AN ASSIGNMENT MODEL FOR DYNAMIC LOAD PLANNING OF INTERMODAL TRAINS

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Abstract - Intermodal terminals are important facilities in the container transport network, providing an exchange of containers between road and rail transport. Numerous factors can affect throughput in such highly integrated systems. These include numbers and types of equipment, physical layout, storage capacity and operating strategies. This study aims to improve operating strategies by developing an analytical tool to assist in load planning of container trains. The problem investigated can be described as a dynamic assignment problem with many uncertain parameters. Numerical investigations focus on tuning the proposed model to deal with the uncertainties.

Keywords - intermodal terminal, assignment problem, load planning, containers

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1. INTRODUCTION

Intermodal terminals are a point of interface between road and rail transport for containerized goods. Containers processed in intermodal terminals vary in length, height, weight and handling requirements. They are carried by trains consisting of a sequence of wagons which vary in length, deck height and carrying capacity. Containers are transferred to/from wagons by a variety of handling equipment such as forklifts, reach stackers and gantry cranes. Terminals are also varied in such aspects as layout, handling equipment, storage, operating policies and volume of containers transshipped.

This study considers a terminal where containers are transferred to and from trucks on a platform directly adjacent to the transshipment tracks. The platform also contains a short-term storage area (buffer) typically one or two rows wide and stacked up to two containers high. Most importantly, unloading and loading of a train are performed simultaneously in the considered terminal.

When a train arrives at the platform, unloading begins immediately. Meanwhile, several trucks may be waiting to collect or deliver containers and continue to arrive randomly within a given window of operation. When a truck arrives carrying an outbound container, it is directed to the platform location adjacent to its assigned wagon. If the wagon has space available, the outbound container is loaded directly from the truck. Otherwise, the outbound container must be placed on the ground and wait until its assigned wagon is unloaded.

The grounding of outbound containers is called double handling, and should be avoided where possible. Double handling of a container requires one additional container move and therefore additional time to complete service of the train. Also, double handling increases the number of containers stored on the platform buffer.
This increases the possibility of shuffling containers in a stack if the bottom container is required. At least one additional move is required for shuffling. Therefore, reduction of double handling is an important aspect in reducing train service time.

The benefit of reduced container handling often comes at the expense of unfavorable weight distribution on the train. It is important to have weight biased towards the front of the train to reduce wear on braking mechanisms. This study proposes several techniques for determining an appropriate balance between container handling and weight distribution. The proposed methods provide an assignment of containers to slots on the train, which is called a load plan.

The considered operating environment makes load planning difficult particularly since truck arrival times are not known in advance. Load plans must therefore be dynamic by adapting as new information becomes available. Before describing the proposed model, the following section gives a review of recent relating to load planning. This is followed by a more detailed description of the considered problem and then a formal definition of the model. Numerical investigations tune the model using simulations of the intermodal terminal.

2. LITERATURE ON INTERMODAL TERMINALS

Bontenkoning et al [1] state that intermodal research is emerging and could be a research field in its own right. Their paper along with Macharis and Bontenkoning [2] provide extensive reviews of literature in the area of intermodal terminals. A brief literature review is provided here, focusing on studies related to load planning.

Possibly the most closely related study to this paper is that of Bostel and Dejax [3]. They propose a mathematical model to minimize container handling at a rail-rail transfer terminal. The model determines the optimum placement of containers on an
origin train, destination train and in short term storage. A heuristic was developed to obtain good solutions to the model.

Another study by Feo and Gonzalez-Velrade [4] considers load planning in the context of a piggyback system. In a piggyback system containers are loaded onto a wagon whilst still mounted on the trailer that carried them into the terminal. This study proposed a model for optimally assigning highway trailers to wagon hitches. They estimate that by using their methodology, a particular network operator could save around $4500 per train. Powell and Carvalho [5] consider a similar but broader version of this problem. They use a logistic queuing network model to ensure that the flow of wagons across the network satisfies the requirements of individual terminals.

A significant factor in load planning is the formation of trains which is influenced by a combination of demand and the available empty railcars. In our study train formation and railcar availability are inputs of the model however several papers have addressed these issues. Chih et al [6] describe a menu driven routing and inventory logistic system (RAILS). This package is based on a mixed integer model using demand forecasts to assign wagons and containers to trains across the network. They use a decomposition heuristic to obtain solutions for a planning horizon of about two weeks. An earlier study by Chih and Van Dyke [7] proposed a dynamic multi-commodity transshipment model to optimize the movement of empty wagons over an intermodal network. More recently, Nozick and Morlok [8] addressed this problem in a broader sense. They proposed a mixed-integer model to determine decisions such as train lengths, engine allocations, traffic routings, equipment pools and work allocation for the terminals. Because of the complexity of their model they developed a heuristic to obtain good solutions in an acceptable time frame.
Another input of the load planning model is the location of wagons on the transshipment tracks. Kozan [9] addresses this problem by evaluating several policies for allocating trains to transshipment tracks. By making certain approximations to intermodal operations, Kozan was able to derive estimates of train handling times. These estimates were used to make comparisons of the track allocation policies.

This study aims to expand the current research of load planning by considering a different terminal system to those of previous studies. Under the considered system load planning problem operates under different objectives and constraints. The following section describes the considered problem in detail.

3. LOAD PLANNING PROBLEM (LPP)

As mentioned previously, containers and wagons come in a variety of lengths in most real world terminals. This study investigates a conceptualized version of the real problem by assuming that all containers have equal length. Each wagon is divided into slots of one container length. For example a wagon with five slots would be capable of holding five containers. A complete load plan consists of a full assignment of containers to slots. By making the assumption of equal length, the model is greatly simplified so that branch and bound techniques can be applied.

3.1. Objectives

The minimum amount of handling for a container occurs when it is transferred directly from the truck to an adjacent wagon. Any additional handling will be considered as excess handling time, which can be separated into two components. Double handling occurs whenever a truck is serviced by handling equipment, and the assigned slot is occupied by an inbound container. The container must be placed on
the ground and loaded onto the train later when its slot becomes vacant. This requires a second handling operation, which adds to the excess handling time.

*Travel* of handling equipment occurs when the slot assigned to a waiting container is changed during subsequent revisions of the load plan. In this case the container will no longer be waiting adjacent to its intended slot and must be carried there by handling equipment. In some cases it may be beneficial to incur travel if double handling or weight distribution are improved as a result.

To reduce wear on braking mechanisms, the *mass distribution* of the train should be biased towards the front. In practice, this is difficult to achieve since trucks carrying heavy containers may arrive before there is any space available at the front of the train. When this occurs, the weight distribution is often compromised to avoid double handling of containers.

### 3.2. Revising the Load Plan

Exact arrival times of containers are not known which is why LPP is dynamic. However many terminals have some knowledge of the distribution of container arrivals before train arrival, during transhipment and after train departure. There are also terminals with pre-notification of truck deliveries and pick-ups. The proposed model can be applied in each of these cases. This is because the distribution of arrivals is not an input nor an assumption to the model. Instead, the model should be used to revise the load plan whenever certain events occur. These are events that change the suitability of the current load plan based on the objectives. The timing and sequence of these events depends on the move sequences of handling equipment, and truck arrival times which are both uncertain parameters to a certain degree.

In the system considered here, events that trigger a revision of load plan include the following: (1) inbound container unloaded from a slot; (2) truck arrives at gate.
For the dynamic LPP model the time horizon is divided into periods of unequal length. Occurrences of events 1 and 2 trigger the beginning of a new period after which the load plan is revised.

3.3. System States

Outbound containers in LPP can exist in one of several states: (O) outside of the terminal; (A) onboard a truck waiting at the terminal gate for direction to a platform location; (K) onboard a truck waiting on the platform; (G) grounded on the platform; or (L) loaded onboard a wagon. All outbound containers will pass through states O, A, K and L. Containers that additionally pass through state G incur double handling.

Slots also exist in one of several states: (N) not vacant and occupied by an inbound container; (V) vacant; or (L) not vacant and loaded with an outbound container. A slot in state V is available to be assigned or loaded with an outbound container. Containers can be assigned to slots in state N but cannot be loaded until the slot becomes vacant. Containers cannot be assigned to slots in state L.

4. MODEL FORMULATION

This study proposes a dynamic assignment model that can be used to determine and revise load plans. Because of uncertainty, solving the model over a rolling horizon cannot guarantee an optimal solution. However, this study aims to demonstrate the effectiveness of this approach. Before describing the proposed model, a general description of assignment problems is given below.

Given a weighted bipartite network $G = (N_1 \cup N_2, A)$ with $|N_1| \leq |N_2|$, the problem is to find a perfect assignment of minimum weight. A perfect assignment can be described as a selection $S$ of arcs in $A$. $S$ defines a unique assignment for every node in $N_1$ that $|S| = |N_1|$. 
In terms of LPP, one node set consists of outbound containers whilst the other consists of slots. It is assumed that there are equal numbers of slots and containers however this can be relaxed. At the beginning of the time horizon, the set of arcs consists of all possible assignments between containers and slots. Periodically, certain arcs are removed to prevent loaded containers and slots from being reassigned.

The following notation is used to describe the dynamic assignment model for LPP. The proposed model will be denoted as (LPA) which stands for load planning assignment model.

**Variables**

- $\rho_{ij}$: 1 if container $i$ is matched to slot $j$ in period $t$, 0 otherwise
- $z_t$: objective cost evaluated in period $t$

**Parameters**

- $T$: number of time periods
- $n$: number of slots and outbound containers
- $S_t^u$: set of slots with status $u$ at period $t$, $u \in \{N,V,L\}$
- $d_j$: position along platform of slot $j$
- $C_t^u$: set of containers in state $u$ at period $t$, $u \in \{O,A,K,G,L\}$
- $m_i$: mass of container $i$
- $A_t$: set of arcs in bipartite graph for period $t$. This set contains arcs linking loaded containers to their corresponding slots as well as arcs linking all remaining containers with the remaining slots.

$$A_t = \{i, j\} \mid (i \in C_t^L, j \in S_t^L, \rho_{ij(t-1)} = 1) \lor$$

$$\quad (i \in C_t^O \cup C_t^A \cup C_t^K \cup C_t^G, j \in S_t^N \cup S_t^V)$$

- $h_{ij}$: arc weight for double handling from $i$ to $j$ at period $t$, $(i, j) \in A_t$.
- $d_{ij}$: arc weight for excess travel from $i$ to $j$ at period $t$, $(i, j) \in A_t$.
- $c_{ij}$: arc weight for mass distribution from $i$ to $j$ at period $t$, $(i, j) \in A_t$. 
scaling parameters for double handling, excess travel and mass distribution respectively.

At the beginning of the time horizon, containers and slots are in their initial states which are given. This is called period 0 and LPA is solved for this initial period. Whenever an event occurs to trigger a new period, the states of containers and slots are updated accordingly and the LPA model is resolved for the new period. In the end period $T$, the model is solved for the last time giving $z_T$, the objective cost of the final load plan. The LPA model for some period $t$ can be described as the following.

$$\min_{ij} z_t = \sum_{i=1}^{n} \sum_{j=1}^{n} (\gamma_i h_{ij} + \gamma_2 d_{ij} + \gamma_3 c_{ij}) \rho_{ij}$$ (1)

subject to

$$\sum_{j \in (i,j) \in A_t^i} \rho_{ij} = 1 \quad i = 1, \ldots, n$$ (2)

$$\sum_{i \in (i,j) \in A_t^j} \rho_{ij} = 1 \quad j = 1, \ldots, n$$ (3)

$$\rho_{ij} = 0 \text{ or } 1 \quad (i,j) \in A_i$$ (4)

The objective function (1) is to minimise the sum of weights for the selected arcs. Constraints (2) and (3) ensure that containers are assigned to exactly one slot and vice versa.

5. OBJECTIVE FUNCTION

Since LPA is a multi-objective problem, the objective function is formulated as a weighted sum. Arc weights are calculated based on the individual objectives which are to minimise excess handling time (double handling and travel) and to optimise the weight distribution of the train. The remainder of this section describes how these arc weights are formulated.
5.1. Weight Distribution

Weight distribution is evaluated by determining the centre-of-mass (CM) of the train. The objective is to minimise the distance of CM from the front of the train. The CM formula for a rod in one dimension is given by

\[ CM = \int_0^L \omega(x) \, dx + \int_0^L \omega(x) \, dx, \]

where \( L \) is the length of the rod and \( \omega(x) \) gives the density of the rod at point \( x \). This formula is applied to a loaded train by considering the train as a one dimensional rod. The calculation is simplified by assuming there are no gaps between adjacent slots. Using the length of a container as the unit of measure, the density function of the train (with no gaps) is given below.

\[
\omega(x) = \begin{cases} 
\sum_{i=1}^n \rho_{ii} m_i, & 0 \leq x \leq 1 \\
\vdots & \\
\sum_{i=1}^n \rho_{jj} m_j, & j-1 \leq x \leq j \\
\vdots & \\
\sum_{i=1}^n \rho_{nn} m_n, & n-1 \leq x \leq n 
\end{cases}
\]  

(5)

Applying the CM formula to this function gives equation (6).

\[
CM_t = \frac{\sum_{j=1}^n (2j-1) \sum_{i=1}^n \rho_{ij} m_i}{2 \sum_{i=1}^n m_i}
\]  

(6)

Minimising the distance of centre-of-mass from the front of the train is equivalent to minimising \( CM_t \). Therefore, it is required from (1) that \( \sum_{j=1}^n \sum_{i=1}^n c_{ij} \rho_{ij} = CM_t \).

By substituting the right-hand-side of (6) for \( CM_t \) the arc weight component for weight distribution, \( c_{ij} \), can be determined.

\[
c_{ij} = \frac{(2j-1)m_j}{2 \sum_{k=1}^n m_k}
\]  

(7)
5.2. Travel

Recall that the travel objective refers to excess movement of handling equipment caused by changing the assigned slot for a waiting container. This excess movement is penalised by using \((8)\) in the arc weights. Note that containers outside of the terminal, that is \(i \in C^O_t\), do not incur any travel penalty.

\[
d_{ij}^t = \begin{cases} 
  d_j - \sum_{q \in \mathcal{C}} d_q \rho_{qij}, & i \in C^H_t, \tau \in \{1, ..., t-1\} \\
  0, & \text{otherwise}
\end{cases}
\]  

\[(8)\]

5.3. Double Handling (Heuristics)

For a container assigned to an occupied slot, it is unknown whether double handling will occur. This is because future move sequences of handling equipment are unknown. The slot may or may not be unloaded before the container is serviced. This section describes how uncertainty is dealt with by the LPA model.

There are two situations where the double handling status of a given container can be known with certainty for the current load plan. Firstly, if the container has been placed on the ground then double handling has definitely occurred. Secondly, double handling will definitely not occur if a vacant slot is assigned and the container has not been grounded. In these situations, the double handling term is weighted 0 or 1 accordingly.

In all other situations double handling is uncertain but an educated guess can be made. Firstly, for any containers still outside of the terminal it is assumed that double handling will not occur. Since their arrival time is unknown, nothing can be inferred about double handling. The assumption is intended to save the vacant slots for containers inside the terminal.
The next group of containers includes those waiting on board trucks that are assigned to occupied slots. The occupied slot may or may not be unloaded before the outbound container is serviced. However, in reality move sequences are somewhat predictable because they give priority to transfers to/from waiting trucks. This is because terminal operators want to minimise the waiting time of their customer’s trucks.

Based on this knowledge, a strategy can be formulated to determine appropriate arc weights for the double handling objective. The double handling penalty for assigning slots with waiting trucks should be lower than those without. This is to reflect the probability of double handling in each case. Equation (9) describes the double handling arc weight for the LPA model. Let $S^K_t (\subseteq S^N_t)$ be the set of occupied slots with trucks waiting for inbound containers.

$$h_{ij} = \begin{cases} 1 & i \in \bigcap_{t=1}^T C^G_t \text{ or } i \in C^A_t \cup C^K_t, j \in S^K_t \setminus S^K_t \\ \omega & i \in C^A_t \cup C^K_t, j \in S^K_t \\ 0 & \text{otherwise} \end{cases}$$ \ (9)

The upper alternative in (9) applies to containers that have already been grounded, or those waiting on board a truck with slot $j$ being occupied and no waiting truck. The second alternative applies to containers waiting on board a truck with slot $j$ being occupied but with a truck waiting. Parameter $\omega$ is an adjustable value such that $0 \leq \omega \leq 1$. This parameter can be viewed as an estimation of the probability that double handling will occur.

5.4. Using the LPA Model

To illustrate how the arc weights change from one period to the next, consider the following simple example. A particular train consists of three slots to house three
outbound containers. Initially all slots are occupied by inbound containers which must be unloaded. Since the $c_{ij}$ terms remain constant from one period to the next they will be ignored and we assume that all outbound containers are of equal mass. It is also assumed that container and slots are 20 feet long.

Consider the following series of events: truck arrives to deliver container 1; truck arrives to collect from slot 2; slot 2 is unloaded to its truck; and finally, container 1 is transferred from truck to slot 2. Table 1 shows the progression of the double handling and travel penalties for arcs relating to container 1. The load plan is revised after the occurrence of each event based on the updated penalty values.

Initially, container 1 is outside of the terminal so that no double handling or travel penalty is incurred. At this point container 1 has been assigned to slot 1. Period 1 begins when a truck arrives to deliver container 1. Since all slots are occupied the full handling penalty is incurred for all slots. When period 2 begins, container 1 is waiting adjacent to slot 1 so that assignment to slots 2 and 3 incur travel penalties of 20 feet and 40 feet respectively. A truck has arrived to collect from slot 2 so that only a partial handling penalty applies to slot 2. Container 1 would be assigned to slot 2 provided $\omega$ was small enough to offset the travel penalty. In the third period slot 2 has been unloaded so no handling penalty is applied to this slot. Finally in period 4 container 1 is loaded to slot 2. All other arcs from container 1 and to slot 2 are removed from the graph. The arc weight for arc (1,2) is now fixed for the remainder of the time horizon.

In practice a realistically sized problem would have about 100 containers. With this many containers, load plan revisions would be triggered about 300 times (100 outbound container truck arrivals, 100 inbound container truck arrivals, 100 inbound containers unloaded). The model must be solved in less than one minute because a
truck may be waiting at the gate for directions or a crane waiting to start its next transfer.

Applying a load planning model within a real terminal would result in two major benefits to terminal operators. Firstly, by improving the mass distribution of trains the wear on braking mechanisms is reduced which reduces the chance of failure and unscheduled maintenance. Secondly, less responsibility is placed on the operators of handling equipment. These operators are often the ones who decide where containers are placed on the train. By removing this responsibility the operators spend less time assessing the situation and can therefore work more quickly. Also, there is less pressure placed on inexperience operators because they are merely following directions from the load plan.

6. LPP SIMULATION MODEL

Using the LPA model in practice would require a real time information system to track the status of all containers, handling equipment and trucks in the terminal. This information would provide inputs to the model for revising the load plan. Once the load plan was revised it would be communicated to the gate for directing trucks and also to the handling equipment. In most terminals existing infrastructure could perform this task using wireless networks and remote terminals. In order to test the LPA model in such an environment a simulation model has been developed.

This model will be described in terms of entities and the relationships between them. Each entity represents a particular player in the process of load planning and container handling. The simulation model includes the following entities: slots; inbound containers; outbound containers; storage area; platform; arrival generator; gate; handling equipment; and the load planner.
The first five entities listed are passive players, only changing state by the actions of other entities. The storage area and platform entities are queues associated with the handling equipment. It is assumed there are no reshuffles in storage and no conflicts on the platform. Arrivals of trucks for pick-up and delivery are generated by the arrival generator. When a truck arrival occurs, the gate entity directs the truck to a specific location on the platform.

The handling equipment behave like forklifts or reach stackers. They can cross each others path but are restricted from performing moves within a given safety margin. Move sequences are generated using simple rules that are discussed in subsequent sections.

The most important entity is the load planner. This entity observes the system for the occurrence of a trigger to revise the load plan. Recall that the triggers are the unloading of a slot, and a truck arriving at the gate carrying a container. When a trigger occurs, the load planner provides input parameters to the LPA model and solves it to obtain a revised load plan. This load plan is then communicated to the handling equipment and gate entities.

Figure 1 gives a snapshot of the system during the simulation of loading operations for a hypothetical train. A truck is shown to have arrived at the gate carrying an outbound container. Its next move will be to the assigned platform location based on the LPA load plan. Meanwhile the arrival generator has generated the next arrival, a truck for the collection of a specific inbound container. The container associated with a given truck is determined randomly.

There are four trucks waiting on the platform, two collecting trucks and two delivery trucks. Truck (ii) is about to be serviced by a handling machine that will perform the following sequence of movements. First it will travel to the truck and lift
the container. Then it will carry the container along the platform to the assigned slot. This movement is penalised by the LPA model. After loading the container the handling vehicle will be ready for the next job. Truck \((iv)\) is a collecting truck that is about to be serviced by a second handling machine.

An inbound container in the storage area is waiting to be transferred to truck \((i)\). The other inbound container in storage must wait for its truck to arrive before being transferred. There is also an outbound container that was earlier placed in the storage area.

There are several outputs produced by the simulation model. The main result is the final load plan and its objective function evaluation \(z_T\). Another important result is a breakdown of the load plan giving the number of double handling operations, carrying distance travelled by handling equipment, and the train’s centre-of-mass. Also produced is a detailed record of entity attributes as the simulation progressed. Thus every movement and activity that occurred during the simulation can be traced.

Inputs to the simulation model include the number of slots and containers. Other inputs are the masses of the outbound containers and the list of slots initially carrying inbound containers. For the truck arrival generator, a probability distribution for truck inter-arrival times is required. An exponential distribution was used for this study. The final input required is a value for the parameter \(\omega\) of equation \(9\) in the LPA model. The values assigned to these input parameters are discussed in a later section.

7. STATIC LPA MODEL

To assess the performance of using the LPA model, it would be useful if the optimal load plan could be determined. This could be made possible by developing a static version of the LPA model. This model would take as its inputs the arrival times of all trucks to the terminal. Additionally the model would have to incorporate a
mathematical model to predict handling equipment move sequences based on a given load plan. Unfortunately, this model would be difficult to formulate, difficult solve and quite inflexible. However, a simplified load planning environment can be developed so that handling equipment move sequences are predictable without creating additional complexity in the model. Thus, the performance of the LPA model within an LPP simulation can be evaluated for a simplified load planning environment.

The simplification used to make move sequences predictable was to assume that move sequences can be determined using truck arrival times only. Under this assumption the move sequences are invariant to load plans. Therefore the move sequences are defined by parameters to the static model, rather than a complex arrangement of variables and constraints. Using this assumption, a static version of LPA (called SLPA) is presented below.

\[
\min z_T = \sum_{(i,j) \in A} (h_{ij} + c_{ij}) \rho_{ij} 
\]

subject to

\[
\sum_{(i,j) \in A} \rho_{ij} = 1 \quad i = 1, \ldots, n
\]

\[
\sum_{(i,j) \in A} \rho_{ij} = 1 \quad j = 1, \ldots, n
\]

\[
\rho_{ij} = 0 \text{ or } 1 \quad (i, j) \in A
\]

where \( A = \{(i, j) | i = 1, \ldots, n, j = 1, \ldots, n\} \)

The main difference between the dynamic and static models is that time subscripts are no longer needed. Another difference is in how the arc weights are calculated. For SLPA the centre-of-mass term \( c_{ij} \) remains unchanged from (7). Since the model is static, there are no revisions of the load plan so that no excess travel of handling
equipment is incurred. Therefore $d_{ij} = 0$ for all containers and slots. Finally, the static double handling term $h_{ij}$ equals one if slot $j$ has not been unloaded before the handling equipment services container $i$.

$$h_{ij} = \begin{cases} 1 & \exists \tau : i \in C_{r-1}^K, i \not\in C_{r}^K, j \in S_{r}^N \\ 0 & \text{otherwise} \end{cases} \quad (14)$$

Note that for SLPA, the sets $C_t^O$, $C_t^A$, $C_t^K$, $S_t^N$, $S_t^L$ and $S_t^V$ are known for all $t$ and are the input parameters for the model. This data can be collected during an LPP simulation and supplied to the SLPA model. Under the prescribed assumption, the static model precisely predicts the objective function evaluated for an LPP simulation of any load plan. This is because the occurrence of double handling will be unchanged for any container-slot assignment regardless of the load plan. The independence of move sequences from load plans has been achieved by imposing the following assumptions on the operating policies of handling equipment:

- handling equipment are dedicated to inbound or outbound containers, not both;
- handling equipment servicing inbound containers are controlled by a deterministic, load plan independent policy;
- handling equipment servicing outbound containers operate based on a first come first served policy (based on truck arrival times);
- service time is constant; and
- grounded outbound containers are loaded onto the train after all outbound container trucks have been serviced.

There are several possibilities for a deterministic control policy for handling equipment servicing inbound containers. For this study, the handling equipment gives highest priority to inbound containers on wagons with trucks waiting. The second
priority is to inbound containers on the ground with trucks waiting. Inbound containers on wagons with no waiting truck are given the lowest priority. When selecting where to go next, handling equipment will service the nearest inbound container of highest priority.

Although these particular assumptions are unrealistic, the underlying structure of the load planning environment remains in tact. Therefore, the performance of the LPA model should be similar when applied with more realistic operating policies. For this reason SLPA is used in numerical experiments to provide a performance benchmark for the dynamic LPA model. After running an LPP simulation, the truck arrival times and move sequences are provided as inputs to the SLPA model. The model is then solved to give the optimal solution to the dynamic problem. This gives a tolerance for the solution quality obtained using the dynamic LPA model.

8. SOLUTION TECHNIQUES

There are several algorithms available for the assignment problem. These include the successive shortest path algorithm, Hungarian algorithm, and the cost scaling algorithm. Of these methods the most efficient is a modification of the cost scaling algorithm which runs in \( O \ n^{5/2} \log(nW) \), where arc weights are bounded by \( W \). Ahuja et al [10] provide a review of these methods and other related network flow algorithms. For this study CPLEX was used to solve both dynamic and static LPA models.

9. NUMERICAL EXPERIMENTS

This section gives the results of simulation experiments to compare LPA solutions for several values of \( \omega \) (recall (9)). These solutions are also compared to those obtained for SLPA and a local search heuristic (LS). The neighbourhood used for
local search is based on an interchange of two containers in the load plan. At each iteration the best possible interchange is selected and the process is repeated until no further improvement can be gained.

This section is divided into three parts. In the first part, a simplified handling model is used so that SLPA can be used as a benchmark. Several values of $\omega$ are compared and the best value is used to evaluate the performance of LS. The second part uses a more realistic handling model and again compares several values of $\omega$ and evaluates LS. In the final part, a range of values for the objective parameter $\gamma_3$ are tested to give an insight into the trade off between handling time and centre-of-mass.

9.1. Dynamic vs. Static

As required for SLPA, a constant service time of $s = 65$ seconds was used for both inbound and outbound dedicated handling equipment. Two handling equipment were assigned, one dedicated to outbound containers and the other to inbound. Operating policies follow the requirements outlined with the static model.

Recall that three scaling coefficients $\gamma_1$, $\gamma_2$ and $\gamma_3$ must be determined to weight the objectives of double handling, excess travel and centre-of-mass respectively. For this study, double handling and centre-of-mass were given equal importance so that $\gamma_1 = \gamma_3 = 1$. The coefficient for excess travel was determined based on the principle that the amount of time wasted on excess travel would be penalized with the same weighting as the amount of time wasted on double handling.

Based on this idea $\gamma_2 = 1/28 \times 1/60 = 1/1680$ where the handling machine travels at about 28 feet/second and double handling wastes one container transfer which takes 60 seconds. All containers were assumed to be 20 feet long. Inter-arrival times of trucks were assumed to be distributed exponentially. An inter-arrival rate $\lambda$ can be
interpreted as one inbound and one outbound truck arriving on average every \( \lambda \) seconds. A value of \( \lambda = 60 \text{s} \) was used for this study.

For the simulation experiments, 10 test problems were randomly generated for each problem size of 50, 100 and 150 slots. Simulations were performed on each test problem using \( \omega \) values of 0, 0.25, 0.5, 0.75 and 1. After identifying the most successful value for \( \omega \), local search was applied to LPA using this value. Table 2 displays the results for this series of simulations. The performance measures displayed include the number of times (out of ten datasets) the best result was obtained, the average objective value (\( z \)), and the average and standard deviation of \( z \) divided by \( z^* \) which is the optimum solution obtained by solving SLPA.

These experiments indicate that a value of \( \omega = 0.25 \) is most suitable for all tested problem sizes under the given handling model. The superior performance of this value became more pronounced as \( n \) was increased. On average, the solution obtained was 15\% to 21\% outside of the optimum solution determined by solving the static SLPA model. CPLEX took 1 to 2 seconds to solve each period in LPA whilst LS took anywhere from 1 second to 30 seconds. All simulations were written in Microsoft Visual C++ and ran on a Pentium 2.4 GHz machine.

The objective values that resulted in Table 2 were expressed in terms of their components which includes excess travel of handling equipment, double handling and centre-of-mass. This information is displayed in Table 3. It can be seen from these results that \( \omega = 0.25 \) did not dominate for any particular performance measure but achieved balanced solutions.
9.2. LPA with Realistic Handling Model

The previous section applied the LPA model within a simplified container handling environment. This has been useful to gain an insight into the solution quality of LPA load plans, which is impractical for a complex environment. Also, the best value for \( \omega \) was determined for the simplified case. However, for a more realistic handling environment the best value for \( \omega \) would probably be different to that from the simplified case. For this reason, the previous experiments were repeated using a realistic model of container handling.

The handling equipment is assumed to service containers in the order of the following priorities.

1. Inbound or outbound containers with waiting trucks (regardless of whether transfer is to/from the ground).

2. Inbound containers to be transferred to the ground, or outbound containers to be transferred from the ground.

Within these priority groups, the nearest container is selected. As a tie breaker an inbound container is favoured over an outbound container. An additional restriction is that two handling machines cannot operate at the same location at the same time. This policy is realistic because in practice, terminal operators give priority to waiting trucks in order to minimise their customers’ waiting time. It is representative of a terminal operating forklifts or reach-stackers since there is no restriction on handling machines passing by each other. For the simulations of this study, two handling machines were assigned.

Table 4 shows the results of these trials. The same performance measures were used as in Table 2 except that \( z^* \) now refers to the best known result rather than the optimum solution. The best known result is simply the best found for all tested values.
of $\omega$ and local search. These simulations suggest that $\omega = 0$ performs best, particularly as $n$ increases. This is not surprising since an inbound container will always be unloaded first if an inbound and outbound truck are both waiting at the same slot.

Table 5 displays the results in terms of the separate objective components. The value $\omega = 0$ was dominant for the measures of excess travel and centre-of-mass. This performance came slightly at the expense of double handling because less emphasis is placed on the utilisation of vacant slots with $\omega = 0$.

9.3. Centre-of-Mass Tradeoff

Reducing the amount of excess handling of containers comes at the expense of mass distribution on the train. In the previous experiments the parameter $\gamma_3$ was given a value of one so that excess handling and centre-of-mass were given equal importance. It may be the case however, that one of these objectives is given greater importance by the decision maker. This next series of simulations considers a range of values for $\gamma_3$ to give an indication of the gains to be made in one objective by sacrificing the other.

Parameter $\gamma_3$ was tested over a range of values between 0 and 10. Ten test problems were generated with $n = 100$ using the realistic handling model. LPA was applied and the results were averaged for each value of $\gamma_3$. Figure 2 displays the averages as a pareto front for excess handling time versus centre-of-mass. Excess handling time is calculated as $\sum_{i=1}^{n} \sum_{j=1}^{n} (mh_{ijT} + vd_{ijT})p_{ijT}$ where $m$ is the transfer time and $v$ is the average traveling velocity. Recall from the previous section that $m = 60s$ and $v = 28$ ft/s.
In the range of $\gamma_3 = 0$ to 5, Figure 2 shows that excess handling could be reduced by about 10% for each slot the centre-of-mass was pushed back. After the reduction reaches about 30%, the trend levels so any further reduction would be quite costly in terms of centre-of-mass. Therefore, the decision maker would most likely operate within the range of $\gamma_3 = 0.5$ to 5.

10. CONCLUSION

This paper has proposed an assignment model (LPA) for dynamically assigning containers to slots on a train at an intermodal terminal. The objectives were to minimise excess handling time and optimise the mass distribution of the train. Many of the parameters are uncertain so the proposed model was designed to operate within a rolling horizon.

Simulation experiments were performed to evaluate the dynamic model under two different operating environments. The first operating environment was a simplified case allowing an optimal solution to be determined. On average, the dynamic LPA model obtained solutions 15% to 21% outside of the optimum. The second operating environment was a more realistic scenario and was used to analyse the tradeoff between excess handling time and mass distribution. This analysis found that significant reduction of excess handling time could be achieved with a relatively small concession in mass distribution.

The numerical investigation also assessed the performance of a local search heuristic which was competitive with exact solution of the LPA model. This suggests that the heuristic would be suitable in a more complex operating environment where exact solutions are impractical due to time constraints.

One assumption of the LPA model was that all containers are of equal size. This is untrue in most intermodal terminals, which also have additional considerations such
as hazardous goods, axle load limits and pin changes. This study has provided a foundation for ongoing research into these issues.

ACKNOWLEDGEMENT

This research was funded by the Cooperative Research Centre for Rail Engineering and Technologies.

REFERENCES


Figure 1: Simulation snapshot for a hypothetical train.
Figure 2 Pareto front for excess handling time vs. centre-of-mass averaged over 10 simulations.
### Tables

**Table 1** Evolution of double handling and travel penalties for container 1.

<table>
<thead>
<tr>
<th>t</th>
<th>Event</th>
<th>$(h_{12}, d_{11})$</th>
<th>$(h_{12}, d_{12})$</th>
<th>$(h_{13}, d_{13})$</th>
<th>Slot assigned</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>train arrives</td>
<td>(0, 0)</td>
<td>(0, 0)</td>
<td>(0, 0)</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>truck arrives to deliver ctr 1</td>
<td>(1, 0)</td>
<td>(1, 0)</td>
<td>(1, 0)</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>truck arrives to collect from slot 2</td>
<td>(1, 0)</td>
<td>(1, 0)</td>
<td>(1, 0)</td>
<td>1 or 2</td>
</tr>
<tr>
<td>3</td>
<td>slot 2 unloaded to truck</td>
<td>(1, 0)</td>
<td>(1, 0)</td>
<td>(1, 0)</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>ctr 1 loaded from truck to slot 2</td>
<td>na</td>
<td>(0, 20)</td>
<td>(1, 40)</td>
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**Table 2** Simulation results of 10 test problems for each $n = 50, 100$ and 150. Best results shown in bold typeface.

<table>
<thead>
<tr>
<th>$n$</th>
<th>measure</th>
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<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>1</th>
<th>LS $\omega = 0.25$</th>
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<td>50</td>
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<td>2</td>
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<td>2</td>
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<td>1</td>
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<tr>
<td></td>
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<td>33.19</td>
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<td>27.26</td>
<td>28.16</td>
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<tr>
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<td>avg.(z/*)</td>
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<td>1.18</td>
<td>1.22</td>
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<td>0.09</td>
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<td>0.11</td>
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</tr>
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<td>0</td>
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<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>avg.(z)</td>
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<td>49.53</td>
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<td>51.96</td>
<td>52.06</td>
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<td>1.21</td>
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<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>150</td>
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<td>0</td>
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<td>1</td>
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<tr>
<td></td>
<td>avg.(z)</td>
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<td>1.24</td>
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**Table 3** Simulation results in terms of objective components including excess travel (TVL), double handling (DH) and centre-of-mass (CM).

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<th>measure</th>
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<th>0.5</th>
<th>0.75</th>
<th>1</th>
<th>LS $\omega = 0.25$</th>
</tr>
</thead>
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<td>0</td>
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<td>100</td>
<td>7</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>150</td>
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<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>TVL(avg)</td>
<td>50</td>
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<td>1266.00</td>
<td>1706.00</td>
<td>2350.00</td>
<td>2638.00</td>
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<td>150</td>
<td>1858.00</td>
<td>3282.00</td>
<td>3892.00</td>
<td>4760.00</td>
<td>6976.00</td>
<td>8510.00</td>
</tr>
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<td>DH(best)</td>
<td>50</td>
<td>5</td>
<td>2</td>
<td>4</td>
<td>9</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td></td>
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<td>0</td>
<td>2</td>
<td>3</td>
<td>7</td>
<td>9</td>
<td>5</td>
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<td>0</td>
<td>1</td>
<td>3</td>
<td>9</td>
<td>6</td>
<td>2</td>
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<tr>
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<td>50</td>
<td>12.50</td>
<td>5.70</td>
<td>5.30</td>
<td>4.70</td>
<td>5.40</td>
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<tr>
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<td>0</td>
<td>0</td>
<td>0</td>
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</tr>
<tr>
<td>CM(avg)</td>
<td>50</td>
<td>19.25</td>
<td>20.32</td>
<td>20.61</td>
<td>20.77</td>
<td>20.74</td>
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<tr>
<td></td>
<td>150</td>
<td>56.85</td>
<td>60.88</td>
<td>64.45</td>
<td>64.94</td>
<td>65.07</td>
<td>62.85</td>
</tr>
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</table>
Table 4 Simulation results of 10 test problems for each \( n = 50, 100 \) and \( 150 \) with realistic handling model. Best results shown in bold typeface.

<table>
<thead>
<tr>
<th>( n )</th>
<th>measure</th>
<th>0</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>1</th>
<th>LS ( \omega = 0 )</th>
</tr>
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<tbody>
<tr>
<td>50</td>
<td>best</td>
<td>7</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
<td></td>
<td>avg.(z)</td>
<td>22.00</td>
<td>21.93</td>
<td>22.63</td>
<td>22.55</td>
<td>22.59</td>
<td>22.66</td>
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<tr>
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<td>avg.(z/z*)</td>
<td>1.03</td>
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<tr>
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<td>s.d.(z/z*)</td>
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<td>avg.(z/z*)</td>
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<td>s.d.(z/z*)</td>
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<td>1</td>
</tr>
<tr>
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<td>avg.(z)</td>
<td>60.66</td>
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<td>61.71</td>
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<td>64.40</td>
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<tr>
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<td>avg.(z/z*)</td>
<td>1.01</td>
<td>1.02</td>
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<td>1.05</td>
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<td>0.01</td>
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<td>0.03</td>
<td>0.05</td>
<td>0.06</td>
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</table>

Table 5 Simulation results in terms of objective components including excess travel (TVL), double handling (DH) and centre-of-mass (CM). Realistic handling model used.

<table>
<thead>
<tr>
<th>measure</th>
<th>( n )</th>
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<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>1</th>
<th>LS ( \omega = 0.25 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>TVL(best)</td>
<td>50</td>
<td>5</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>100</td>
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<td>2</td>
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<td>2</td>
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<td>150</td>
<td>9</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>1</td>
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<td>8</td>
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<td>4</td>
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<td>5</td>
<td>7</td>
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<td>1.90</td>
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<td>CM(best)</td>
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