A fast-converging space-time adaptive processing algorithm for non-Gaussian clutter suppression

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1. Introduction

Space-time adaptive processing (STAP) refers to combined spatial beamforming and temporal filtering of radar/sonar returns in phased array systems. It uses multiple antenna elements followed by tapped-delay-lines to coherently process multiple pulses, thus providing superior ability to suppress jammers and clutters while preserving desired signal target [1]. Since its introduction, STAP has been rigorously researched and has been shown to provide significant performance gains in interference suppression and target detection [2]. Many STAP algorithms deal with common scenarios where clutters and noises are complex Gaussian, which leads to mathematically tractable solutions [1–3]. However, recent studies and field measurements have found [4–10] that heavy-tailed non-Gaussian clutters often occur in backscatters from mountain tops, dense forest canopy, rough sea surfaces, and manmade concrete objects, etc. These radar clutters are spiky, impulsive in nature and can cause significant performance degradation in STAP and target detection.

Several statistical models have been used to describe the impulsive non-Gaussian clutter environment including the compound complex Gaussian [4], the generalized complex Gaussian [5], and the complex alpha-stable [10,11]. The compound complex Gaussian model is often used in practice [4], where the clutter/noise process is modeled as the product of two random processes: $X = \sqrt{\tau} \cdot G$, with $\tau$ being the texture and $G$ the speckle. If $\tau$ follows the gamma distribution and $G$ the...
Fig. 1. The envelop probability density function (pdf) of the compound K clutters, in comparison to complex Gaussian clutters whose envelop is Rayleigh (a special case of compound K with $\nu = \infty$). The compound K distributions exhibit much heavier tails than Rayleigh clutter.

Fig. 2. The auto-covariance functions (ACF) of Rayleigh, gamma texture, and compound K clutters. A larger $\lambda$ indicates that the ACF decays slower.

complex Gaussian, then the envelop of $X$, defined as the magnitude $R = |X|$, will be the compound K distribution with the probability density function (pdf) given as [4,5]

$$f_R(r) = \frac{2}{\sigma \Gamma(\nu)} \left( \frac{r}{2\sigma} \right)^\nu K_{\nu-1} \left( \frac{r}{2\sigma} \right)$$

where $\nu$ is the shape parameter, $K_\nu()$ is the modified Bessel Function of the second kind of order $\nu$, and $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$ is the gamma function. The pdfs of the compound K distribution with different shape parameters are plotted in Fig. 1, where all envelop distributions have a normalized second moment $E(R^2) = 2$, and the simulated compound K clutters are generated by the nonlinear memoryless transform (NMLT) method [8,9]. The tails of the compound K pdfs are much higher than the Rayleigh distribution which is the envelope pdf of the complex Gaussian process.

Besides the heavy-tailed distribution, non-Gaussian clutters also exhibit different auto-covariance properties. The Rayleigh distributed speckle component, modeling the short-term fluctuation of the scatters, has a very fast-decaying auto-covariance function (ACF) spanning only a few slow-time samples. The gamma distributed texture component, on the other hand, represents modulations of the local power of the clutters and often has a slowly-decaying ACF. The ACF of the gamma texture is commonly modeled as $R_G(t) = \exp(-t/\lambda)$, where $\lambda$ is on the order of several hundred samples, as shown in Fig. 2 in comparison with the ACFs of the Rayleigh and compound K clutters.

For Gaussian clutter suppression, the minimum variance distortionless response (MVDR) method is commonly employed to minimize the output power with constraints at target location and Doppler frequency. In practice, the sample matrix inversion (SMI) or normalized least mean square (NLMS) algorithm is often used for iterative adaptation. In contrast, a fractionally lower-order moments (FLOM) adaptive algorithm and its normalized version (NFLOM) have been proposed in [10]
and [18] for suppressing heavy-tailed non-Gaussian clutters. The FLOM and NFLOM algorithms differ from the MVAR and NLMS schemes in that they minimize the $p$-th order moment ($0 < p \leq 2$) of the output signal rather than its variance ($p = 2$), thus reducing the detrimental effects of spiky clutter samples.

The FLOM algorithm, also called the mean $p$-norm (LMP) algorithm in the adaptive filtering literature [13,14], has been shown to be effective in many non-Gaussian interference environments. It has been applied to seismic system identification, synthetic aperture radar (SAR), magnetoencephalography, and medical ultrasonic imaging, etc., where Bernoulli–Gaussian models [15,16] or symmetrical alpha-stable models [13,14,17] are often used for impulsive interference with an algebraic tail. It has been shown that the FLOM algorithm performs significantly better than the conventional LMS (2nd-order norm) algorithm. The effectiveness of the FLOM algorithm in less heavy-tailed compound K models remains unclear. Furthermore, the majority of the existing results focus on the deviation of the adaptive weights from the system impulse response, which is of the primary concern in system identification type of applications [13–16]. The output mean square error (MSE) and signal-to-interference-and-noise-ratio (SINR) are largely ignored, which are more important measures in STAP or other phased array applications [18–20].

In this paper, we first evaluate the NFLOM algorithm for STAP applications where compound K clutters are present simultaneously with high-power, wideband jammers. We measure the performances in terms of excess MSE, misalignment (of weights), beampatterns, and output SINR. We also investigate the effect of the auto-covariance function (ACF) of compound K clutters on these performances. Our results show several new aspects of the NFLOM algorithm:

1. For the NFLOM with a smaller order $p$ close to 1, the misalignment exhibits faster initial convergence as well as lower steady-state error than that of a larger-$p$ NFLOM. This result is in agreement with the ones reported for system identification applications in heavy-tailed interference [21].
2. The excess MSE of a small-$p$ NFLOM converges much faster than that of a larger order NFLOM, but the steady-state error is slightly higher too, resulting in slightly lower average SINR in the converged STAP output. This result is true for both Gaussian and non-Gaussian clutters.
3. The ACF of the compound K clutter has strong impact on the convergence of the large-$p$ NFLOM algorithms. For misalignment, a higher ACF leads to faster initial convergence but quicker slowing down than a lower ACF, resulting in a higher steady-state error. The high ACF result has not been reported elsewhere, even for system identification applications. For the excess MSE, a lower ACF slows the convergence speed significantly than a higher ACF, and the converged steady-state error is also higher, especially for large-$p$ NFLOM algorithms.
4. The ACF of the compound K clutter has less impact on the performances of the NFLOM with $p = 1$ than on a larger-$p$ NFLOM. With $p = 1$ (i.e., the normalized sign algorithm (NSA)), the steady-state error of the excess MSE remains almost the same for different ACF and different impulsiveness (shape parameters). This indicates strong robustness of the NSA algorithm against changing, impulsive clutters.
5. The beampatterns of the NFLOM and NLMS algorithms differ significantly only at time instants when very large, spiky clutter samples occur. The NLMS algorithm uses more degrees of freedom to track the impulsive clutter samples and leaks more power of jammers and noise to the output. The NFLOM ($p < 2$) algorithm places less emphasis on the spiky clutter components and achieves slightly better SINR. However, the spiky clutter components are suppressed less severely and may require a nonlinear preprocessor before matched filter detection [18].

Based on the observation that the NFLOM algorithm with a smaller order $p$ converges faster but to a lower steady-state MSE than that of a large-order $p$, we propose a new variable-order (VO) FLOM algorithm to solve the conflicting goals of fast convergence and low steady-state error. The new VO-FLOM algorithm uses a small order $p$ at the beginning of the adaptation and gradually increases the order to a large $p$ to achieve both fast convergence and low steady-state MSE. The proposed algorithm is also evaluated in both complex Gaussian and compound K clutter scenarios. The results show that the new VO-FLOM algorithm outperforms the plain NFLOM and NLMS algorithms in both MSE and misalignment.

2. Conventional STAP and its NLMS algorithm

Consider an arbitrary radar array antenna consisting of $M$ elements with the $m$-th element located at $\Theta_m$ in a spherical coordinate system. Coherent bursts of $K$ pulses are transmitted at a constant pulse repetition frequency (PRF) $f_r = 1/T_r$, where $T_r$ is the pulse repetition interval (PRI). Radar returns are collected over a coherent processing interval (CPI) of length $K T_r$. Within each PRI, there are $L$ time (range) samples collected to cover the range interval. This multidimensional data set can be visualized as an $M \times K \times L$ cube of complex samples [1]. For STAP performed in the space-slow-time domain, denote the received samples at range bin $l$ as $u_{k,m}(t)$ with slow-time index $k = 1, 2, \ldots, K$, and array element index $m = 1, 2, \ldots, M$, and the sampling time index $t$. Let $N = M \times K$, then the $N \times 1$ concatenated space-time sample vector is

$$U(t) = \left[ u_{1,1}^T(t), \ldots, u_{1,K}^T(t), \ldots, u_{N,1}^T(t) \right]^T,$$

$$u_{k,m}(t) = \left[ u_{k,1}(t), u_{k,2}(t), \ldots, u_{k,M}(t) \right]^T,$$
The radar return vector \( \mathbf{U}(t) \) is a mixture of the target echo (\( \mathbf{U}_s \)) with the uncorrelated jammer (\( \mathbf{U}_j \)), uncorrelated clutters (\( \mathbf{U}_c \)), and background noise (\( \mathbf{U}_n \)):
\[
\mathbf{U}(t) = \mathbf{U}_s(t) + \mathbf{U}_j(t) + \mathbf{U}_c(t) + \mathbf{U}_n(t),
\]
where
\[
\begin{align*}
\mathbf{U}_s(t) &= S(t) \mathbf{b}(\omega_s) \otimes \mathbf{a}(\Theta_s), \\
\mathbf{U}_j(t) &= \sum_{i=1}^{N_j} S_{ji} \mathbf{g}_{ji} \otimes \mathbf{a}(\Theta_{ji}), \\
\mathbf{U}_c(t) &= \sum_{i=1}^{N_c} S_{ci} \mathbf{b}(\omega_{ci}) \otimes \mathbf{a}(\Theta_{ci}),
\end{align*}
\]
where the point source \( S(t) \) is located at the spherical coordinate \( \Theta_s = (r_s, \theta_s, \phi_s) \) with \( r_s, \theta_s, \) and \( \phi_s \) denoting the radial distance, azimuth angle, and elevation angle, respectively. The normalized angular Doppler frequency is \( \omega_s = 2\pi f_s/f_r \). The operator \( \otimes \) denotes the Kronecker matrix product. The spatial and temporal steering vectors are, respectively,
\[
\begin{align*}
\mathbf{a}(\Theta_s) &= [1, e^{-j\Omega(t_{12}-\tau_{12})}, \ldots, e^{-j\Omega(t_{1N}-\tau_{1N})}]^T, \\
\mathbf{b}(\omega_{ci}) &= [1, \ldots, e^{-j\omega_{ci}t}, \ldots, e^{-j(K-1)\omega_{ci}}]^T,
\end{align*}
\]
where \( \Omega \) is the operating frequency of the phased array, and \( \tau_{ms} = |\Theta_m - \Theta_s|/c \) is the propagation delay from the signal source to the \( m \)-th array element with \( c \) being the wave propagation speed.

The \( N_j \) jammers \( S_{ji} \) are at locations \( \Theta_{ji} \) with gains \( \mathbf{g}_{ji} = [g_{ji}(1), \ldots, g_{ji}(k), \ldots, g_{ji}(K)]^T \). The \( N_c \) independent clutter patches are uniformly distributed in a circular ring/sphere around the radar platform [1] with the \( i \)-th patch at \( \Theta_{ci} \) and having a Doppler frequency \( \omega_{ci} \) proportional to its angular location. The receiver noise \( \mathbf{U}_n \) appears as a uniform noise floor throughout the angle-Doppler plane.

The STAP system consists of a tapped-delay-line attached to each array element. Let \( \mathbf{W} \) be the concatenated weight vector of the STAP processor, then the output of the STAP \( y(t) \) can be expressed in a matrix form as
\[
y(t) = \mathbf{W}^H \mathbf{U}(t),
\]
where the superscript \( (\cdot)^H \) denotes conjugate transpose (or Hermitian transpose).

For the Gaussian clutter environment, the minimum variance distortionless response (MVDR) method is commonly used for adapting the weight vector \( \mathbf{W} \). That is to minimize the second-order moment of the output signal subject to steering constraints
\[
\min_{\mathbf{W}} E[|y(t)|^2], \quad \text{subject to} \quad \mathbf{C}^H \mathbf{W} = \mathbf{h},
\]
where \( E[\cdot] \) is the expectation operator, \( E[|y(t)|^2] = \mathbf{W}^H \mathbf{R}_{uu} \mathbf{W} \), and \( \mathbf{R}_{uu} \) is the covariance matrix of the concatenated input vector \( \mathbf{U} \). The matrix \( \mathbf{C} \) is a set of linear constraints and \( \mathbf{h} \) is the desired response vector. For example, a simple point constraint [18] may be chosen as \( \mathbf{C} = \mathbf{b}(\omega_s) \otimes \mathbf{a}(\Theta_s) \) and \( \mathbf{h} = \mathbf{1} \), which enforces a unit gain response at the target location \( \Theta_s \) and the Doppler frequency \( f_s \). The optimal solution to the constrained minimization problem (7) is well-known assuming that the covariance matrix \( \mathbf{R}_{uu} \) has a full rank [3]:
\[
\mathbf{W}_{\text{opt}} = \mathbf{R}_{uu}^{-1} \mathbf{C}^H (\mathbf{C}^H \mathbf{R}_{uu}^{-1} \mathbf{C})^{-1} \mathbf{h}. \tag{8}
\]

Direct implementation of (8) requires the knowledge of the covariance matrix of the array input vector and the sample matrix inversion (SMI) method is often employed in practice [2]. Alternatively, the weight vector \( \mathbf{W}_{\text{opt}} \) can be decomposed into two orthogonal components: a fixed beamformer \( \mathbf{W}_q \) and an unconstrained adaptive weight vector \( \mathbf{W}_q \). They are determined by
\[
\begin{align*}
\mathbf{W}_q &= \mathbf{C}(\mathbf{C}^H \mathbf{C})^{-1} \mathbf{h}, \\
\mathbf{W}_{\text{opt}} &= (\mathbf{C}^H \mathbf{R}_{uu} \mathbf{C})^{-1} \mathbf{C}^H \mathbf{R}_{uu} \mathbf{W}_q,
\end{align*}
\]
where \( \mathbf{C}_q \) is termed the signal blocking matrix. It is orthogonal to \( \mathbf{C} \) satisfying \( \mathbf{C}^H \mathbf{C}_q = \mathbf{0} \). This decomposition, as shown in Fig. 3, is known as the generalized sidelobe canceller (GSC) and \( \mathbf{W}_q \) can be iteratively adapted by the normalized least mean square (NLMS) algorithm as
\[
\mathbf{W}_q(t + 1) = \mathbf{W}_q(t) + \mu_q e(t) x^*(t) x(t) + \delta,
\]
where \( x(t) = \mathbf{C}_q^H \mathbf{U}(t) \), the error signal is defined by \( e(t) = y(t) - [\mathbf{W}_q - \mathbf{C}_q \mathbf{W}_q]^H \mathbf{U}(t) \). The step size \( \mu_q \) controls the rate of convergence and the regularization parameter \( \delta \) prevents the numerical instability when the inputs are small [3].
Equivalently, the NLMS algorithm in (11) is the same as

$$W(0) = W_q = C(C^H C)^{-1} h,$$

$$W(t+1) = B \left[ W(t) - \mu \frac{y^*(t)U(t)}{U^H(t)BU(t) + \delta} \right] + W_q,$$

where the superscript $*$ denotes the conjugate, $\mu$ is the step size and

$$B = I - C(C^H C)^{-1} C^H.$$  

(13)

3. The proposed NFLOM algorithms for STAP

In the severe, impulsive clutter environment, the conventional STAP algorithm suffers from performance loss due to two reasons: one is the high probability of outliers in the received samples; another is the large eigenvalue spread of the sample covariance matrix. An approach to combat these problems is the FLOM algorithm which minimizes the $p$-th order moment rather than the variance of the STAP output [10,18]

$$\min_{W} E\left[ |y(t)|^p \right], \text{subject to } C^H W = h.$$  

(14)

There is no closed-form solution for the optimal coefficients that minimizes the cost function, but a gradient descent method is available. Similar to the NLMS algorithm, the NFLOM algorithm is iteratively adaptive as

$$W_a(0) = 0,$$

$$W_a(t+1) = W_a(t) + \mu_a \frac{|e(t)|^{p-2} e^*(t)x(t)}{\sum_i |x_i(t)|^{p} + \delta},$$

or equivalently

$$W(0) = W_q = C(C^H C)^{-1} h,$$

$$W(t+1) = B \left[ W(t) - \mu \frac{|y(t)|^{p-2} y^*(t)U(t)}{\sum_i |x_i(t)|^{p} + \delta} \right] + W_q$$  

(16)

where $x_i(t)$ are the elements of the blocking matrix output $x(t) = C_a^H U(t)$. Other parameters are the same as those in the NLMS algorithm (12).

The NFLOM algorithm reduces to the NLMS algorithm when $p = 2$ and to the normalized sign algorithm (NSA) when $p = 1$ [11,12]. Our numerical analysis has found [18,20] that, when the order $p$ is smaller, the NFLOM algorithm converges faster but exhibits larger steady-state mean square errors (MSE). This phenomenon is observed in both Gaussian and heavy-tailed clutters, as will be shown in Section 4.

The conflicting goals of fast convergence and low steady-state error of the NFLOM algorithms motivates a new variable-order NFLOM algorithm to achieve both fast convergence and low steady-state MSE by varying the order $p$. Intuitively, the variable order NFLOM algorithm shall start with a small order, for example $p = 1$, and then gradually increases to $p = 2$. A straightforward approach for order switching is to estimate the excess MSE of the NFLOM algorithm in windows of size $D$ and then compare the MSE to the previous window. If the difference exceeds a threshold, the order $p$ is increased. The proposed variable-order NFLOM follows the procedures:
1. Choose $P = \{P_l\} = \{P_{\text{min}} : \Delta P : P_{\text{max}}\}$. Set $l = 1$ and the initial order as $p = P_l$;
2. Select the estimation window size $D$ and the threshold $T_h$; Set the output energy of the previous window $E_0 = D P_U$,
   where $P_U$ is the total power of the input signal $U(t)$;
3. Adapt the filter coefficients $W(t)$ based on (16) using the current order $p$. Estimate the output energy of the current window as $E_1 = \sum_{i=1}^{D} |y(i)|^2$;
4. Compare $E_0$ to $E_1$. If $E_1 - E_0 > D T_p^2$, then increment $l$ and update the order $p$ to $P_l$;
5. Set $E_0 = E_1$ and repeat Steps 3–4 until $p = P_{\text{max}}$.

The parameter selection of the algorithm determines the convergence rate and the steady-state MSE. The threshold $T_h$ can be set at the 1% to 10% of the signal-to-noise-ratio (SNR) or clutter-to-noise-ratio (CNR) level. The window size is normally chosen at several hundred to several thousand samples. The selection of $P = \{P_l\} = \{P_{\text{min}} : \Delta P : P_{\text{max}}\}$ is rather flexible with $P_{\text{min}} \geq 1$ and $P_{\text{max}} = 2$ for complex Gaussian clutters. For heavy-tailed clutters, slightly smaller $P_{\text{min}}$ and $P_{\text{max}}$ normally provide better results.

### 4. Performance analysis

A linear phased array example was used to demonstrate the performances of the NFLOM and the VO-FLOM algorithms. The array consisted of $M = 10$ equally spaced elements at half wavelength of the operation frequency. The coherent pulse interval (CPI) was $K = 7$ and a fixed range bin was used for the STAP. The target signal had a power of 0 dB with respect to the background noise and its angle of arrival (AoA) was $20^\circ$ with respect to the axis of the array. The normalized Doppler frequency of the target was fixed at 0.25. The noises were independent among antenna elements and CPI taps with white Gaussian spectrum. Two wideband jammers presented at AoA of $-20^\circ$ and $+50^\circ$, respectively. Both jammers had a full Doppler spectrum and a total power of 30 dB. In addition, many clutters impinged on the array from different AoAs which were uniformly distributed between $-180^\circ$ and $180^\circ$. The Doppler frequencies of the clutters depended on their AoAs. The envelop of the clutters was either Rayleigh (complex Gaussian clutter) or compound K with a total average power of 30 dB.

![Fig. 4. Beampatterns of the NLMS and NFLOM algorithms in compound K clutters ($\nu = 0.5, \lambda = 100$) when impulsive clutter samples were encountered. In comparison, the beampattern of the MVDR scheme was computed by the weight vector optimized over all clutter samples. (a) MVDR: placed deep nulls at both jammer locations and at the clutter ridge therefore passing the target signal with high SINR; (b) NLMS: used many degrees of freedom on suppressing the impulsive clutter components but let the jammers and other clutter components leak through; (c) and (d) NFLOM with $p = 1.5$ and $p = 1.7$: maintained deep nulls at jammer locations by placing less emphasis on the impulsive clutter components, thus achieving better output SINR.](image-url)
The compound K clutters were simulated using the nonlinear memoryless transformation (NMLT) method \[9,22\] and the auto-covariance function (ACF) of the gamma texture was \( R(t) = \exp(-t/\lambda) \) with a large \( \lambda \) indicating high ACF. For performance comparison, the fixed order NFLOM used a step size \( \mu = 0.002 \) and the regulation parameter \( \delta = 20 \text{CNR} \).

The STAP algorithms can be evaluated by the output beampattern defined as

\[ \Psi(\Theta, f_0) = |W_{\text{opt}}^H b(f_0) \otimes a(\Theta)|^2. \]  

(17)

The convergence performance is commonly evaluated by the excess MSE \( J_{\text{ex}}(t) \) and misalignment \( M(t) \) defined as \[3\]

\[ J_{\text{ex}}(t) = E\left[|W^H(t)U(t)|^2\right] - J_{\text{min}}, \]  

(18)

\[ M(t) = 20 \log_{10} \frac{|W(t) - W_{\text{opt}}|}{|W_{\text{opt}}|}. \]  

(19)

and \( J_{\text{min}} = E(|W_{\text{opt}}^H U(t)|^2) \). The MVDR optimal solutions were used as the common base for comparison for all clutter scenarios, although the NFLOM algorithms are designed to minimize the lower order moments. This means that, if measured

\[ \text{Fig. 5. The convergence curves of the NFLOM algorithms in complex Gaussian clutters and compound K clutters } (\nu = 0.7, \lambda = 100). \text{ Two wideband jammers and background noises were also present. The total powers of clutters and jammers were 30 dB above the background noise, respectively. Ensemble average of 100 trials is used in all curves. (a) MSE in Gaussian clutter: a smaller order NFLOM converges faster, but to a larger steady-state error. (b) Misalignment in Gaussian clutter: a smaller } p \text{ NFLOM converges faster and achieves lower error norm. (c) MSE in K clutters: orders close to 1 have the similar convergence as those in Gaussian clutter, exhibiting robustness against impulsive clutters; orders close to 2 have the similar initial convergence speed as those in Gaussian clutters, but higher steady-state errors. (d) Misalignment in K clutters: similar to those in Gaussian clutters but with slightly higher error norms.} \]
in terms of $L_p$ norms or $p$-th error moments, the NFLOM performance would be better than these second-order performance measures.

4.1. Performances of the NFLOM algorithm

The beampatterns of the MVDR, NLMS, and NFLOM schemes are plotted in Fig. 4, which were obtained in impulsive K clutters with shape parameter $\nu = 0.5$ and an ACF function $R_G(t) = \exp(-t/\lambda)$ with $\lambda = 100$. The beampattern of the MVDR scheme was computed by the weight vector optimized over all clutter samples thus providing the best performance with deep nulls placed at both jammer locations and at the clutter ridge. It passed the target signal with unit gain and achieved high SINR. The beampatterns of the NLMS and NFLOM were computed by their weights $W(t)$ when a large spiky clutter component was present at $t$. The NLMS used many degrees of freedom on suppressing the impulsive clutter components, but let the jammers and other clutter components leak through. The NFLOM with $p = 1.5$ and $p = 1.7$ both maintained deep nulls at jammer locations by placing less emphasis on the impulsive clutter components, thus achieving better output SINR than the NLMS algorithm.

The convergence curves of the NFLOM algorithms in complex Gaussian clutters and compound K clutters are plotted in Fig. 5, where the MSE and misalignment curves are the ensemble average over 100 independent trials. In Gaussian clutters, the MSE curves (Fig. 5(a)) show that a smaller order $p$ NFLOM converges faster, but to a larger steady-state error. The

![Fig. 6. Effects of ACF of K clutter ($\nu = 0.7$) on mean square error of the NFLOM algorithm. The ACF of the gamma texture is $R_G(t) = \exp(-t/\lambda)$ with $\lambda = 10, 100, \text{ and } 300$, respectively. The MSE of the NFLOM converges faster and to a lower steady-state error in clutters with higher auto-covariance (larger $\lambda$) for all orders of $p$. The effects of ACF on the convergence of the NFLOM with $p = 1$ is the smallest. Note the change of $x$-axis scale in different sub-figures.](image-url)
misalignment curves (Fig. 5(b)) show that a smaller order $p$ NFLOM converges faster and achieves lower error norm. For compound K clutters, the MSE curves (Fig. 5(c)) show that, when $p$ is close to 1, the curves are very similar to those in Gaussian clutters, exhibiting high robustness against impulsive clutters; when $p$ is close to 2, the initial convergence speed is similar to those in Gaussian clutters, but the steady-state errors are higher. The misalignment curves (Fig. 5(b)) behave similar to those in Gaussian clutters but with slightly higher error norms.

It is found that the ACF of the texture component of compound K clutters has significant impacts on the convergence of the NFLOM with $p > 1$. The MSE and misalignment curves in K clutters with three ACF parameters $\lambda = 10, 100,$ and 300 are compared in Figs. 6 and 7, respectively. In terms of MSE, the NFLOM converges faster and to a lower steady-state error in clutters with higher auto-covariance (larger $\lambda$) for all orders, but the effects of ACF on $p = 1$ is the smallest. In terms of misalignment, a larger $\lambda$ leads to a very fast initial convergence but slows down significantly afterwards. A small $\lambda$ leads to a smooth convergence and to a lower steady-state error. A smaller $p$ NFLOM has a lower steady-state error of misalignment than a larger $p$. Effects of the ACF on the performance of the NFLOM with $p = 1.5$ is between those of $p = 1.25$ and $p = 1.75$, and the curves are omitted here.

The output SINR was also evaluated for orders $p = [0.9 : 2]$ in compound K clutters with different shape parameters. If the SINR is computed on output samples from $t = 5 \times 10^4$ to $t = 10 \times 10^4$, the best SINR was achieved by the NFLOM.
Fig. 8. Output SINR as a function of fractional order $p$ in compound K clutters with different shape parameters and ACF parameters. (a) and (b) SINR computed at iterations $t = (5:10) \times 10^4$. The best SINR was achieved by the NFLOM with $1.5 \leq p \leq 1.8$, a few dB better than the NLMS ($p = 2$) algorithm. (c) and (d) SINR computed after all algorithms converged. The NFLOM with $p > 1.5$ performed well. The NFLOM algorithms achieved better SINR in clutters that have higher auto-covariance. The NLMS algorithm performed better than the lower-order NFLOM if converged.

4.2. Performances of the VO-FLOM algorithm

The conflicting goals of fast convergence and low steady-state error with the fixed order NFLOM can be met simultaneously by the variable-order FLOM algorithm. The effectiveness of the VO-FLOM algorithm is illustrated in Fig. 9 for three clutter scenarios: a complex Gaussian and two compound K scenarios with $\nu = 2$ and $\nu = 0.7$, respectively. Both compound K scenarios had an ACF parameter $\lambda = 100$. The MSE and misalignment curves are also the ensemble average of 100 trials for three clutter scenarios: Gaussian and two compound K cases with $\nu = 0.7$ and $\nu = 2$, respectively, both with $\lambda = 100$. The parameters for the VO-FLOM algorithm were: $P_{\text{min}} = 1, P_{\text{max}} = 2, \Delta P = 0.1, D = 1000$, and the threshold $T_k = 0.01\text{SNR}$. For the excess MSE, the VO-FLOM algorithm converges very fast at around $t = 2 \times 10^4$ sample iterations; while the NLMS...
algorithm does not converge at $10^5$ sample iterations. The steady-state errors of the VO-FLOM algorithm were the same as those achievable by the fixed-order NFLOM. For the misalignment, the VO-FLOM converges even faster than the MSE, at $t = 1 \times 10^4$ sample iterations. The steady-state errors of the misalignment were comparable with those achieved by the NLMS algorithm. The change of the order $p$ may vary from trial to trial and the results of a representative trial are shown in Fig. 9(c), indicating the initial time indexes when the corresponding order is adapted. It is observed that the change of the orders is faster in more impulsive clutters than that in Gaussian clutters. This behavior very well matches the convergence behavior of the excess MSE curves.

With the parameter $P_{\text{max}} = 2$, the VO-FLOM algorithm exhibits much higher excess MSE in impulsive clutter scenarios than that in Gaussian clutters. If this parameter is selected slightly smaller than 2, the converged VO-FLOM can achieve slightly higher performances in terms of higher output SINR, lower MSE, and better robustness against impulsive clutters. This is illustrated in Fig. 10 with the example of $P_{\text{max}} = 1.8$. The MSE and beampattern of the VO-FLOM algorithm in compound K clutters are plotted in Fig. 10. The proposed algorithm can effectively suppress the clutters and jammers by placing deep nulls at jammer locations and the clutter ridge, as shown in Fig. 10(b). The VO-FLOM also converges to a lower steady-state error than the fixed-order NFLOM, as shown in Fig. 10(a).

5. Conclusion

We have evaluated the excess MSE, misalignment, and output SINR performances of the NFLOM algorithms for STAP applications in compound K clutters. The results show that the excess MSE is a better performance measure for phased array applications than the misalignment which is used more often in system identification applications, because the MSE is tightly related with output SINR while the deviation of the weight vector from the MVDR optimal weights plays a less
Fig. 10. The MSE and beampattern of the VO-FLOM with $P_{\text{max}} = 1.8$ in compound K clutters. The K clutters are with $\nu = 2$ and $\lambda = 100$. A variable-order FLOM adaptive algorithm has also been proposed for phased array signal processing, which starts with a small order for fast convergence and increases the order after the excess MSE stops decreasing. The VO-FLOM algorithm improves upon the NFLOM and NLMS algorithm in that the weight adaptation is proportional to a variable $p$-order moment of the error rather than a fixed order moment or the mean square error. The proposed algorithm achieves an excellent compromise between fast initial convergence and low steady-state errors by taking the advantages of small and large order NFLOM algorithms. The excess mean squared error (MSE) curves have been evaluated for both Gaussian clutter and non-Gaussian, heavy-tailed clutter scenarios. The results show that the proposed VO-FLOM converges much faster than the plain NFLOM and NLMS algorithms and achieves the same steady-state error as the NLMS algorithm.

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