we define the negated number of activities assigned to a gate as the ‘slack’ of a gate:
   \[ sl_3^M := -|\{i \in V | M_i = M\}| \quad \forall M \in \mathcal{M}\{M^0\} \]
   In case of a feasible solution, we can constitute $sl^1, sl^2 \geq 0$ and $sl^3 \leq 0$. To achieve robustness a preferably big slack is favored in all three cases. We will now define a membership function which describes for every slack to what degree it can be considered to having sufficient slack.

To simplify matters, we regard the membership functions $\gamma^k : \mathbb{R} \rightarrow [0, 1]$ of $sl^k$ as continuous, partial linear functions. Let $\underline{h}^k \in \mathbb{R}$ be the biggest value for which $\gamma^k (sl^k) = 0$ holds and let $\overline{h}^k \in \mathbb{R}$ be the smallest value for which $\gamma^k (sl^k) = 1$ holds. Let us define
   \[ \gamma^k(sl^k) := \begin{cases} 0, & \text{if } sl^k \leq \underline{h}^k, \\ 1, & \text{if } sl^k \geq \overline{h}^k, \\ \frac{sl^k - \underline{h}^k}{\overline{h}^k - \underline{h}^k}, & \text{if } \underline{h}^k < sl^k < \overline{h}^k, \end{cases} \quad k = 1, 2, 3 \]
   as shown in Figure 6. Of course, this is a strongly simplified description of the membership function but it conforms to illustrate the procedure.

Maximizing the (weighted) sum of all grades of membership can now be regarded as a robustness objective which can easily be integrated into the existing objective function $z(M)$. The new objective is constructed as follows:
Fig. 6. Membership function describing to what degree slack $s^k$ is sufficient to achieve robustness.

\[
    z(M) := \min \sum_{i=1}^{6} x_i z_i,
\]

where

\[
    z_4 := -\sum_{i \in V} \gamma^1(s_i^1)
\]

\[
    z_5 := -\sum_{\{(i,j) \in V \times V | M_i = M_j \neq M_0 \vee (i, M_i, j, M_j) \in \text{shadow}\}} \gamma^2(s_{(i,j)}^2)
\]

\[
    z_6 := \sum_{M \in M \setminus \{M^0\}} \gamma^3(s_M^3)
\]

with appropriate weights $x_4, x_5, x_6$.

If we consider two slack variables, usually the state in which both variables have the same size is preferred to the state in which one variable is bigger but the sum remains the same. This leads to the assumption that $\gamma^k$ should be chosen as a concave curve within the interval $[0, 1]$ instead of being linear. Furthermore, there does not need to exist an $s^k$ with $\gamma^k(s^k) = 1$.

6 Conclusion and future work

This paper proposes two methods to incorporate robustness into a flight gate assignment problem. To accomplish the goal, the general flight gate assignment problem is reviewed first as well as the disruptions that often occur in operation of flight gate assignment.

Disruptions are main factors preventing airline to operate as planned and costs involved in deviation from original plan are often very large. Therefore disruption management is introduced to airlines to deal with disruptions. Disruption management is defined in this paper as the efforts both to avoid and recover from the disruptions, which are strategies of robust scheduling and recovery.
Recovery strategy is taken to maximize the profits and come back to the original plan as soon as possible after disruptions occur. And the survey for recovery strategies is focused on ARP for it receives the most significant attention among the recovery strategies. The way to solve ARP such as the delaying, swapping, and canceling is applied to flight gate recovery problem as well as the analysis for decision costs. Although recovery strategy can save much for airlines, it is still a high-cost process, while the robustness is a more active strategy taken in planning process with the goal to avoid the disruptions. Our emphasis is on two approaches of robust scheduling. The first is to draw out plans with higher degree of overlap ensuring some substitutes at certain points, and the second uses a fuzzy membership function to penalize the schedule which is liable to cause disruptions. Some other robust methods have been applied to the GAP and the relative research is reviewed.

Then the two robust approaches, namely the overlap method and the fuzzy set approach, are introduced into a flight gate assignment model. The overlap method is applied to the gate assignment model for measuring the number of available flight gates for arrival and departure activities, more available gates at the same time indicating more robust schedule, and the index of robustness defined as the percentage of robustness is incorporated into the new objective. While in the second model, fuzzy membership functions for \( p_i, p_{ij}, \) and \( N_M \) are designed and added to the original objective to give penalties when relevant attributes approach the limitations, which ensures more robust schedule will be obtained. No algorithm is proposed in the two models.

It will be significant to draw out more robust schedules as flight gates are scarce and expensive resources, while not much researches have been reported on robust flight gate assignment schedule as yet. Therefore, more robust methods are to be investigated to get more feasible and effective schedules.

Some algorithms have been developed for GAP while little has been for robust GAP, which is an essential and interesting research direction to enable the robust scheduling feasible for airline daily operation.

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Disruption management in flight gate scheduling


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