Equity-Based Incentives and Supply Chain Buy-Back Contracts

Tinglong Dai
dai@cmu.edu
Posner Hall A19B, Tepper School of Business
Carnegie Mellon University
Pittsburgh, PA 15213

Zhaolin Li
erick.li@sydney.edu.au
Room 490, Merewether Building (H04)
Discipline of Business Analytics
The University of Sydney Business School
Sydney, NSW 2006, Australia

Daewon Sun
dsun@nd.edu
Department of Management
359 Mendoza College of Business
University of Notre Dame
Notre Dame, IN 46556

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a Correspondence author.
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ABSTRACT
We analyse the effect of equity-based incentives in a supply chain with a downstream firm and an upstream supplier. By using the operational decision as a signal to influence external investors’ beliefs, the downstream firm’s manager intends to maximize a convex combination of the interim share price and the terminal cash flows. We show that equity-based incentives create a side-effect. Specifically, with a universal buy-back contract, the deadweight loss of signalling induced by equity-based incentives could spread throughout the supply chain and cause chain-wide damages. To mitigate such undesirable consequences, we propose a new mechanism to eliminate the inefficiency. We derive the optimal mechanism that maximizes the downstream firm’s profits subject to the constraint that the supply chain efficiency is not undermined. In contrast to the full-information benchmark, this mechanism gives positive surplus to the supplier.

Key words: Buy-back contract, Equity-based incentives, Information asymmetry, Signalling, Supply chain management.

INTRODUCTION
Over the past several decades, equity-based incentives have increased considerably in both new-economy and old-economy firms. For example, Bebchuk and Crinstein (2005) found that the aggregate compensation paid by the US public companies to their top five executives, during 1993 to 2003, added up to about $350 billion. The equity-based compensation was 37% of the total compensation to top five executives of the S&P 500 firms in 1993 and increased to 55% by
Equity-Based Incentives and Buy-Back Contracts

2003. The use of equity-based incentives has been an important research topic since the influential work of Holmstrom and Tirole (1993). By linking rewards and penalties to the rise and fall of a firm’s share price, equity-based incentives would overcome many problems associated with other forms of incentives.

Equity-based incentives may, however, create some new problems. A well-known side-effect is that managers may take actions to boost short-term share prices. These actions sometimes reduce long-term firm value and (may) hurt shareholders’ interests. For example, Bebchuk and Stole (1993) showed that managers under-invest in long-run projects relative to the efficient profit-maximizing level if a financial market only observes the productivity of those projects. Conversely, managers over-invest in long-run projects if the financial market only observes the investment level. Branderburger and Polak (1996) showed that managers who care about short-term share prices may take actions suggested by external investors regardless of all the superior information they have. Other wasteful actions taken by managers to influence short-term share prices have been well-documented in the signalling literature (see the literature surveys conducted by Riley, 2001 and Sobel, 2009).

Inventory is one of the drivers of supply chain performance (Chopra & Meindl, 2010) and inventory flow is the most important flow in supply chains that drives information and financial flows. Due to the development of information technologies and the current climate of freedom of information, investors can observe a firm’s operational activities through various information sources. For instance, investors can learn about the inventory build-up of a firm either from the firm’s financial statements or from sales forecasts provided by the firm’s strategic supplier. Because an inventory decision may entail substantial cash flow consequences, it can be used as a credible signal to influence investors’ perceptions. The extant empirical results (e.g.,
Equity-Based Incentives and Buy-Back Contracts

Oyer, 1998; Hendricks & Singhal, 2009; Kedia & Phillippon, 2009) have found that firms may distort inventory decisions to boost short-term share prices.

While there are many supply chain coordination mechanisms, buy-back contracts are commonly used in practice and are one of the most well-studied supply chain contracts in the literature (see Chung, Talluri, & Narasimhan, 2010 and the references therein). Typically, a buy-back contract specifies that the supplier buys back any unsold inventory for some agreed-upon buy-back price higher than the salvage value (Chopra & Meindl, 2010). Because a buy-back contract can redistribute the overstock cost between the retailer and the supplier, the retailer’s inventory level can be restored to the supply chain optimum. However, with the presence of equity-based incentives, a buy-back contract can provide a convenient tool to managers who want to inflate the sales figures. For instance, Wang and Zipkin (2009) demonstrated that with buy-back contracts, both upstream and downstream firms may have incentives to over-sell and over-order, and Lai, Debo, and Nan (2011) showed that upstream firms may inflate their sales in the current period and compensate “over-buying” downstream firms by using a buy-back contract as camouflage.

In this paper, we consider an inventory signalling model with equity-based incentives and buy-back contracts. We examine how inventory decisions could be used as a signal and study the related consequences on supply chain management. To the best of our knowledge, the extant signalling literature usually considers a single firm. Therefore, it remains unclear i) how the signalling behaviour of one firm could affect all the other firms in the same supply chain, ii) whether the supply chain can be coordinated with traditional buy-back contracts, and iii) if not, how the supply chain contracts should be redesigned to mitigate any undesirable consequences.

To answer our main research questions discussed above, we consider a supply chain that
comprises a downstream firm (hereafter the firm) and the firm’s exclusive supplier. We make four key assumptions in our analysis: First, there exists asymmetric information regarding the external customer demand. Second, the firm’s manager (hereafter the manager) has some preference for favourably affecting external investors’ perceptions of the firm value. Third, the inventory decision made by the firm is publicly observable and serves as a signal to convey information to investors. Finally, the firm and its exclusive supplier sign a buy-back contract. With these four assumptions, we first analyze the interactions between the manager and the financial market. The manager is endowed with some private information regarding the external customer demand and uses such information to make the inventory decisions. We use a binary random variable to describe the realization of the demand information. When the realization of this random variable takes a higher (or lower) value, the firm has a higher (or lower) demand, and we refer to the firm as a strong (or weak, respectively) firm. However, we assume that the independent verification of such information is impossible, too costly, or otherwise unavailable. Therefore, with the absence of other forms of credible disclosures, external investors attempt to infer the manager’s private information from the observed inventory decision and determine the firm’s interim value, conditional on all the information available at that time. We assume that the manager wants to maximize a convex combination of the interim and terminal firm value and, as a result, may wish to distort the publicly observable inventory decision to influence investors’ beliefs regarding the demand outlook. In our analysis, we derive the separating equilibrium and show that, when the managers’ interest in the interim share price exceeds a threshold, a strong firm’s manager may over-order to distinguish his/her firm from a weak one.

This over-ordering behaviour has several important managerial implications. First, it causes chain-wide damage under a one-size-fits-all buy-back contract. In some cases, where
Equity-Based Incentives and Buy-Back Contracts

over-ordering behavior exists, a wholesale price contract can outperform a universal buy-back contract because the signalling behaviour (i.e., over-ordering) offsets the adverse effects of double marginalisation and makes the wholesale price contract more profitable to the supply chain. Second, to mitigate the undesirable consequence of the signalling behaviour, we need a new coordination mechanism. In this paper, we develop a mechanism to maximize the supply chain profit when a downstream firm is tempted to distort its inventory decision. In particular, we propose a menu of buy-back contracts. On the menu, a buy-back contract designed for the strong firm gives the firm a bigger proportion of the total chain profit than does the other buy-back contact designed for the weak firm. However, the contract designed for the strong firm has a fixed charge, which is high enough to deter the weak firm but low enough to lure the strong one. Therefore, the proposed mechanism allows the firm to signal in multiple dimensions: contract type and order quantity. We show that when our new coordination mechanism is offered, the inventory decision is not distorted and hence, the supply chain’s efficiency is maximized. Furthermore, after deriving the coordinating mechanism that maximizes the firm’s profits, we find that the supplier receives a positive surplus. This is in contrast to the full-information benchmark where the profit-maximizing (downstream) firm would take all the supply chain profits and give no surplus to the supplier. Due to the desire to signal, the firm would leave some money on the table to the supplier, which could be interpreted as a signalling cost of the firm. The supplier efficiency, however, remains intact because such signalling cost affects the wealth transfer between the firm and the supplier and does not affect the supply chain total profit.

Our research is related to the literature on signalling and supply chain buy-back contracts. Instead of exhaustively reviewing the literature on these two streams, we briefly review the most related results.
The extant signalling literature has thoroughly analysed the signal-dimensional and single-firm signalling behaviours. We refer readers to Riley (2001) for a general review on this topic and Sobel (2009) for in-depth technical discussions on solving the generic signalling equilibrium. We extend this stream of literature by building a multi-dimensional signalling model in a supply chain framework. Specifically, we consider that the firm can signal by choosing different contract types and different order quantities.

In the supply chain literature, Pasternack (1985) was the first to show that a buy-back contract can re-distribute the supply chain over-stock costs between the retailer and the supplier, who otherwise would not share such costs under a wholesale-price contract, and thus restore the retailer's inventory level to the supply chain optimum. Subsequent research on this topic considered such issues as preventing brand-image loss (e.g., Padmanabhan & Png 1995), price-setting retailers (e.g., Emmons & Gilbert 1998), effort-dependent demand (e.g., Taylor 2002), new-product introduction (e.g., Li & Gao 2008), weather-related demand uncertainty (e.g., Chen & Yano 2010), and laboratory evidence (e.g., Katok & Wu 2009). For updated literature on supply chain contracting, readers can refer to Chung, Talluri, and Narasimhan (2010). To the best of our knowledge most of the existing literature on supply chain buy-back contracts did not consider investors’ perceptions.

The works done by Arya and Mittendorf (2010), Lai, Debo, and Nan (2011), and Lai, Xiao, and Yang (2011) are closely related to our research. Arya and Mittendorf (2010) showed that the accounting rule used for assessing the value of unsold inventory (“lower of historical
Equity-Based Incentives and Buy-Back Contracts

cost or market value”) can mitigate the supply chain inefficiency. Lai, Debo, and Nan (2011) considered that the upstream firm is the source of distortions (i.e., sending excessive inventory to the downstream to inflate sales in the short run) and did not explore how to mitigate such inefficiency. In contrast, we consider that the downstream firm is the source of distortions and develop a multi-dimensional screening mechanism to mitigate the inefficiency. The working paper by Lai, Xiao, and Yang (2011), hereafter LXY, independently analysed a model similar to ours. They propose a menu of contracts instead of a single buy-back contract to restore the supply chain efficiency. The differences between LXY and ours are the following. First, each buy-back contract on their menu has three parameters whereas our contract has only two parameters. We are able to obtain two linear incentive-compatibility constraints (we refer readers to Proposition 2 and 3 in later sections). These linear incentive-compatibility constraints facilitate the derivation of the optimal mechanism explicitly. Second, their mechanism and ours lead to different surplus to the firm and the supplier. Under their optimal mechanism, the supplier takes all the supply chain profits whereas under our optimal mechanism, both parties earn surplus. This is because the manager’s desire to signal will force the profit-maximizing firm to share the supply chain profit with the supplier as a signalling cost. Finally, in terms of the methodology, we adopt the approach described in Sobel (2009) to define the equilibrium rigorously and apply Sobel’s algorithm to solve the equilibrium efficiently.

The remainder of the paper is organized as follows. We first describe the model and then characterize the separating equilibrium and discuss other equilibria. After that, we analyse the adverse effect of signalling on the supply chain and develop a mechanism to mitigate the deadweight loss of signalling. Finally, we conclude the paper. All the proofs are presented in Appendix A, and Appendix B discusses other equilibria of our signalling game.
THE MODEL

In this section, we present a model of inventory signalling with three periods and no discounting. The model involves three risk-neutral players: a manager of a publicly traded (downstream) firm, a group of anonymous investors referred to as the financial market, and the firm’s exclusive supplier. Figure 1 provides a timeline depicting the sequence of events and key assumptions.

Now, we provide detailed discussions on related assumptions and events in each period. At Period 1, we assume that the firm’s manager privately receives the information about the firm’s customer demand. We denote the manager’s information (also called the firm type) by \( t \in \{h,l\} \). Following the convention, we call the firm with the realized private information \( t = h \) the strong firm; otherwise, the firm is designated as the weak firm. We use \( \gamma \) to represent the prior probability that the firm is strong and \( (1 - \gamma) \) to represent the probability that the firm type is weak. The firm’s customer demand conditioned on the realized firm type \( t \) is denoted by a non-negative random variable \( D_t \) with a strictly increasing probability distribution function \( F_t(\cdot) \) for \( t \in \{h,l\} \). We assume that the demand conditioned on \( t = h \) is stochastically larger than the demand conditioned on \( t = l \) (i.e., \( F_h(x) \leq F_l(x) \) for all \( x \geq 0 \)). Furthermore, although all the probability distributions are common knowledge, we assume that only the manager observes the realization of firm type \( t \). After receiving the private information, the manager needs to determine the inventory level \( q \geq 0 \), which becomes public information at Period 2. Finally, we assume that the supplier produces the end product at a cost of \( c \) per unit.
Equity-Based Incentives and Buy-Back Contracts

At Period 2, a round of trading in the firm’s shares occurs. The market value of the firm’s shares at Period 2, which we refer to as the firm’s interim share price, is conditioned on the publicly observed inventory level $q$. For simplicity, we assume that the firm has exactly one share and its dividend policy is to pay out all earnings as dividends at Period 3. As such, at the equilibrium, the financial market i) determines the firm’s share price based on the expected cash flow, conditioned on all the available information at that time, and ii) takes into account the fact that the firm’s manager might distort the operations.

At Period 3, the inventory is delivered to the firm and the external customer demand is realized. After that, the terminal cash flow is realized. Now we derive terminal cash flow functions for our signalling game. First, the terminal cash flow generated by the firm at Period 3 is given by

$$\pi_i(t, q, D_t) = r \min(q, D_t) - wq + b(q - D_t)^+ = (r - b) \min(q, D_t) - (w - b)q,$$  \hspace{1cm} (1)

where $r$ is the retail price ($r > c$), $w$ is the wholesale price, and $b$ is the buy-back price, and $(x)^+ = \max(0, x)$. We assume that any unmet demand is lost, and the salvage value of the unsold inventory is assumed to be negligible. By taking the expectation with respect to $D_t$, we have the expected cash flow denoted by $T_i(t, q)$ as

$$E\pi_i(t, q, D_t) = (r - b)E \min(q, D_t) - (w - b)q = T_i(t, q).$$ \hspace{1cm} (2)

To ensure that the optimal order quantity for the firm is finite, we assume that a typical regularity condition ($r > w > b$) holds. Then, for any given $w$ and $b$, we observe that the maximizer of $T_i(t, q)$ is

$$q_i^* = F^{-1}_r \left( \frac{r - w}{r - b} \right).$$
where $F^{-1}_i(\cdot)$ is the inverse of the probability distribution function $F_i(\cdot)$. For expositional simplicity, we now define a profit difference function as

$$\delta_i(q) = T_i(h,q) - T_i(l,q) = (r-b)[E\min(q,D_h) - E\min(q,D_l)].$$

Note that because $F_h(x) \leq F_l(x)$ for all $x \geq 0$, it can be verified that $\delta_i(q)$ is non-negative and is weakly increasing in $q$. In other words, $T_i(t,q)$ satisfies the single-crossing condition, which implies that with stronger demand outlook, it is less costly for a strong firm to over-order than it is for a weak firm.

Next, it is easy to see that the supplier’s terminal cash flow at Period 3 is

$$\pi_2(t,q,D_t) = (w-c)q - b(q-D_t)^+. $$

By combining the profit functions of the firm and supplier, we can express the total supply chain profit for the standard integrated system as

$$\pi(t,q,D_t) = \pi_1(t,q,D_t) + \pi_2(t,q,D_t) = r\min(q,D_t) - cq. \quad (3)$$

Thus, the expected total chain profit can be written as

$$T(t,q) = E\pi(t,q,D_t) = rE\min(q,D_t) - cq, \quad (4)$$

which is strictly concave in $q$ and becomes negative when $q$ is sufficiently large. Note that the optimal inventory level that maximizes the supply chain profit is

$$q_* = F^{-1}_r\left(\frac{r-c}{r}\right),$$

and we define profit difference function as

$$\delta(q) = T(h,q) - T(l,q) = r[E\min(q,D_h) - E\min(q,D_l)].$$

It can be verified that $\delta(q)$ is non-negative and is weakly increasing in $q$, i.e., $T(t,q)$ satisfies the single-crossing condition (Sobel, 2009).
Equity-Based Incentives and Buy-Back Contracts

Finally, following the prevalent approach to modeling equity-based incentives, we assume that the utility of managers is a linear combination that assigns a weight of $\alpha$ to their firm’s interim share price and a weight of $1-\alpha$ to the terminal cash flow, where $\alpha \in (0,1)$ is common knowledge. A larger value of $\alpha$ implies that managers are more concerned about the interim share price. Such an assumption on the managers’ objective has been widely used in literature, for example, Holmstrom and Tirole (1993) showed that the optimal managerial incentive contract includes components that depend on short-term share prices and the terminal cash flows. Furthermore, we emphasize that the parameter $\alpha$ has several different interpretations in literature. For instance, a manager of a firm faces pressure from two types of shareholders: i) shareholders who need a quick liquidation (prior to the realization of cash flow) and ii) others who intend to hold the shares until cash flow is realized. In this case, this modeling approach implies that the first type of shareholders accounts for $\alpha \times 100\%$ of all the shareholders (Hayes & Schaefer 2009). The second possible interpretation is that a manager may be concerned with maintaining a good reputation. For example, the manager may receive a job offer from a rival firm (with probability $\alpha$) and the offer is usually linked to the manager’s reputation or the interim share price (Hirshleifer 1993). Another interpretation is provided by Bebchuk and Stole (1993). They considered that a manager’s payoff is $f+v_1V+v_2T_1(t,q)$, where $f$ is the fixed wage, $V$ is the interim share price, and $v_1$ and $v_2$ are the numbers of share options expired at later stages. By letting $\alpha=v_1/(v_1+v_2)$, we observe that maximising $f+v_1V+v_2T_1(t,q)$ is equivalent to maximizing $\alpha V+(1-\alpha)T_1(t,q)$.

CHARACTERIZATION OF EQUILIBRIUM
Equity-Based Incentives and Buy-Back Contracts

Now, we characterize the equilibrium of the proposed signaling game. To this end, we define the pricing action \( a \) as the posterior belief of the financial market about the firm type being \( h \) when the observed inventory decision is \( q \). For any given pricing action \( 0 \leq a \leq 1 \), the interim share price is

\[
V(a, q) = a T_h(h, q) + (1-a) T_l(l, q) = a \delta_h(q) + T_l(l, q),
\]

and the manager’s utility is

\[
U(t, q, a) = a V(a, q) + (1-a) T_l(t, q).
\]

Next, let \( S(t, q) \) be the probability that the manager with private information \( t \) orders \( q \) units of inventory and \( R(q, a) \) be the probability that the financial market takes the pricing action \( a \) after observing the order quantity \( q \) (note that \( S \) is a function of the firm type \( t \) and \( R \) is a function of the observed order quantity \( q \)). Now we are ready to define a Bayesian Nash equilibrium of our signaling game by following Sobel (2009).

**Definition 1**: The strategies \( (S^*, R^*) \) form a Bayesian Nash equilibrium if and only if for all \( t \in \{h, l\} \), the following conditions are satisfied.

i) \( S^*(t, q) > 0 \) implies

\[
\sum_{q \in [0,1]} U(t, q, a) R^*(q, a) = \max_{q \in [0,1]} \sum_{q \in [0,1]} U(t, q', a) R^*(q', a),
\]

ii) If, for some \( q, \ S^*(h, q) \gamma + S^*(l, q)(1- \gamma) > 0 \), then \( R^*(q, a) > 0 \) implies

\[
a = \Pr(t = h \mid q) = \frac{\gamma S^*(h, q)}{\gamma S^*(h, q) + (1- \gamma) S^*(l, q)}.
\]

The meanings of the two conditions (equations (6) and (7)) are as follows. Condition (6) means that the manager’s inventory ordering policy maximizes his/her utility given the equilibrium pricing action. Condition (7) requires that the financial market updates the posterior
Equity-Based Incentives and Buy-Back Contracts

belief by using Bayes’ rule when the order quantity is chosen by the manager with a positive probability. Note that the equilibrium refinements require that if the financial market believes that certain order quantity $q$ will never be chosen by the manager, i.e.,

$$S(h,q)\gamma + S(l,q)(1-\gamma) = 0,$$  \hspace{1cm} (8)

then this order quantity should never be observed at the equilibrium.

**Separating Equilibrium**

A separating equilibrium is a benchmark outcome for signalling games. If a separating equilibrium exists, then it is possible for a manager to share his/her private information fully with the financial market in spite of having a potential conflict of interest.

We first define some notations. Suppose that a firm type is weak and the financial market takes an incorrect pricing action of $a=1$. Then, the utility of the weak firm’s manager is

$$U(l,q,1) = \alpha V(l,q) + (1-\alpha)T_l(l,q) = \alpha T_l(h,q) + (1-\alpha)T_l(l,q),$$

which is strictly concave in $q$ and becomes negative when $q$ is sufficiently large. Note that the function $U(l,q,1)$ constitutes an upper bound on the optimal utility that the weak firm’s manager can get at the equilibrium. Let $U^*(l) = T_l(l,q^*_l)$, which represents the maximum expected cash flow that the weak firm can generate. Next, we see that, because i) $U^*(l)$ is strictly concave in $q$ and negative when $q$ is sufficiently large and ii) $U(l,q^*_l,1) > U(l,q^*_l,0) = T_l(l,q^*_l) = U^*(l)$, there must exist two thresholds $q_l$ and $\bar{q}_l$ (where $q_l < \bar{q}_l$) such that $U(l,q,1) \leq U^*(l)$ if $q \geq \bar{q}_l$ or $q \leq q_l$. We illustrate these two thresholds in Figure 1.

**INSERT FIGURE 1 ABOUT HERE!**
Now, we construct the separating equilibrium by following the procedures described in Section 6.1 of Sobel (2009). The results are summarized below and the proof is given in Appendix A.

**Proposition 1:** There exists a unique threshold \( \alpha_1 \) defined as

\[
\alpha_1 = \frac{T_1(l,q^d_i) - T_1(l,q^d_h)}{T_1(h,q^d_h) - T_1(l,q^d_h)}.
\]

(i) When \( \alpha \leq \alpha_1 \), a strong firm orders \( q^*_h = q^*_h \) units and a weak firm orders \( q^*_i = q^*_i \) units in the separating equilibrium. The utility of the strong firm’s manager is \( U^*(h) = T_1(h,q^d_h) \) and that of the weak firm’s manager is \( U^*(l) = T_1(l,q^d_l) \).

(ii) When \( \alpha > \alpha_1 \), a strong firm orders \( q^*_h = q^*_h \) units and a weak firm orders \( q^*_i = q^*_i \) units in the separating equilibrium. The utility of the strong firm’s manager is \( U^*(h) = T_1(h,q^d_h) \) and that of the weak firm’s manager is \( U^*(l) = T_1(l,q^d_l) \).

We explain the intuition of Proposition 1 as follows. When a firm is revealed to be weak (e.g., the weak firm’s manager orders some \( q \) that a strong firm’s manager never orders), the best response of the financial market is \( BR(l) = 0 \). Hence, the utility of the weak firm’s manager is \( U(l,q,0) = T_1(l,q) \). Recall that \( q^*_i \) is the maximizer of \( T_1(l,q) \). By ordering \( q^*_i \) units of inventory, and if the firm type is indeed revealed to be weak, the weak firm’s manager attains a utility of \( T_1(l,q^*_i) = U^*(l) \), which is a lower bound on the optimal utility that the manager can get. If the strong firm commits to order a sufficiently large quantity \( (q \geq q^*_l) \), the maximum utility that the
Equity-Based Incentives and Buy-Back Contracts

weak firm’s manager can get by mimicking is \( U(l, q_l) \), which is less than \( U^*(l) \). Hence, the weak firm’s manager finds it unprofitable to mimic, which implies that the order quantity \( q^d_{l} \) indeed maximizes his/her utility provided that the strong firm commits to order a sufficiently large quantity \( q \geq \bar{q}_l \). It should be noted that the manager prefers higher interim share price. Because the share price equals the expected terminal cash flow, the single-crossing property implies that with the same order quantity, the strong firm is expected to generate a higher terminal cash flow and hence a higher interim share price. Thus, the manager of the weak firm has incentives to mimic whereas the manager of the strong firm does not want to pretend to be weak. Therefore, when deriving \( q^*_h \), we need to impose an incentive-compatibility (IC) constraint to ensure that the weak firm does not mimic, i.e., deviations should not increase the utility of the weak firm’s manager at the equilibrium. In particular, the IC constraint would require that the order quantity is sufficiently high enough to deter any mimicking. It should be emphasized that although ordering sufficiently low can deter mimicking, it is not profitable for the strong firm (i.e., \( T_l(h, \bar{q}_l) > T_l(h, q_{l}) \)). Therefore, the concavity of the expected terminal cash flow function \( T_l(h, q) \) implies that the optimal order quantity for the strong firm must be the larger value of \( q^d_{h} \) and \( \bar{q}_l \).

Other Equilibria

In signalling games, there could be two other possible equilibria: Partial-Pooling and Pooling equilibria. We provide a detailed analysis for deriving the partial-pooling and pooling equilibria in Appendix B, and in this section we briefly present our main findings. First, in Appendix B we demonstrate that the strong firm’s manager is strictly better off at the separating equilibrium than
at the partial-pooling equilibrium while the weak firm’s manager receives the same utility at both equilibria. Hence, the partial-pooling equilibrium is not preferred. Second, there could also be a pooling equilibrium where all the firms order the same quantity. But such a pooling equilibrium does not satisfy the stability requirement in the Intuitive Criterion, which is one of the most influential criteria for equilibrium refinements. In particular, the Intuitive Criterion requires that “out-of-equilibrium beliefs place zero weight on types that can never gain from deviating from a fixed equilibrium outcome.” This non-technical description is adopted from Sobel (2009), and the technical details of this criterion can be found in Cho and Kreps (1987) as well as in Appendix B. Here, we provide an intuitive explanation of this result. Note that equation (5) suggests that the incremental difference in the share price between the strong and weak firms is \( \delta_i(q) \), which is positive and increasing in \( q \). Because only the strong firm can possibly gain from deviations, a slightly higher order quantity will be interpreted as coming from the strong firm in the pool (i.e., investors put zero weight on the weak firm when they observe an order quantity that is slightly higher than the common order quantity). Therefore, the strong firm’s manager always has incentives to deviate from a pooling equilibrium. This implies that the pooling equilibrium fails to satisfy the Intuitive Criterion.

In summary, we demonstrate that the partial-pooling equilibrium is dominated by the separating equilibrium and that the pooling equilibrium is not stable. Therefore, the separating equilibrium characterized in Proposition 1 is the most plausible, and in the following sections, we only focus on the separating equilibrium.

DEADWEIGHT LOSS OF SIGNALLING WITH BUY-BACK CONTRACTS
Equity-Based Incentives and Buy-Back Contracts

To quantify the side-effect of buy-back contracts in signalling games, we study a benchmark case with full information, i.e., no signalling and no information asymmetry. If \( q_i^d = q_i \), the supply chain profit is maximized and we say that the supply chain is coordinated. The following Lemma derives a one-size-fits-all buy-back contract that coordinates the supply chain.

**Lemma 1:** If the wholesale price \( w \) and the buy-back price \( b \) satisfy the following condition

\[
\frac{r - w}{r - b} = \frac{r - c}{c},
\]

then \( q_i^d = q_i \), i.e., the optimal inventory level in the decentralized system is the same as that in the integrated system regardless of the firm type \( t \).

Lemma 1 is a well-known result in the buy-back contract literature. Hereafter, we call a buy-back contract satisfying equation (10) a **coordinating contract**. Next, for expositional simplicity, we define \( \rho \in (0,1) \) as the profit split ratio such that

\[
\rho = \frac{r - w}{r - c} = \frac{w - b}{c},
\]

where \( \frac{r - w}{r - c} = \frac{w - b}{c} \) is implied by equation (10). We note that the overstock cost of a firm is \( w - b \) and the understock cost is \( r - w \); whereas the supply chain overstock cost is \( c \) and the supply chain understock cost is \( r - c \). Hence, equation (11) implies that a firm shares \( \rho \times 100\% \) of the supply chain understock and overstock costs. Hereafter, instead of specifying \( w \) and \( b \) for a buy-back contract, it is sufficient to describe a coordinating buy-back contract by using the profit split ratio \( \rho \) only.
Lemma 2: If a buy-back contract satisfies equation (10), then \( \pi_i(t, q, D_t) = \rho \pi_i(t, q, D_t) \) and 
\( T_i(t, q) = \rho T(t, q) \) for any \( t \in \{h, l\} \) and \( q \geq 0 \).

Lemma 2 indicates that under a coordinating contract, the downstream firm takes \( \rho \times 100\% \) of the total supply chain profit both ex ante (before \( D_t \) is realized) and ex post (after \( D_t \) is realized). Accordingly, the expected cash flow generated by the type-\( t \) firm is 
\( T_i(t, q) = \rho T(t, q) \), which satisfies the single-crossing and concave properties. Now we are ready to investigate the impact of a buy-back contract in signal games.

Corollary 1: Define \( \alpha_2 \) as
\[
\alpha_2 = \frac{T(l, q_l) - T(l, q_h)}{T(h, q_h) - T(l, q_h)},
\]
(12)
and \( \bar{q} \) as the larger of the two solutions to \( U(l, q, l) = T(l, q, l) \), where
\[
U(l, q, l) = \alpha V(l, q) + (1 - \alpha)T_i(l, q) = \alpha \rho T(h, q) + (1 - \alpha)\rho T(l, q).
\]
Then, the optimal order quantities and profits with a buy-back contract are as follows:
(i) When \( \alpha \leq \alpha_2 \), a strong firm orders \( q_h^* = q_h \) units of inventory and a weak firm orders \( q_l^* = q_l \) units of inventory in the separating equilibrium. The expected supply chain profit is \( T(h, q_h) \) when the firm is strong and \( T(l, q_l) \) when the firm is weak.
(ii) When \( \alpha > \alpha_2 \), a strong firm orders \( q_h^* = \bar{q} \) units of inventory and a weak firm orders \( q_l^* = q_l \) units of inventory in the separating equilibrium. The expected supply chain profit is \( T(h, \bar{q}) \) when the firm is strong and \( T(l, q_l) \) when the firm is weak.
Note that case (i) of Corollary 1 is the most desirable outcome, in which the supply chain efficiency is maximized and so is the interim share price. Such a scenario (we call it the first-best) occurs when a manager’s weight on the interim share price is sufficiently low. However, notice that a sufficiently low $\alpha$ implies that the component of equity-based compensation is also reduced significantly. In the current labor market, it is difficult for a firm to attract good managerial candidates without offering a lucrative equity-based compensation (see Bebchuk & Crinstein 2005 and the references therein). In Section 5, we will show that by offering type-dependent buy-back contracts instead of a one-size-fits-all contract, the first-best can be achieved for any $\alpha$. Next, in case (ii) of Corollary 1, we observe that the supply chain efficiency is undermined by the deadweight loss of signalling, i.e., $T(h, q_s) > T(h, \bar{q})$. Because the supplier and the firm take $(1 - \rho) \times 100\%$ and $\rho \times 100\%$ of the supply chain profit (ex ante and ex post), both the supplier and the firm suffer. This result demonstrates that a universal buy-back contract can transfer the deadweight loss from downstream to upstream, which causes chain-wide damages.

To further investigate the degree of this side-effect, we compare the performance of a universal buy-back contract with that of a traditional wholesale price contract. To derive the equilibrium solutions of a wholesale contract, we set $b=0$. For clarity, we summarize the key expressions and discuss some important characteristics as follows:

- When $b=0$, the expected terminal cash flow of the type-$t$ firm becomes
  \[ T^0_t(t, q) = rE \min(q, D_t) - wq, \]
  where we use the superscript 0 to indicate that the buy-back price is zero.
- The cash flow of the supplier is $(w-c)q$. 

Equity-Based Incentives and Buy-Back Contracts
Equity-Based Incentives and Buy-Back Contracts

- The total expected supply chain profit function $T(t,q)$ is unaffected by the values of $b$ and $w$.
- Let $q_t^0 = F_t^{-1}\left(\frac{r-w}{r}\right)$ be the maximizer of $T_t^0(t,q)$. Define $\bar{q}$ as the larger of the two solutions to
  \[ U^0(l,q,l) = T_t^0(l,q_t^0), \]
  where $U^0(l,q,l) = \alpha T_t^0(h,q) + (1-\alpha) T_t^0(l,q)$. Because the wholesale price contract does not satisfy equation (10), in general, $T_t^0(t,q)$ is not proportional to the total supply chain profit.
- We define another threshold on $\alpha$:
  \[ \alpha_0 = \frac{T_t^0(l,q_t^0) - T_t^0(l,q_t)}{T_t^0(h,q_t^0) - T_t^0(l,q_t)}. \]

Using these key expressions, we can investigate the separating equilibrium with a traditional wholesale contract. Because the single-crossing and concave properties are preserved, we can use the same approach in deriving Proposition 1 to characterize the separating equilibrium.

**Corollary 2**: For a traditional wholesale contract, there exists a unique separating equilibrium specified below:

(i) When $\alpha \leq \alpha_0$, a strong firm orders $Q_t^0 = q_t^0$ units of inventory and a weak firm orders $Q_t = q_t^0$ units of inventory.
Equity-Based Incentives and Buy-Back Contracts

(ii) When $\alpha > \alpha_0$, a strong firm orders $Q_h^0 = q^0$ units of inventory and a weak firm orders $Q_l^0 = q_l^0$ units of inventory.

Because Corollaries 1 and 2 characterize the two equilibria with universal buy-back and whole-sale contracts, now we can investigate whether the coordinated chain or the uncoordinated chain performs better. To this end, we note that the ex ante total supply chain profit (before the private information is observed by the manager) under a universal buy-back contract is

$$ET^b = T(h, q_h^*)\gamma + T(l, q_l^*)(1 - \gamma),$$

where $q_h^*$ and $q_l^*$ are derived from Corollary 1. Next, under a wholesale price contract, we see that the ex ante total supply chain profit becomes

$$ET^0 = T(h, Q_h^0)\gamma + T(l, Q_l^0)(1 - \gamma),$$

where $Q_h^0$ and $Q_l^0$ are derived from Corollary 2. By comparing $ET^b$ and $ET^0$, one can find whether a buy-back contract can enhance the supply chain efficiency.

To illustrate the deadweight loss, we present a numerical example with the following parameters: i) With probability $\gamma = 0.5$, the firm is strong and the external demand is exponential with a mean value of 1. ii) With probability $1 - \gamma = 0.5$, the firm is weak and the external demand is exponential with a mean value of 0.5. iii) The retail price is $r = 3$ and the production cost is $c = 1$. This parameter setting yields the relevant supply chain profit functions of the coordinated chain with a buy-back contract as

$$T(h, q) = 3[1 - \exp(-q)] - q$$

and

$$T(l, q) = 1.5[1 - \exp(-2q)] - q.$$
Equity-Based Incentives and Buy-Back Contracts

One can easily show that the maximizers of $T(t,q)$ are $q_h^* = 1.0986$ and $q_l^* = 0.5493$ and the critical threshold is $\alpha_2 = 0.3240$. Now we consider the uncoordinated chain with $w=1.2$. The relevant supply chain profit functions of the wholesale contract are

$$T_i^w(h,q) = 3[1 - \exp(-q)] - 1.2q$$
and
$$T_i^w(l,q) = 1.5[1 - \exp(-2q)] - 1.2q,$$
and the maximizers of $T_i^w(t,q)$ are $Q_h^0 = 0.9163$ and $Q_l^0 = 0.4581$. We observe that the optimal order quantities are smaller than those in the coordinated chain because of the double marginalization. Finally, we have the critical threshold as $\alpha_0 = 0.3514$ for the uncoordinated chain.

Figure 3 demonstrates that $ET^b$ is decreasing in $\alpha$ but $ET^0$ is not. In particular, when $\alpha$ is sufficiently high, $ET^b < ET^0$. For example, when $\alpha=0.8$, the ex ante total profit of the supply chain that uses the universal buy-back contract is $ET^b = 0.558$ whereas the ex ante total profit of the supply chain using the wholesale price contract is $ET^0 = 0.636$, which is 13.9% higher. This numerical example provides an important managerial insight: Signalling mitigates the understocking problem caused by double marginalisation whereas a universal buy-back contract makes the supply chain more susceptible to operational distortions caused by signalling.

**SUPPLY CHAIN COORDINATION MECHANISM**

As illustrated in Figure 3, it is easy to see that reducing $\alpha$, for instance, giving a manager fewer share options expired on the interim date, will mitigate the deadweight loss of signalling. However, this is not always feasible in reality because equity-based compensations are widely
Equity-Based Incentives and Buy-Back Contracts

adopted. In this section, we propose a new supply chain coordination mechanism without reducing $\alpha$. Specifically, we consider a “screening” mechanism (we refer the readers to Riley (2001) for the discussion on linkages between signalling and screening and Rochet & Stole (2003) for an updated survey on this research stream). It offers two different buy-back contracts, where contract 1 specifies that the profit split ratio is $\rho_l$ and contract 2 specifies that the profit split ratio is $\rho_h$ ($\rho_h \geq \rho_l$) and the fixed charge is $K \geq 0$. Note that the wholesale price and the buy-back price of the proposed mechanism can be easily determined by equation (11) for any given profit split ratios. The most important feature of this mechanism is that it has an extra dimension (i.e., the contract type) for a manager to signal.

By choosing the parameters ($\rho_h$, $\rho_l$, and $K$) properly, at the equilibrium, a strong firm should accept contract 2 and order $q_h$ units whereas a weak firm should accept contract 1 and order $q_l$ units. We emphasize that at this equilibrium, the total supply chain profit will be maximized and the interim share price, which is equal to the expected cash flow, is also maximized. Hence, this mechanism can coordinate the supply chain with the presence of equity-based incentives.

To ensure that this equilibrium is a Nash equilibrium, we need to check whether two IC constraints are satisfied such that no one would gain by deviating from the equilibrium. Toward this end, consider the best response of the strong firm when the proposed mechanism is offered. For expositional simplicity, let $T(h,q_h) = T_h$ and $T(l,q_l) = T_l$ be the optimal expected supply chain profit attainable when the firm is strong and weak, respectively. First, we observe that by choosing the equilibrium strategy (i.e., accept contract 2 and order $q_h$ units), the strong firm’s manager attains an expected utility of $\rho_h T_h - K$, which equals the optimal expected cash flow of
Equity-Based Incentives and Buy-Back Contracts

the strong firm. Second, we need to consider the other case where the strong firm’s manager accepts contract 1. Suppose that the manager deviates from the equilibrium by accepting contract 1 and ordering $q$ units. By choosing contract 1, the strong firm generates an expected terminal cash flow of $\rho_T(h,q)$. Note that the interim share price is no higher than $\rho_T(h,q)$. Clearly, we see that

$$a\rho_T(h,q) + (1-\alpha)\rho_T(h,q) \leq \rho_T'\rho_h,$$

because $q_h$ is the maximizer of $T(h,q)$. Therefore, we obtain an upper bound on the utility attainable to the strong firm’s manager who deviates. To ensure that the strong firm has no incentives to deviate, the following IC constraint must be satisfied:

$$\rho_h - K \geq \rho_T h.$$  (14)

In other words, if the parameters are chosen such that equation (14) is satisfied, the strong firm does not have any incentives to deviate, and hence will choose contract 2 at the equilibrium.

Next, we analyze the best response of the weak firm. First, by playing the equilibrium strategy (i.e., accept contract 1 and order $q_l$ units), the weak firm’s manager gets an expected utility of $\rho_T l$, which equals the optimal expected cash flow of the weak firm. For expositional simplicity, we define

$$\tilde{T} = \max_q \{aT(h,q) + (1-\alpha)T(l,q)\},$$

which is a constant independent of contract terms. Second, consider the weak firm’s deviation, i.e., it mimics the strong firm by accepting contract 2 and ordering $q$. We note that i) the share price is no higher than $\rho_h T(h,q) - K$ (this could happen when the financial market mistakenly believes that the weak firm is strong) and ii) the terminal cash flow of the weak firm that accepts contract 2 is $\rho_h T(l,q) - K$. Clearly, the manager can do no better than
This implies that $\rho\tilde{T} - K$ is an upper bound on the utility attainable to the weak firm’s manager who deviates. Therefore, to prevent the weak firm from deviating, the following IC constraint must be satisfied:

$$\rho T_i \geq \rho \tilde{T} - K .$$  \hfill (15)

To summarize, when a mechanism satisfies the two IC constraints specified in equations (14) and (15), the firm will truthfully reveal its type by choosing the equilibrium contract. Therefore, at the equilibrium, the inventory order quantity is not distorted.

**Proposition 2**: There always exist contract parameters (i.e., $\rho_h$, $\rho_l$, and $K$) that satisfy the IC constraints (14) and (15). Hence, the first-best can be achieved.

Proposition 2 states that the proposed mechanism is feasible and offers some important managerial insights. First, note that a key feature of the proposed mechanism is that the fixed charge on the strong firm cannot be too high (i.e., the IC constraint (14) requires $\rho_h T_h - \rho_l T_h \geq K$) or too low (i.e., the IC constraint (15) requires $K \geq \rho_h \tilde{T} - \rho_l T$). In other words, when the fixed charge is too high, the strong firm may switch to the contract designed for the weak firm. In contrast, if the fixed charge is too low, the weak firm may switch to the contract designed for the strong firm. Second, because the contract type is sufficient to signal the firm type, the inventory ordering decision will not be distorted. In addition, the inventory decision made by a firm is also optimal for the supply chain because each buy-back contract on the menu is a coordinating contract satisfying equation (10). Third, we note that there are many possible combinations of
Equity-Based Incentives and Buy-Back Contracts

$\rho_h, \rho_I, \text{ and } K$ that can satisfy the IC constraints (14) and (15) and that these two IC constraints are linear in $\rho_h, \rho_I, \text{ and } K$.

The remaining question is that “among all the combinations of $\rho_h, \rho_I, \text{ and } K$ that satisfy the IC constraints (14) and (15), which combination would maximize the firm’s ex ante profit?” Note that the firm’s ex ante profit (before receiving the private information) is

$$U_1 = \gamma(\rho_h T_h - K) + (1 - \gamma)\rho_I T_I,$$  \hfill (16)

which is a linear function of $\rho_h, \rho_I, \text{ and } K$. By solving a linear programming (LP) problem with equation (16) as the objective function and inequalities (14) and (15) as the constraints, we derive the optimal mechanism.

**Proposition 3**: The optimal mechanism that maximizes the firm’s ex ante profit without undermining the supply chain efficiency is the following:

$$\rho_h^* = 1, \rho_I^* = \frac{T_h - \tilde{T}}{T_h - T_I}, \text{ and } K^* = \frac{\tilde{T} - T_I}{T_h - T_I}.$$

The optimal mechanism has the following features: First, when the firm is strong, the supplier gives up all the supply chain profit to the firm in exchange for a fixed payment. Second, when the firm is weak, the supplier takes some of the supply chain profits but does not receive any fixed payment. In the full-information benchmark, the two IC constraints are not needed because the firm type is known by the financial market. The information-symmetric mechanism that maximizes the firm’s profit is trivial, i.e., $\rho_h^* = 1, \rho_I^* = 1, \text{ and } K^* = 0$ because equation (16) is increasing in $\rho_h$ and $\rho_I$ but decreasing in $K$. The supplier obtains no surplus whereas the firm
Equity-Based Incentives and Buy-Back Contracts

takes all the supply chain profits. With asymmetric information regarding the firm type, the supplier’s ex ante profit is

\[ U_2 = \gamma (K^* + (1 - \rho_h^*) T_h) + (1 - \gamma)(1 - \rho_i^*) T_i = \frac{\gamma T_h (\bar{T} - T_h) + (1 - \gamma) T_i (T_h - \bar{T})}{T_h - T_i} > 0. \]

In other words, with asymmetric information regarding the firm type, the profit-maximizing firm would leave some money on the table to the supplier. The surplus given to the supplier can be thought of as a signalling cost of the firm. Because such signalling cost only affects the wealth transfer between the two parties, the supply chain profit remains intact.

Before we conclude this section, we illustrate the savings after implementing the optimal coordination mechanism by using a numerical example. Consider the data that we use to construct Figure 3. When \( \alpha = 0.8 \), it can be verified that \( T_h = 0.9014 \), \( T_i = 0.4507 \), and \( \bar{T} = 0.7771 \). The optimal mechanism of Proposition 3 has the following parameters: \( \rho_h = 1 \), \( \rho_i = 0.2758 \), and \( K = 0.6528 \). The ex ante supply chain profit will be restored to 0.676, the first-best level. As we mentioned before, when using a universal buy-back contract the ex ante supply chain profit is \( ET^b = 0.558 \), which is 17.46% lower than the system optimum. When using a wholesale price contract with \( w = 1.2 \), the ex ante supply chain profit is \( ET^0 = 0.636 \), which is 5.9% lower than the system optimum.

CONCLUSIONS

This study is motivated by the observation that a firm’s operational decisions could have profound implications for a financial market. Because equity-based incentives are widely used in practice, researchers and practitioners need to understand the complicated interactions among equity-based incentives, operational decisions, and supply chain contracts.
Equity-Based Incentives and Buy-Back Contracts

To investigate the dynamics of these important factors, we have formulated a multi-stage, inventory signalling model where i) a downstream firm in a supply chain may distort the inventory decision to signal, ii) the supply chain is coordinated by a traditional buy-back contract, and iii) the manager receives equity-based compensations. By identifying the equilibrium strategy, we have illustrated that due to equity-based incentives, the manager may distort the operational decision to send a signal to external investors. The operational distortion causes an undesirable side-effect, which spreads the deadweight loss of signalling all over the supply chain. Such a deadweight loss of signalling could be significant to the supply chain entities and the shareholders. To remedy the inefficiency caused by equity-based incentives, we have proposed a new coordination mechanism that offers a menu of two different buy-back contracts. We have showed that the proposed mechanism can maximize supply chain efficiency because the manager does not need to distort the operational decision to signal.

There are many ways to extend our study. For example, in this paper we have considered the inventory decision as a signalling device. Another plausible device for signalling might be the capacity decision. While some technical changes are required to model capacity decisions as the signal, we believe that our main findings still hold because there are many similarities between the two formulations for inventory and capacity decisions. Next, in our current model, we have assumed that the firm types are binary. Generalizing our results to the case with more than two firm types would increase our understanding of the research problem. Note that Sobel (2009) provided an efficient algorithm to solve the signalling equilibrium with multiple types, which paves the way to solving the coordinating mechanism. Furthermore, the impact of risk-aversion and the release of public signals, such as analyst forecasts, would also be important future research directions.
APPENDIX A: PROOF OF PROPOSITIONS 1, 2, AND 3

Proof of Proposition 1

We construct the separating equilibrium by following the procedures described in Section 6.1 of Sobel (2009), to which we refer readers for the comprehensive and detailed procedures. The first step is to optimize \( U(l, q, BR(l)) \), where \( BR(l) \) is the best response of the financial market when the firm type is weak. Clearly, \( BR(l) = 0 \). Hence,

\[
U(l, q, BR(l)) = U(l, q, 0) = T_i(l, q),
\]

which is maximized at \( q = q_i^d \). In other words, at the separating equilibrium, the weak firm orders \( q_i^d \) units.

The second step is to optimize \( U(h, q, BR(h)) \), where \( BR(h) \) is the best response of the financial market when the firm type is strong, subject to the IC constraint:

\[
U(l, q, BR(h)) \leq U^*(l) = U(l, q_i^d, 0) = T_i(l, q_i^d).
\]

Note that \( BR(h) = 1 \) and hence \( U(h, q, BR(h)) = T_i(h, q) \) is concave in \( q \). To satisfy the above IC constraint, it requires that \( q \) is either sufficiently low (\( q \leq q_i \)) or sufficiently high (\( q \geq q_i \)). By the definitions of \( q_i \) and \( q_i \) (please refer to Figure 2), we observe that

\[
U^*(l) = \alpha T_i(h, q_i) + (1 - \alpha) T_i(l, q_i) = \alpha T_i(h, q_i) + (1 - \alpha) T_i(l, q_i).
\]

After some algebra, we find

\[
T_i(h, q_i) = U^*(l) + (1 - \alpha) \left[ T_i(h, q_i) - T_i(l, q_i) \right] = U^*(l) + (1 - \alpha) \delta_i(q_i),
\]

and

\[
T_i(h, q_i) = U^*(l) + (1 - \alpha) \left[ T_i(h, q_i) - T_i(l, q_i) \right] = U^*(l) + (1 - \alpha) \delta_i(q_i).
\]

The single-crossing condition of \( \delta_i(q) \) implies that \( T_i(h, q_i) > T_i(h, q_i) \). Note that \( T(h, q) \) is increasing over the interval \( (0, q_i) \) because \( q_i < q_i \), therefore, we can conclude that the order
quantity of the strong firm cannot be less than $q_1$. Using the concavity of $U(h, q_1)$, we find that the strong firm’s optimal order quantity is $q^*_h = q^d_h$ if $q^d_h \geq \bar{q}_1$ or $q^*_h = \bar{q}_1$ if $q^d_h < \bar{q}_1$.

Now we demonstrate that $\alpha$ determines whether $q^d_h \geq \bar{q}_1$ or $q^d_h < \bar{q}_1$. Using the definition of $\alpha$, we have

$$\alpha = \frac{T_1(l, q^d_h) - T_1(l, q^d_H)}{T_1(h, q^d_h) - T_1(l, q^d_H)}$$

and the fact that $q^d < q^d_H$, we see that if $\alpha \leq \alpha_1$, then

$$\alpha[T_1(h, q^d_h) - T_1(l, q^d_H)] \leq T_1(l, q^d_H) - T_1(l, q^d_h) = U^*(l) - T_1(l, q^d_h),$$

which is equivalent to

$$\alpha T_1(h, q^d_h) + (1 - \alpha)T_1(l, q^d_h) \leq U^*(l).$$

Figure 2 depicts that for $q^d < q < \bar{q}_1$, it holds that $\alpha T_1(h, q) + (1 - \alpha)T_1(l, q) > U^*(l)$. Therefore, the inequality (A1) implies $q^d_h \geq \bar{q}_1$ (i.e., case i) of Proposition 1). On the other hand, if $\alpha > \alpha_1$, then

$$\alpha T_1(h, q^d_h) + (1 - \alpha)T_1(l, q^d_h) > U^*(l),$$

which implies that $q^d_h < \bar{q}_1$ (i.e., case ii) of Proposition 1).

Finally, it is clear that i) the weak firm’s manager never orders more than $\bar{q}_1$ and hence, any order quantity $q > \bar{q}_1$ must come from the strong firm, and ii) the strong firm’s manager never orders less than $q^d_i$ and hence, any order quantity $q < q^d_i$ must come from the weak firm. Thus, the best response rule of the financial market is to take the action $BR(l)=0$ if the order quantity $q > \bar{q}_1$ and $BR(h)=1$ if $q < q^d_i$. This completes the proof.

**Proof of Proposition 2**
Equity-Based Incentives and Buy-Back Contracts

We need to prove the existence of contract parameters (i.e., $\rho_h$, $\rho_i$, and $K$) that satisfy the IC constraints (14) and (15). To this end, we note that because $\rho_h(T_h - \tilde{T}) \geq \rho_i(T_h - T_i)$ implies $\rho_h T_h - \rho_i T_h \geq \rho_h \tilde{T} - \rho_i T_i$, we can always find $K$ such that $\rho_h T_h - \rho_i T_h \geq K \geq \rho_h \tilde{T} - \rho_i T_i$. It is then trivial to verify that the IC constraints (14) and (15) are satisfied. Next, because $T_h \geq \tilde{T} \geq T_i$, we see that there always exist parameters $(\rho_h, \rho_i)$ such that $\rho_h(T_h - \tilde{T}) \geq \rho_i(T_h - T_i)$ is satisfied. Therefore, any $K$ between $\rho_h \tilde{T} - \rho_i T_i$ and $\rho_h T_h - \rho_i T_h$ will satisfy the IC constraints (14) and (15) and can achieve the first-best.

**Proof of Proposition 3**

For clarity, we replicate the LP problem as follows.

$$\max_{\rho_h, \rho_i, K} U_i = \gamma(\rho_h T_h - K) + (1 - \gamma) \rho_i T_i,$$

subject to the following constraints:

$$\rho_i T_h - \rho_h T_h \leq -K, \quad (A2)$$

$$\rho_h \tilde{T} - \rho_i T_i \leq K, \quad (A3)$$

$$\rho_h \leq 1. \quad (A4)$$

The Lagrangian of the optimization problem is

$$L = \gamma(\rho_h T_h - K) + (1 - \lambda) \rho_i T_i - \lambda_1(\rho_i T_h - \rho_h T_h + K) - \lambda_2(\rho_h \tilde{T} - \rho_i T_i - K) - \lambda_3(\rho_h - 1),$$

where $\lambda_1$, $\lambda_2$ and $\lambda_3$ are the Lagrange multipliers associated with constraints (A2), (A3), and (A4), respectively. Optimizing the Lagrangian with respect to $\rho_h$, $\rho_i$, and $K$, we find

$$-\lambda_2 \tilde{T} + (\gamma + \lambda_1)T_i - \lambda_3 = 0, \quad (A5)$$
Equity-Based Incentives and Buy-Back Contracts

\[(1 - \gamma + \lambda_2)T_i - \lambda_i T_h = 0, \quad \text{(A6)}\]
\[\lambda_2 - \lambda_i = \gamma. \quad \text{(A7)}\]

Solving the linear system of equations (A5) to (A7), we find that
\[\lambda_i = \frac{T_i}{T_h - T_i}, \quad \lambda_2 = \frac{T_i}{T_h - T_i} + \gamma, \quad \text{and} \quad \lambda_3 = (T_h - \bar{T}) \left( \frac{T_i}{T_h - T_i} + \gamma \right).\]

All the Lagrangian multipliers are positive. Hence, constraints (A2) to (A4) must be binding.

When constraints (A2) to (A4) are binding, we obtain \(\rho^*_h = 1, \rho^*_f = \frac{T_h - \bar{T}}{T_h - T_i}, \text{ and } K^* = \frac{T_h - \bar{T}}{T_h - T_i} T_h.\)

**APPENDIX B: OTHER EQUILIBRIA**

**Partial-Pooling Equilibrium**

In this section, we characterize the partial-pooling equilibrium for the proposed signalling game and then demonstrate that it is dominated by the separating equilibrium described in Proposition 1.

To characterize the partial-pooling equilibrium, we first show that the strong firm does not randomize the order quantity. Suppose that the strong firm’s ordering strategy is \(1 > S(h, q) > 0\) for some \(q\). In other words, the strong firm may order \(q\) units at the equilibrium with a positive probability. First, according to equation (7), we see that the pricing action taken by the financial market is
\[a = \Pr(t = h \mid q) = \frac{\rho S(h, q)}{\rho S(h, q) + (1 - \gamma) S(l, q)},\]
which is increasing in \(S(h, q)\). Second, equation (5) implies that the interim share price \(V(a, q)\) is increasing in \(a\), hence, it must be increasing in \(S(h, q)\) as well. Third, recall that the utility of the strong firm’s manager is \(U(h, q, a) = \alpha V(a, q) + (1 - \alpha) T_i(h, q)\), which is also increasing in \(S(h, q)\).
Finally, using equation (6), we observe that if ordering $q$ maximizes the manager’s utility, he/she would increase $S(h,q)$ to 1. Therefore, the strong firm does not have incentives to randomize the order quantity. Let $q_e$ be the order quantity of the strong firm in a partial-pooling equilibrium such that $S(h,q_e)=1$.

Next, we analyze the weak firm’s ordering policy. Because pooling with the strong firm would lead to a higher share price, the weak firm might order $q_e$ units with a positive probability. For expositional simplicity, we write $s=S(l,q_e)>0$. First, we note that if the weak firm does not order $q_e$ units, its type will be revealed because the strong firm orders $q_e$ units with probability 1. Therefore, it is clear that the best action for the weak firm that does not order $q_e$ units is to order $q_e^d$ units. Second, by ordering $q_e^d$ units, the weak firm’s manager attains a utility of $U^*(l)$. In addition, we see that the pricing action taken by the financial market is decreasing in $s$, which means that the weak firm cannot increase $s$ indefinitely. Third, equation (6) would imply that the weak firm is indifferent between ordering $q_e$ and $q_e^d$ units. Let

$$a_e = \Pr(t = h | q_e) = \frac{\gamma}{\gamma + (1 - \gamma)s}$$

be the equilibrium pricing action taken by the financial market when the observed order quantity is $q_e$. Then, we observe that the indifference condition implies that

$$U(l, q_e, a_e) = aV(a_e, q_e) + (1 - a)T_l(l, q_e) = U^*(l).$$  \(B1\)

Finally, by expanding $V(a_e, q_e)$, one can solve $s$ to obtain the ordering policy of the weak firm, and this will complete the characterization of the partial pooling equilibrium, where the strong firm orders $q_e$ units with probability 1 and the weak firm orders $q_e$ units with probability $s$.

Now, we demonstrate that the strong firm’s manager is strictly better off with the separating equilibrium while the weak firm’s manager receives the same utility at both equilibria.
Equity-Based Incentives and Buy-Back Contracts

While there is a continuum of $q_e$ (i.e., there are multiple partial-pooling equilibria characterized above), we show that any partial-pooling equilibrium is dominated by the separating equilibrium. First, note that Proposition 1 indicates that the weak firm’s manager obtains a utility of $U^*(l)$, which is the same as he/she gets at the partial-pooling equilibrium. Second, Proposition 1 also shows that the strong firm’s manager attains a utility that is at least $T_i(h, \bar{q}_1)$. Furthermore, the proof of Proposition 1 implies that $q_e$ cannot be greater than $\bar{q}_1$; otherwise, $s=0$ (please refer to the proof of Proposition 1). Using these two observations and equation (B1), we find that the utility of the strong firm’s manager is

$$U(h, q_e, a_e) = \alpha V(a_e, q_e) + (1-\alpha)T_i(h, q_e) = \alpha V(a_e, q_e) + (1-\alpha)T_i(l, q_e) + (1-\alpha)\delta_i(q_e)$$

$$= U^*(l) + (1-\alpha)\delta_i(q_e) < U^*(l) + (1-\alpha)\delta_i(\bar{q}_1) = T_i(h, \bar{q}_1),$$

where the last inequality is implied by the single-crossing condition of $\delta_i(q)$. Therefore, we see that the strong firm’s manager does no worse at the separating equilibrium than at the partial-pooling equilibrium.

**Pooling Equilibrium**

By definition, in a pooling equilibrium, all the firms order the same quantity denoted by $q_p$. In our model setting, we first see that $q_p$ cannot be greater than $\bar{q}_1$; otherwise, $S(l, q_p)=0$. Next, when the order quantity $q_p$ is observed, the financial market takes the pricing action $a=\gamma$ because no inference about the firm type can be drawn beyond the prior belief. Hence, the utility of the strong firm’s manager is

$$U(h, q_p, \gamma) = \alpha V(\gamma, q_p) + (1-\alpha)T_i(h, q_p),$$

(B2)

and that of the weak firm’s manager is
Equity-Based Incentives and Buy-Back Contracts

\[ U(l, q_p, \gamma) = \alpha V(\gamma, q_p) + (1-\alpha)T_i(l, q_p). \]  \tag{B3}  

Again, we emphasize that there is a continuum of \( q_p \) that can be chosen.

Now we prove that the pooling equilibrium cannot satisfy the Intuitive Criterion (Cho & Kreps 1987, page 202). For clarity, we replicate the Intuitive Criterion below:

**Definition B1**: (The Intuitive Criterion) For each out-of-equilibrium message \( m \), form the set \( S(m) \) consisting of all type \( t \) such that \( u'(t) = \max_{r \in BR(T(m),m)} u(t,m,r) \). If for any message \( m \), there is some type \( t' \) (necessarily not in \( S(m) \)) such that \( u'(t') = \min_{r \in BR(T(m),S(m),m)} u(t',m,r) \), then the equilibrium outcome is said to fail the Intuitive Criterion.

First, in our game setting, the Intuitive Criterion means the following. If an order quantity \( q \neq q_p \) is observed, the financial market assesses that which type gains by ordering this out-of-equilibrium quantity. Second, Definition B1 implies that all the types that gain by ordering \( q \neq q_p \) is in the set \( S(q) \). Furthermore, if any type \( t' \) not in \( S(q) \) will not be better off by switching to this order quantity \( q \), then the equilibrium is said to fail the Intuitive Criterion because the type in \( S(q) \) has incentives to deviate.

Next, note that because our model has only two types, testing the Intuitive Criterion is not very complicated. Suppose that \( \varepsilon > 0 \) and the strong firm orders \( q_p + \varepsilon \) units. Then, the maximum payoff to the strong firm’s manager is

\[ U(h, q_p + \varepsilon, BR(h)) = U(h, q_p + \varepsilon, l) = T_i(h, q_p + \varepsilon). \]
Equity-Based Incentives and Buy-Back Contracts

Note that i) the term $V(\gamma, q_p)$ is less than $V(1, q_p)$ because $\gamma < 1$ and ii) $V(a, q_p)$ is increasing in $a$. In addition, from equation (B2), we observe that $U(h, q_p, \gamma) < T_i(h, q_p)$. Therefore, we see that when the disturbance $\varepsilon$ in the order quantity is small, it holds that

$$T_i(h, q_p + \varepsilon) - T_i(h, q_p) > U(h, q_p, \gamma),$$

which implies that the strong firm can gain by deviating to order a slightly higher order quantity. On the other hand, it is easy to show that the weak firm does worse by deviating. Specifically, one can show that

$$U(l, q_p + \varepsilon, BR(l)) = U(l, q_p + \varepsilon, 0) = T_i(l, q_p + \varepsilon) < U(l, q_p, \gamma).$$

In summary, a slightly higher order quantity $q$ will be interpreted as coming from the strong firm. The single-crossing condition of $\delta_l(q)$ implies that the weak firm finds less profitable to mimic when the inventory level increases. Because the strong firm with higher demand can afford to order more while the weak firm cannot, the strong firm always has incentives to break away from the pool. Hence, the pooling equilibrium does not pass the Intuitive Criterion.
Equity-Based Incentives and Buy-Back Contracts

References


Equity-Based Incentives and Buy-Back Contracts


Equity-Based Incentives and Buy-Back Contracts


### Table 1: Summary of Relevant Literature and Our Contributions

<table>
<thead>
<tr>
<th>Supply chain buy-back literature</th>
<th>Relevant Results</th>
<th>Gaps in Literature</th>
<th>Our Contributions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>The use of buy-back contracts to coordinate supply chains</td>
<td>Operational decision is disconnected from the financial market.</td>
<td>Presenting a multi-stage model to explore the linkages among equity-based incentives, supply chain contracts, and signalling.</td>
</tr>
</tbody>
</table>

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<tr>
<th>Signalling literature</th>
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<th>Gaps in Literature</th>
<th>Our Contributions</th>
</tr>
</thead>
<tbody>
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<td></td>
<td>Single-dimensional signalling (e.g., Bebchuk &amp; Stole, 1993; Hayes &amp; Schaefer, 2009).</td>
<td>Multi-dimensional signalling</td>
<td>Developing a mechanism to mitigate supply chain inefficiencies after characterising the multi-dimensional equilibrium.</td>
</tr>
</tbody>
</table>
Equity-Based Incentives and Buy-Back Contracts

The prior belief of the firm’s demand outlook is formed.

The firm’s manager receives private information about the demand and decides how much inventory to order.

The supplier delivers the products. The operational decision of the firm’s manager is observed by the financial market. Investors update their belief about the demand outlook.

A round of trading of the firm’s shares takes place and the interim share price is determined.

The demand is realized. The firm uses the available inventory to satisfy the demand. Unmet demand is lost and unsold inventory is returned according to the contract.

The terminal cash flow is re-distributed.

Figure 1: Timeline of the model
Figure 2: Illustration of $q^*$ and $\bar{q}_i$

*Inventory level $q$*
Equity-Based Incentives and Buy-Back Contracts

Figure 3: Comparison of Total Supply Chain Profits