Two-part tariffs in the online gaming industry: The role of creative destruction and network externalities

Kieron Meagher *, Ernie G.S. Teo

School of Economics, University of South Wales, Sydney, NSW 2052, Australia

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Abstract

Playing computer games online is a fast growing, billion dollar industry which has received little academic attention. The industry exhibits a number of interesting economic features. The industry structure is determined by creative destruction as in Aghion and Howitt (1992) [Aghion, P., Howitt, P. 1992. A model of growth through creative destruction, Econometrica, 60(2), 323–351], with game makers experiencing market power within a genre until the game is superceded. Furthermore, the attractiveness of playing a game online depends on the existence of opponents (positive network externalities) while technical and reputational problems eventually arise (negative network externalities). We model the choice of two-part tariffs by a monopolist under creative destruction and network externalities and derive conditions for the multiple equilibria which currently exist in the industry.

JEL classification: C70; D21; D42; L11; L12; L86

Keywords: Computer games; Internet; Two-part tariff; Creative destruction; Network externalities

* Corresponding author. Tel.: +61 2 9385 1145; fax: +61 2 9313 6337.
E-mail address: k.meagher@unsw.edu.au (K. Meagher).

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1. Introduction

Multiplayer online games (MPOGs) are a form of computer games that can be played online by many players. The player can interact and play the games together with other people. MPOGs can refer to any genre of games ranging from simple card games or arcade style games to graphically intensive role playing games. Every time a player goes online to play the game, the firm needs to provide server space to allow the player to play. MPOG firms also need to provide continuous support for their customers. There has been increased interest in MPOGs by major computer game companies in recent years. The number of major MPOGs available on the market has more than doubled in the past three years (Woodcock, 2003).

MPOGs are gaining mainstream popularity; more consumers are getting into these games. The market that was once dominated by hardcore gamers is now evolving to include a wider range of consumers. Reviews of online games that used to appear only in hardcore gamer magazines, such as the PC Gamer, now appear in mainstream magazines. For example, the recent launch of The Sims Online was featured in magazines such as Time, USA Today and Entertainment Weekly.

Results (IDSA, 2002, 2003) from annual surveys$^1$ in the US showed that the percentage, of most frequent game players$^2$ who played online games, had increased from 18% in 1999 to 24% in 2000, 31% in 2001 and 37% in 2002.

Industry reports (DFC Intelligence: 1: 2002; 1: 2003) forecasted that the worldwide online game market would grow from $875 million in 2002 to over $5 billion in 2008. It was also forecasted that the usage of online games would reach 35 billion hours by 2008, an increment of 450% from 2002.

It can be observed that firms in the MPOG industry use different pricing strategies. Some firms use a combination of access fees and subscription fees while others only charge one of them. It can be seen that these firms do not all follow the rule from Oi (1971) where access fees are used to fully extract consumer surplus and subscription fees are set to marginal costs. Table 1 illustrates this.

What makes MPOG firms different? One characteristics of MPOG is that there are strong network externalities involved. The utility derived by a game player largely depends on the number of other people playing the same game online. Since the server capacity hosted by a MPOG firm is limited, large increases in network inevitably lead to congestion [see Reiter (1999) for more on congestion and overcrowding in networks].

Another interesting characteristic is the rate of creative destruction. As the computer games industry is a fast evolving one, games are replaced or consumers get bored with a game relatively fast. This is an important aspect as the lifespan of the product depends on the rate of creative destruction [see Aghion and Howitt (1992) and Schumpeter (1942) for more on creative destruction].

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$^1$ Conducted by gathering data from more than 1500 nationally representative households in the US that have been identified as owning either a video game console or a personal computer used to run entertainment software.

$^2$ Most frequent players are defined as a person who plays PC games for more than 5 hours a week.
The main aim of this paper is to realistically analyze the two-part pricing model, which will result in the firm being able to use different combinations of prices. For this purpose, we incorporate the effects of network externalities and creative destruction into the two-part pricing monopoly model. The main insight from this model is that the firm will be able to fully extract consumer surplus as long as they follow a parametric and inverse relationship between access fees and subscription prices. This allows subscription prices to be low when access fees are high and vice versa.

There had been much research on two-part pricing and their usage in the field of industrial organization. Oi (1971) was the first to suggest that monopolies should use it as a form of price discrimination to fully extract surpluses from consumers. Oi found that by using a discriminatory two-part tariff, where per unit price equals marginal costs and all consumers’ surpluses are extracted using a lump sum tariff (or an access fee), the monopolist could make the most profits.

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The monopoly model is used for simplicity’s sake. It is also reasonable to assume that most MPOG firms have some kind of monopoly power, as games are usually unique from each other.
As observed in real life, two-part prices are not exclusive to monopolies. Recent research on two-part prices and competition include Harrison and Kline (2001), where it is found that the ability of firms to extract consumer surplus with an access fee decreases as the number of firms in competition increases. This means that firms that face competition could still use two-part pricing to their advantage if the market is not too competitive. In Crèmer (1984), heterogeneous consumers are assumed and results similar to Oi were found.

In Hoerger (1993), a model of two-part prices with experience goods is presented. Hoerger looks at two-part pricing for experience goods in a monopoly scenario. Experience goods are goods whereby the consumer will have to try the good before he knows the quality. Quality differs for individual consumers. Hoerger’s results are that the high-quality firm would charge new customers the same or a lower price than a low quality firm would charge. It would then charge repeat customers a price that would yield zero repeat sales if it were a low quality firm. By charging a high repeat price, the firm is able to signal high quality and earn positive profits for repeat purchases. Hoerger showed that in presence of experience goods, a monopoly would be able to make profits through subscription fees, which signals quality at the same time. This is in contrast with the results from Oi (1971) where a monopolist uses access fees to extract consumer surpluses and charges marginal costs for unit prices. The difference in results might be due to the inclusion of experience goods; access fees are driven down in order to get consumers to try the good, which leaves subscription fees as a signal of quality.

Two-part prices are sometimes necessary due to the nature of the good. These goods are usually used periodically by consumers and each use would require some form of resources that the firm provides. Some examples of goods and services that are used periodically are telephone services, country clubs, gyms and Internet services. Through observation, not all firms offering goods that exhibit these qualities will use access fees to extract consumer surplus. One such example is the MPOGs industry.

2. The model

2.1. Utility

Due to network externalities and congestion effects, the per period utility from playing the game online is first increasing with the number of other consumers and then decreasing after a point of congestion. To represent this we use quadratic function formulation. The representative consumer’s utility from playing the game when there are \( N \) online players in total is

\[
u(N) = \frac{\alpha_1 N - \frac{\alpha_2}{k} N^2}{},\]

where \( \alpha_1 \) represents the intensity of the positive network externality, that is utility derived from having a pool of opponents. \( \alpha_2 \) represents the intensity of congestion.
(the negative network externality). \(k\) represents the capacity choice of the firm and is made prior to the startup of the business. We initially take \(k\) as fixed and exogenous for the life of the games before considering the firm’s investment choice in Proposition 2.

\(u(N)\) is a quadratic function as shown in Fig. 1. In reality \(N\) must be a positive integer and under our specification the first consumer would receive strictly positive utility, \(u(1) > 0\), so no one buying is not an equilibrium. For technical convenience we shall ignore integer problems and assume \(N > 0\) is real. Moving away from \(N = 0\), \(u(N)\) is increasing until the point of congestion, where it starts to decrease.

An important characteristic of the computer game industry is creative destruction. The firm with the leading game in a genre experiences significant market power, but only until such time as the game is superceded. Following Aghion and Howitt (1992), we model replacement as a probabilistic process so that the number of periods that the game will last in the market is random. This randomness is represented by \(q\), which is the probability of the consumer continuing to play the game in the next period. By assuming that \(q\) is fixed and constant across time, we are also assuming that consumers are homogeneous and have the same preferences. Let \(d\) be common discount rate, then as is common in infinite period games for notational conciseness we define \(\delta = d \times q\), as the pseudo-discount factor.

Assume that the firm charges two prices, an access fee (\(A\)) for period zero’s purchases and a subscription fee (\(p\)) for per period use in each subsequent period. Let utility be quasi-linear in income. By taking expected values, indirect utility can be written as

\[\text{4 We assume that the firm cannot change its capacity choice (k) after the investment stage and that consumers are homogeneous. N becomes constant across time. Consumers would buy until the next consumer would get negative utility and N ends up at where V(N,A,p) = 0, or when the market is exhausted and reaches a limit. This assumption holds, as the rational consumer would buy the game if he can gain positive utility from buying it.}\]
\[ V(N,A,p) = E \left[ U(N) - A - \sum_{t=1}^{\infty} \delta^t p \right] = \frac{[x_1 N - \frac{x_2}{k} N^2]}{(1-\delta)} - \left[ A + \frac{\delta p}{(1-\delta)} \right], \]  

where \( U(N) \) is the sum of \( u(N) \) over infinite periods given a fixed \( N \).

2.2. Profit

Assuming a constant marginal cost of \( c \), and normalizing the fixed cost to zero, the firm’s profit function, \( \Pi(N,A,p) \), is given by

\[ \Pi(N,A,p) = [N(A-c)] + \frac{\delta N(p-c)}{(1-\delta)} = N \left[ (A + \frac{\delta p}{(1-\delta)}) - \frac{c}{(1-\delta)} \right]. \]  

Note that the utility function we have represented is for a monopoly scenario, the firm is isolated in its market niche and consumers can choose to either buy or not buy the game. Consumers are also assumed to be homogeneous and have the same utility functions. Since consumers are making a discrete choice, we have discontinuous demand at the individual level. The number of consumers buying determines the industry demand function. The utility from the outside option of not playing the game is normalized to zero. When \( V(N,A,p) \geq 0 \), \( N \) consumers would be willing to buy. When \( V(N,A,p) < 0 \), no consumer buys. Since firms are risk-neutral, they would not enter the market when \( V(N,A,p) < 0 \), for all levels of \( N \). Therefore, this paper will only consider the case when \( V(N,A,p) \geq 0 \).

**Lemma 1.** Given that the firm is allowed to choose the number of consumers they want to sell to, they would choose the maximum possible as they are profit maximizing.

If the firm can restrict the number of units sold, it will choose an \( N \), which gives the most profits. Since individual consumer utility is quadratic, firms want to extract all possible consumers surplus by making \( N \) the maximum \( N \), i.e., the right intercept of the quadratic utility function. We can solve for the maximum \( N \) by equating \( V(N,A,p) \) to zero, and finding the greater root

\[ N^{\text{max}}(A,p) = \frac{z_1 k + \sqrt{z_1^2 k^2 + 4 z_2 k ((\delta - 1) A - \delta p)}}{2 z_2}. \]  

The firm maximizes profits by choosing prices given that consumer utility is zero and at the right intercept of the quadratic utility function. In the model, the firm is allowed to choose quantity and prices, subject to the industry demand function.

**Proposition 1.** When the firm chooses \( A \) and \( p \) (for given capacity \( k \)), the following conditions hold:

(i) In order for the monopolist to make positive profits and hence be in operation, marginal costs have to satisfy
\( c < \frac{x_1^2 k}{4x_2}, \) \hspace{1cm} (5)

(ii) Given (i), for each \( A^* \in \left[ 0, \frac{x_1(x_1k + G) + 3x_2c}{9x_2(1-\delta)} \right] \) there is an equilibrium two-part tariff and quota \((A^*, p^*(A), N^*)\) where

\[ G = \sqrt{x_1^2 k^2 - 3x_2 kc}, \] \hspace{1cm} (6)

\[ p^*(A) = \frac{x_1(x_1k + G) + x_2(9\delta A^* - 9A^* + 3c)}{9x_2\delta}, \] \hspace{1cm} (7)

and

\[ N^* = \frac{x_1k + G}{3x_2}. \] \hspace{1cm} (8)

Each of the multiple equilibria yield the same expected profits

\[ II^* = \frac{x_1^2(2kG + 2x_1k^2) - x_2(9x_1kc + 6cG)}{27x_2^2(1-\delta)}. \] \hspace{1cm} (9)

**Proof.** See Appendix A.

The multiple equilibria, we find above differs from the standard finding of uniqueness in other two-part pricing models such as Oi (1971). In those papers, second period prices are found to be marginal cost. In the model shown here, the second period price is not necessarily equal to marginal cost. This parametric set of prices can also explain why firms in the MPOG industry choose different combinations of access fees and subscription prices. Firms choose different combinations of \( A \) and \( p \) as they are on different points of the parametric equation. For example, firms can choose to charge a high subscription fee, which will result in a low access fee if they set prices according to the parametric equation.

If we look at the effects of the externalities on \( p^*, A^*, N^* \) and \( \pi^* \), we find the following:

\[ \frac{\partial p^*}{\partial x_1} > 0, \quad \frac{\partial A^*}{\partial x_1} > 0, \quad \frac{\partial N^*}{\partial x_1} > 0, \quad \frac{\partial\pi^*}{\partial x_1} > 0, \quad \frac{\partial c^*}{\partial x_2} < 0, \quad \frac{\partial c}{\partial x_2} < 0, \quad \frac{\partial N^*}{\partial x_2} < 0, \quad \frac{\partial x^*}{\partial x_2} < 0. \]

This shows that an increase in positive externalities enables the firm to charge higher prices, reach more consumers and attain higher profits. Likewise, the firm can do it by reducing the negative externalities attached to their product instead. This could be things like making the game more enjoyable, reducing the likelihood of congestions and upgrading software to manage large groups better.

**2.3. Capacity**

The capacity choice \( (k) \), the firm makes, has an impact on the prices that the firm will charge and profits that will result. It is interesting to look at these effects. When we look at the derivatives of optimal profits and prices, we find that when \( k \) increases, profits increases and subscription price increases. The upper limit on access fees decreases with \( k \).
\[
\frac{\partial \Pi^*}{\partial k} = \frac{9c \alpha_2 \left( x_2 c - x_1 \sqrt{x_1^2 k^2 - 3 x_2 kc} \right) - x_1^2 k \left( 15 x_2 c - 4 x_1^2 k - 4 x_1 \sqrt{x_1^2 k^2 - 3 x_2 kc} \right)}{27 x_1^2 (\delta - 1) \sqrt{x_1^2 k^2 - 3 x_2 kc}} > 0, 
\]

(10)

\[
\frac{\partial p^*}{\partial k} = \frac{x_1 \left( 2 x_1 \left( x_1 k + \sqrt{x_1^2 k^2 - 3 x_2 kc} \right) + 3 x_2 c \right)}{18 \delta x_2 \sqrt{x_1^2 k^2 - 3 x_2 kc}} > 0. 
\]

(11)

\[
A^* \text{ increases with } k \text{ if set on its upper bound } A \in \left[ 0, \frac{x_1 (x_1 k + \sqrt{x_1^2 k^2 - 3 x_2 kc}) + 3 x_2 c}{9 x_2 (1 - d q)} \right] 
\]

\[
\frac{\partial A}{\partial k} = \frac{x_1 \left( 2 x_1 \left( \sqrt{x_1^2 k^2 - 3 x_2 kc} - x_1 k \right) + 3 x_2 c \right)}{18 \delta x_2 (1 - \delta) \sqrt{x_1^2 k^2 - 3 x_2 kc}} > 0. 
\]

(12)

The fact that ex post profits increases with the capacity choice of the firm \((k)\) is interesting; this is because there are no costs attached to \(k\). Costs of \(k\) are sunk in this stage. Since the costs of \(k\) are sunk, the bigger the \(k\) gets, the more people the firm is able to sell to. Therefore, firms have to make the profit maximizing choice of \(k\) given their budget constraints in the investment stage.

2.4. The investment stage

Comparative statistics on \(k\) suggest that we can take a step back and look at the investment stage where the firm decides on the capacity choice of \(k\). In this stage, the costs of \(k\) will be considered; this is variable with \(k\). \(k\) is fixed after the investment stage.

**Proposition 2.** Assuming a capacity cost function of \(C(k)\) in the investment stage and that the marginal costs of increasing \(k\) will decrease as \(k\) increases, there would exist a unique \(k^*\) that will maximize profits if \(\frac{\partial^2 C}{\partial k^2} > \frac{\partial^2 \Pi}{\partial k^2} > 0\).

**Proof.** When we assume a cost function for \(k\) in the investment stage, total profits can be written as, \(T(k) = \Pi^*(k) - C(k)\). The firm will maximize total profits by choosing \(k\). Since \(\Pi^*(k)\) and \(C(k)\) are separable, the first order conditions of the problem will be solved where marginal benefits of increasing \(k\) are equal to the marginal costs of increasing \(k\), \(\frac{\partial \Pi^*}{\partial k} = \frac{\partial C}{\partial k}\).

For there to be a unique maximum, the second-order derivative has to be negative. \(\frac{\partial^2 T(k)}{\partial k^2} < 0\). As \(\Pi^*(k)\) and \(C(k)\) are separable, \(\frac{\partial^2 T(k)}{\partial k^2} = \frac{\partial^2 \Pi^*}{\partial k^2} - \frac{\partial^2 C}{\partial k^2}\). Therefore, \(\frac{\partial^2 \Pi^*}{\partial k^2} < \frac{\partial^2 C}{\partial k^2}\) needs to be satisfied for there to be a \(k^*\) that would maximize total profits \((T)\). The second-order derivative of profits on \(k\) is \(\frac{\partial^2 \Pi^*}{\partial k^2} = \frac{x_1^2 (8 x_1^2 k + 8 x_1 \sqrt{x_1^2 k^2 - 3 x_2 kc} - 12 x_2 c) - 9 x_2^2 c^2}{54 x_1^2 (1 - \delta) \sqrt{x_1^2 k^2 - 3 x_2 kc}} > 0\). Therefore, there will only be a \(k^*\) that maximizes total profits when \(\frac{\partial^2 \Pi^*}{\partial k^2} > \frac{\partial^2 C}{\partial k^2} > 0\). \(\square\)
From our results, we find that $\frac{\delta^2 C}{\delta k}$ (the rate that marginal costs decrease with $k$) is a determinant of the existence of $k^*$. A factor that would affect the rate that costs are decreasing is technology. As technology improves, costs per consumer per period could significantly decrease at a much faster rate as the amount of capital investment increases. Therefore, as technology improves the firm will become less restricted by the size of capital investment.

2.5. Welfare

Since consumers face negative utility when $N$ gets large, one would assume that if prices and $N$ were restricted from the monopoly level, total social welfare might increase. Since there are positive externalities attached to $N$, having a $N$ larger than the monopoly level might also improve social welfare. As we prove in the next section, this is not true in the monopoly case.

Proposition 3. The firm’s profit maximizing choices are welfare maximizing.

Proof. The total welfare (surplus) in the market is given by, $W = \text{consumer surplus} + \text{producer surplus}$.

$$W(N, A, p) = N(V(N, A, P)) + II(N, A, p),$$

$$W(N, A, p) = N^* \left\{ \left( x_1 N - \frac{x_2}{k} N^2 - A \right) + \frac{\delta[x_1 N - \frac{x_2}{k} N^2 - p]}{(1 - \delta)} \right\}$$

$$+ \left\{ [N(A - c)] + \frac{\delta[N(p - c)]}{(1 - \delta)} \right\}$$

$$= \frac{N[Nx_1 k - N^2 x_2 - c]}{k(1 - \delta)} = W(N).$$

Note that prices have cancelled out from the equation, as prices are simply transferred from consumers to producers. This means that total surplus will only depend on $N$. Therefore, the social welfare planner will maximize total surplus by

$$\max_N W(N) = \frac{N[Nx_1 k - N^2 x_2 - c]}{k(1 - \delta)}.$$  

By solving the first order conditions for $N$, we find that there are two solutions for $N$

$$N_{W1} = \frac{x_1 k + \sqrt{x_1^2 k^2 - 3x_2 kc}}{3x_2} \quad \text{and} \quad N_{W2} = \frac{x_1 k - \sqrt{x_1^2 k^2 - 3x_2 kc}}{3x_2}.$$  

These $N$s are the same as $N_1$ and $N_2$ in Proposition 1. Since $N_{W1} = N^*$, we need to prove that the social planner will choose $N_{W1}$ over $N_{W2}$ for Proposition 3 to be true. The second-order derivatives are
By looking at the second-order derivatives, we can conclude that \( N_{W1} \) maximizes \( W(N) \) and \( N_{W2} \) minimizes \( W(N) \). Therefore, the social planner will choose \( N_{W1} \), which is the same as \( N^* \).

The profit maximizing \( N \) is also the welfare maximizing one. This is true because this is a monopoly model with inelastic individual demand. The monopolist is able to extract all surpluses in the market via the use of two-part prices. Therefore, to restrict prices or quantity would only result in dead weight losses that would reduce overall welfare. The social planner would choose not to intervene in this case and allow the monopolist to choose prices and quantity.

3. Discussion

If consumers do not infer anything from the prices (combination of \( A \) and \( p \)) set by the firm and they do not see the prices as a signal of the good’s quality, we can infer the firm’s strategy when there is asymmetric information. If the firm expects \( q \) to be different from consumer’s expectations, the firm can strategize and get more profits. The results of the basic model indicate that as long as \( A^* \) is kept within its boundaries, profits can be maximized. This gives the firm flexibility to charge the level of prices by using a level of \( A^* \) that suits their needs.

If the firm knows that their game is of high quality (the game is good enough to remain attractive to players for a long period) and that consumers are not expecting such quality, the firm might choose to set a low access fee. For example, the firm might choose to charge marginal costs for it. When \( A^* = c \), the corresponding subscription fee would be

\[
p = \frac{1}{9} \frac{a_2(9c\delta - 6c) + a_1^2k + \sqrt{a_1^2k + (a_1^2k - 3a_2c)}}{a_2}\delta.
\]  

(16)

The firm does so because, if their expectations were right, consumers would play for more periods than expected. Since consumers have to pay \( p \) for all subsequent periods, making \( A \) low would make \( p \) correspondingly higher and give the firm a higher level of profits.

Conversely, if the firm knows that the game that they have is not as good as consumers expect (and consumers will stop playing it earlier than they expected), it might choose to extract all the surplus in the first period with access fees and set \( p^* = c \). This results in
\[ A = \frac{1}{9} \frac{\alpha_2(-3c + 9c\delta) - \alpha_2^2k - \sqrt{\alpha_2^2k + (\alpha_2^2k - 3\alpha_2c)}}{\alpha_2(-1 + \delta)}. \]  

(17)

Since the firm had extracted all the surplus, given consumers’ expectations, in the first period, profits will no longer be affected if consumers change their expectations and decide to play for less periods.

4. Conclusion

The model presented can help explain why firms in the same industry, differ in their pricing structures. Certain firms choose to charge both a one-time access fee and a periodic subscription fee. Some firms only charge either one. We found that there is a parametric and inverse relationship between access fees and subscription prices. The reason why firms choose different combinations of \( A \) and \( p \) are simply because they are maximizing profits but are on different points of the parametric solution. The model had also found a limit to marginal costs that would restrict entry. Subsequent period prices are not always set to marginal costs. This result is novel and represents a departure from Oi’s model and other two-part pricing models. It is also consistent with observations from the MPOG industry as seen from the figures shown in Table 1. Firms in the industry sometimes choose to charge a low \( A \) that could be zero in combination with some \( p \) or vice versa. The results from this paper indicate that this could be because the firms faces network externalities and creative destruction.

We also concluded that when deciding what capacity investment \( (k) \) to set in the investment stage, the rate that marginal costs \((\text{of } k)\) decrease with \( k \) would determine if there is an optimal level of \( k \). If it is decreasing at a faster rate than marginal profits are increasing with \( k \), there would exist \( k^* \) that would maximize total profits. Otherwise, firms would choose as big a \( k \) as possible.

Further analysis also found that the profit maximizing quantity is also the social welfare maximizing quantity. This is interesting as it indicates that a monopoly is welfare maximizing as well as profit maximizing when there are congestion effects with two-part prices.

A model that looks at network externalities and congestion effects with a two-part tariff such as the one presented in this paper had not been looked at before. Therefore, the results from this paper are significant not only because it applies to a large and growing industry, but also because it contributes to the economic analysis of two-part prices as a whole.

The case where the firm does not charge consumers for playing their games is not covered by this paper and involves advertising revenue from website visits. The model used in this paper had not considered competition and heterogeneous consumers; future analysis that includes these characteristics would provide a more realistic representation of the two-part pricing model.
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Appendix A

Proof of Proposition 1. When the firm chooses $N$, it is restricting the number of consumers by limiting the number of units sold, that is the firm can ration $N$. The firm’s profit maximization problem is

$$\max_{N,A,p} \pi(N,A,p) = N \left( A + \frac{\delta p}{(1-\delta)} \right) - \frac{c}{(1-\delta)}$$

subject to

$$V(N,A,p) = \left[ \frac{a_1 N - \frac{a_2 N^2}{k}}{(1-\delta)} \right] - \left[ A + \frac{\delta p}{(1-\delta)} \right] \geq 0. \quad (18)$$

From Lemma 1, the firm will choose the highest possible $N$ that will keep consumer utility positive. Therefore, the utility constraint becomes an equality and

$$V(N,A,p) = \left[ \frac{a_1 N - \frac{a_2 N^2}{k}}{(1-\delta)} \right] - \left[ A + \frac{\delta p}{(1-\delta)} \right] = 0. \quad (19)$$

Rearrange the utility constraint as

$$\frac{[a_1 N - \frac{a_2 N^2}{k}]}{(1-\delta)} = [A + \frac{\delta p}{(1-\delta)}]$$

and substitute into the profit function, the problem becomes

$$\max_{N,A,p} \pi(N,A,p) = N \left[ \frac{(a_1 N - \frac{a_2 N^2}{k}) - c}{(1-\delta)} \right] = \pi(N). \quad (20)$$

The only variable that remains in the profit function after the substitution is $N$. Solving for the maximum, we get:

$$\frac{\partial \pi}{\partial N} = \frac{2a_1 N - \frac{3a_2 N^2}{k} - c}{(1-\delta)} = 0 \Rightarrow (N_1, N_2)$$

$$= \left( \frac{a_1 k + \sqrt{a_1^2 k^2 - 3a_2 c k}}{3a_2}, \frac{a_1 k - \sqrt{a_1^2 k^2 - 3a_2 c k}}{3a_2} \right), \quad (21)$$

$$\frac{\partial^2 \pi}{\partial N^2} = \frac{2a_1 - \frac{6a_2 N}{k}}{(1-\delta)}, \quad (22)$$

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Note: Positive constraints on prices are not included in the problem as prices can be bounded (without loss of generality) with a limit that will ensure positiveness in the solution. See later parts of this section for proof.
\[ \frac{\partial^2 \pi}{\partial N_1^2} = \left( -\frac{2\sqrt{\alpha_1^2 k^2 - 3\alpha_2 c k}}{k(1 - \delta)} \right), \quad \frac{\partial^2 \pi}{\partial N_2^2} = \left( \frac{2\sqrt{\alpha_1^2 k^2 - 3\alpha_2 c k}}{k(1 - \delta)} \right), \] (23)

where \( N_2 \) can be eliminated, as \( \frac{\partial^2 \pi}{\partial N_2^2} \) is positive which means it is a minimum point. Therefore the profit maximizing \( N, N^* = N_1 \) as \( \frac{\partial^2 \pi}{\partial N_1^2} \) is negative.

The bounds on \( A \) \((\in (\overline{A}, \underline{A})\)) can be found by using the conditions that \( A \) and \( p \) are positive. By substituting in \( N^* \) into the price and profit functions, we find that

\[ A \in \left[ 0, \frac{\alpha_1 \left( \alpha_1 k + \sqrt{\alpha_1^2 k^2 - 3\alpha_2 c k} \right) + 3\alpha_2 c}{9\alpha_2(1 - \delta)} \right], \] (24)

\[ p^*(A) = \frac{\alpha_1 \left( \alpha_1 k + \sqrt{\alpha_1^2 k^2 - 3\alpha_2 c k} \right) + \alpha_2(9\delta A - 9A + 3c)}{9\alpha_2 \delta}, \] (25)

\[ \Pi^* = \frac{\alpha_1^2 \left( 2k \sqrt{\alpha_1^2 k^2 - 3\alpha_2 c k} + 2\alpha_1 k^2 \right) - \alpha_2 \left( 9\alpha_1 k c + 6c \sqrt{\alpha_1^2 k^2 - 3\alpha_2 c k} \right)}{27\alpha_2^2(1 - \delta)}. \] (26)

When we set \( \Pi^* > 0 \) and rearrange it, we find that \( c \) would have to satisfy, \( c < \frac{\alpha_1^2}{4\alpha_2} \) to give positive profits. Therefore, if \( c \) exceeds the above value the firm should not entered the market, as it would be making negative profits. \( \square \)

The same set of results can also be found by assuming \( V(N,A,p) \geq 0 \) and using Kuhn Tucker methods. The firm is allowed to freely choose \( N \) and it is found that the firm will still choose the maximum \( N \).

References


