Adaptive multiuser receiver with joint channel and time delay estimation of CDMA signals based on the square-root unscented filter

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Abstract

The paper discusses an adaptive multiuser receiver for CDMA systems in which the scaled unscented filter (SUF) and the square root unscented filter (SURF) are used for joint estimation and tracking of the code delays and multipath coefficients of the received CDMA signals. The proposed channel estimators are more near-far resistant than the conventional extended Kalman filter (EKF) and present lower complexity than the conventional particle filter (PF) based methods. To present meaningful performance measures, the modified Cramer–Rao lower bound (CRLB) and computational complexity metrics are derived for the proposed and existing channel estimators. Computer simulation results demonstrate the superior performance of the proposed channel estimators.

The proposed estimators are also shown to exhibit lower complexity relative to the PF.

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1. Introduction

In wireless communications, data transmissions experience distortions due to the propagation channel. The performance of data detection strongly depends on the quality of the channel parameter estimates, such as the symbol timing, carrier phase, time delay and channel gain (amplitude). The problem of estimating efficiently the time delays and channel coefficients remains a challenge for the practical implementation of multiuser detectors under any CDMA conditions such as near-far problems, multiple access interference, and so on.

Thus far, several delay estimation algorithms have been proposed based on signal-subspace techniques, such as the MUSIC approach in [1,2] and SVD-based methods in [3,4]. However, these algorithms implicitly assume time-invariant delays over the observation times. Alternative delay acquisition methods [5] have been reported based on MMSE criteria. All these algorithms are inherently nonrecursive.

Adaptive multiuser receivers for CDMA systems have been previously developed based on the EKF [6,7] to estimate jointly the time delays and channel coefficients. In many cases, EKF-based estimators diverge [7,8], since the
measurement model presents a highly nonlinear nature with respect to the time delay, and it relies on an approximate first-order Taylor series expansion of the nonlinear terms around the mean values. Note also that the EKF in [7,8] might be also prone to divergence at low SNR. Although EKF has high accuracy in practice, there are three main drawbacks associated with the usage of EKF. First, the linearization can produce a highly unstable filter performance if the time-step intervals are not sufficiently small. Second, the derivation of the Jacobian matrices is nontrivial in most applications and often leads to significant implementation difficulties. Third, sufficiently small time-step intervals usually produce a high computational overhead as the number of calculations demanded for the generation of the Jacobians, the predictions of state estimate and covariances are large.

Thus, many different algorithms have been developed to compensate for the EKF’s major limitations. A general class of algorithms, commonly referred to as particle filters (PF), has recently received increased attention. Such methods have already been successfully applied to various applications in the fields of controls, statistics, and digital communications, in particular, demodulation in fading channels [9,10], and detection in synchronous CDMA systems [11]. Recently, Iltis [12] proposed a particle filtering method for estimation of only the channel coefficients and code delays, while the estimation of the unknown symbol sequence is obtained through a standard algorithm. Punskaya [13] reported a PF-based scheme for joint estimation of the channel coefficients, code delays, and symbols in flat Rayleigh fading channels. The PF partially overcomes the three major disadvantages of the EKF. However, the huge computational complexity of PF represents a significant drawback. The number of samples required by PF-based methods to achieve reliable estimates might be in the order of several hundreds for two- or three-dimensional problems. Such problems necessitate PF-based algorithms that are several orders of magnitude much more computationally complex than EKF. In addition to the computational expense, the issue of sample depletion is common to PF-based methods. Re-sampling has been used to reduce this problem [14], but there still remains a large tradeoff between computational expense and filter stability. Finally, the most common PF strategy is to sample from the transition prior distribution due to its simplicity. Since the prior proposal distribution conveys no information from observations in proposing new samples, its use is often ineffective and leads to poor filtering performance.

Due to the above-mentioned limitations associated with EKF (divergence) and PF (huge computational expense), this paper proposes the SUF and SRUF-based methods as the primary choice for channel estimator. The SUF [15] and SRUF [16]-based methods have been employed to tackle the nonlinearity and to demonstrate their effectiveness in terms of divergence reduction [15,16]. The SUF and SRUF address this problem by using a deterministic sampling approach. The deterministic sample points avoid the random sampling errors introduced by the PF, and, therefore, dramatically reduce the number of points required to achieve the same transformation accuracy. We give a detailed comparison of the performance of proposed channel estimators and PF- and EKF-based channel estimator in multiuser CDMA systems.

This paper proposes a novel joint multiuser detection and channel estimation algorithm. Novel SUF and SRUF-based multiuser receivers are developed. The SUF and SRUF-based channel estimation and tracking schemes are combined with the proposed multiuser receiver to reduce the errors caused by linearization and to achieve a blind channel tracking scheme. Since the unknown symbols are discrete-valued, and channel coefficients and time delays are complex and continuous-valued, the proposed multiuser receiver is implemented in two alternating steps: a channel estimation step, followed by a detection step of the unknown symbols. Such operation is similar to the decision directed (DD) mode. Furthermore, a two-stage receiver structure is employed with the prediction stage followed by a correction stage. Each stage presents a channel estimation (complex channel coefficients and time delays) step followed by symbol detection. The channel estimation, achieved by the robust recursive filters (e.g., SUF and SRUF), is incorporated within the proposed multiuser detector, in a mode similar with the parallel interference cancellation (PIC) based multiuser detector [17]. Notice further that the proposed channel estimators can be applied in a variety of multiuser detectors that require estimates of channel coefficients and time delays. Few analytical treatments such as the CRLB and computational complexity figures of merit for EKF, SUF, SRUF, and PF in CDMA applications are available. Generally, unknown parameters (e.g., channel coefficients) in many wireless digital communications are complex-valued and time-varying. A simple approach is proposed herein to derive the modified CRLB of unknown parameters with time-varying complex values.

The subsequent sections of this paper are organized as follows. Section 2 introduces the signal and channel model used throughout the paper, and a description of the state-space model. Section 3 provides a description of the nonlinear filtering methods (EKF, PF, SUF and SRUF) that are used for estimating the channel parameters. Section 4 explains the derivation of the modified CRLB and the computational complexity figures of merit associated with the proposed
channel estimator. The results from computer simulations are presented in Section 5. Finally, Section 6 provides concluding remarks.

2. Problem formulation

2.1. System and channel model

Assuming that each of $K$ users transmits over an $M$-path fading channel, the received signal is given by

$$\mathbf{r}(l) = \sum_{k=1}^{K} \sum_{i=1}^{M} c_{k,i}(l)d_{k,m_i}a_k(l - m_i T_b - \tau_{k,i}(l)) + n(l),$$

(1)

where $c_{k,i}(l)$ represent the complex channel coefficients, $d_{k,m_i}$ is the $m$th symbol transmitted by the $k$th user, $m_i = \lfloor (l - \tau_k(l))/T_b \rfloor$, $T_b$ denotes the symbol interval, $a_k(l)$ stands for the pulse shaping PN spreading waveform used by the $k$th user, $\tau_{k,i}(l)$ is the time delay introduced by the $i$th path of the $k$th user, and $n(l)$ represents the additive white Gaussian noise (AWGN), which is assumed to have zero mean and variance $R = \sigma_n^2 = E[|n(l)|^2] = 2N_0/T_s$, with $T_s$ and $l$ standing for the sampling time and discrete time index, respectively.

2.2. State-space model

Let the unknown parameters be represented by the following $2KM \times 1$ vector

$$\mathbf{x} = [\mathbf{c} \quad \mathbf{\tau}]^T,$$

(2)

where $\mathbf{c} = [c_{11}, \ldots, c_{1M}, \ldots, c_{K1}, \ldots, c_{KM}]^T$ and $\mathbf{\tau} = [\tau_{11}, \ldots, \tau_{1M}, \ldots, \tau_{K1}, \ldots, \tau_{KM}]^T$. From [6,12], the state model can be expressed as

$$\mathbf{x}(l+1) = \mathbf{F}(l)\mathbf{x}(l) + \mathbf{v}(l),$$

(3)

where $\mathbf{F}(l) = \text{diag}(\mathbf{F}_c(l), \mathbf{F}_\tau(l))$ represents the $2KM \times 2KM$ state transition matrix, $\mathbf{v}(l) = [\mathbf{v}_c^T(l) \quad \mathbf{v}_\tau^T(l)]$ is $2KM \times 1$ noise vector with zero mean and covariance matrix $\mathbf{Q}(l) = \text{diag}(\mathbf{Q}_c(l), \mathbf{Q}_\tau(l))$, and $\text{diag}(\bullet)$ denotes a diagonal matrix. The matrices $\mathbf{F}_c(l)$ and $\mathbf{Q}_c(l)$ account for the Doppler spread [18].

The scalar measurement model follows from the received signal model of (1)

$$z(l) = h(\mathbf{x}(l)) + n(l),$$

(4)

where the measurement $z(l) = r(l)$, and

$$h(\mathbf{x}(l)) = \sum_{k=1}^{K} \sum_{i=1}^{M} c_{k,i}(l)d_{k,m_i}a_k(l - m_i T_b - \tau_{k,i}(l)).$$

The scalar measurement $z(l)$ is a nonlinear function of the state $\mathbf{x}(l)$. It is assumed that the noise vectors $\mathbf{v}(l)$ and $n(l)$ are individually and mutually uncorrelated with correlation matrices

$$E[\mathbf{v}(i)\mathbf{v}(j)^T] = \mathbf{Q}_c \delta_{ij}, \quad E[n(i)n(j)^T] = R \delta_{ij}, \quad E[\mathbf{v}(i)n(j)^T] = 0,$$

(5)

where $\delta_{ij}$ stands for the two-dimensional Kronecker delta function, and the noise variance is assumed to be $R = \sigma_n^2$.

2.3. Detection and estimation objectives

Since the unknown symbols are discrete-valued and channel parameters (channel coefficients and code delay) are (complex or real) continuous-valued, the proposed receiver assumes two steps: channel estimation and unknown symbol detection. The symbols $\mathbf{d}$, the channel coefficients and time delay $\mathbf{x}$ are unknown. In reference with the modeling equations (3) and (4), the goals of this work are two-fold:

(i) Obtain an maximum a posterior (MAP) estimator (decoder) of the symbols

$$\hat{\mathbf{d}}(z) = \arg\max_{\mathbf{d}} p(\mathbf{d}|z, \hat{\mathbf{x}}).$$

(6)
(ii) Derive the MAP or minimum mean square error (MMSE) estimates of the channel coefficients and code delays
\[ \hat{x} = E\{x|z, \hat{d}\}, \]  
with estimated error covariance
\[ P = E \left[ \left( x(l) - \hat{x}(l|l) \right) \left( x(l) - \hat{x}(l|l) \right)^T \right] |z, \hat{d}. \]

However, the proposed problem does not admit a closed-form analytical solution. Therefore, a novel receiver structure is proposed. Detection of the symbol \( d \) is performed by assuming that the channel and delay estimates \( \hat{x} \) are available as depicted in Fig. 1. The proposed receiver assumes an iterative mode of operation where the acquisition of channel and data symbols is performed in a sequential manner. The next section provides a more detailed description of the proposed receiver.

3. Novel adaptive multiuser receiver and robust channel estimators

Although we approximate the composite data, fading amplitude, and time delay by a Gaussian autoregressive (AR) process, the AR process does not account for the finite alphabet nature of the data symbols. Therefore, a novel
approach is presented herein to incorporate the SUF and SRUF with the multiuser detector depicted in Fig. 1. The proposed novel multiuser detector is configured in a similar mode as the two-stage PIC multiuser detector. The proposed multiuser receiver relies also on a predictive structure similar to that of the Kalman filter. This is necessary to enable the SUF and SRUF to provide a new channel estimate for the next iteration. The recursive nature of the Kalman filtering is used to track the channel and detect unknown symbols. Notice that estimation of $x$ and detection of $d$ steps are conducted in an alternating recursive fashion. The SUF and SRUF channel estimators, denoted by SUF$(\bullet)$ and SRUF$(\bullet)$, respectively, at time $l$ yield suboptimum nonlinear estimators of the time-varying part of the channel coefficients and time delay. The prediction and correction steps at the $l$th iteration for the proposed receiver are:

(1) Prediction
   (A) Estimation step: Obtain rough estimate, $\hat{x}_{l|l-1}$, using the state equation:
   $$\hat{x}_{l|l-1} = F\hat{x}_{l-1}.$$  
   (B) Detection step: Make an initial estimate of the current transmitted symbols, $\tilde{d}_{l|l-1}$, using the observation vector, $z_l$, and $\hat{x}_{l|l-1}$
   $$\tilde{d}_{l|l-1}(z) = \arg \max_{d} p(d|z_l, \hat{x}_{l|l-1}).$$

(2) Correction
   (A) Estimation step: Refine the channel estimate using the SUF and SRUF, and the estimated symbols to yield $\bar{x}_l$
   $$\bar{x}_l = SUF(\tilde{d}_{l|l-1}, z_l) \text{ or } SRUF(\tilde{d}_{l|l-1}, z_l).$$
   (B) Detection step: Re-estimate symbols, $\tilde{d}_l$, from $\bar{x}_l$
   $$\tilde{d}_l(z) = \arg \max_{d} p(d|z_l, \bar{x}_l).$$

The next subsection briefly outlines the key elements used in the implementation of the EKF, SUF, SRUF and PF as channel estimators. To encourage the use of the proposed model, we now turn our attention to the channel estimators and derivation of channel tracking algorithms based on Eqs. (3) and (4). The following subsections will examine in detail the implementation of two main existing channel estimators and the two proposed channel estimators.

3.1. Extended Kalman filter for channel tracking

We will develop the EKF with a state vector $x$ consisting of both real and complex-valued channel state variables. Using only the linear expansion terms, it is easy to derive the following update equations. A full derivation of the EKF recursions is beyond the scope of this paper. The time and measurement update formulas, based on state-space equations (3) and (4), are described at time $l$ by a system of equations (see [19])

$$\begin{align*}
\bar{x}(l+1|l) &= F\bar{x}(l), \\
P_{xx}(l+1|l) &= FP_{xx}(l)F^T + Q, \\
K(l+1) &= P_{xe}(l+1|l)P_{ee}^{-1}(l+1|l), \\
P_{xx}(l+1) &= P_{xx}(l+1|l) - K(l+1)P_{ve}(l+1|l)K^T(l+1),
\end{align*}$$

(8)

where $\bar{x}(l+1|l) = g(\bar{x}(l+1|l))$, $v(l+1|l) = z(l+1) - \bar{z}(l+1|l)$, $P_{ve}(l+1|l) = HP_{xx}(l+1|l)H^T + \sigma_n^2$, $K$ is known as the Kalman gain, $Q$ denotes the variance of the process noise, $\sigma_n^2$ stands for the variance of the measurement noise, $H = \nabla h_{|x(l+1)=\bar{x}(l+1|l)}$ is the Jacobian of the measurement model, and $v$ represents the innovation.

3.2. Scaled unscented filter for channel tracking

To improve the accuracy, consistency and efficiency of EKF algorithms applied to CDMA channel estimation, we introduce next the SUF. This technique and its variations [15] have been widely used in engineering and physical sciences to estimate parameters from noisy data.

The SUF can be summarized briefly as follows. For each measurement time $l$, a set of deterministically selected points, referred to as sigma points, are used to approximate the distribution of the previous state estimates from time
A scheme that satisfies this requirement is \[15\]:

- \(\bar{l}\) weighted, and used to update the estimates in conjunction with the new observation at time \(P\) chart:

\[
\optimal
\]

is a secondary scaling parameter which is usually set to either 0 or 3 \(\kappa\) to yield \(\bar{\lambda}\) are then projected forward in time using the linear state function in Eq. (3) and weighted after the transformation to yield \(\bar{x}_i|l|\) and \(P_i|l|\). Then, the same sigma points are projected using the measurement function in Eq. (4), re-weighted, and used to update the estimates in conjunction with the new observation at time \(l\) to yield \(\bar{x}_i|l|\) and \(P_i|l|\). Since this approach takes into account the statistical properties of the prior random variable, the resulting linearization error tends to be smaller than that of a truncated Taylor-series linearization. For in-depth knowledge, technically more rigorous examination of the topic, the reader is directed to other sources (e.g., \[15\]).

The above mentioned sigma point transformation algorithm, which capitalized on repeated applications of a transformation technique known as the scaled unscented transformation, is computationally efficient. This is because the sigma points are selected according to a deterministic scheme (instead of a random sampling scheme as in the PF).

The key of the SUF amounts to obtaining the sigma-points. A set of \(2n + 1\) weighted points where \(\Phi = \{W_i, X_i\}\) (such that \(\sum_{i=0}^{2n} W_i = 1\)) are chosen to reflect certain properties on \(x\), where \(n\) denotes the dimension of \(x\). A selection scheme that satisfies this requirement is \[15\]:

\[
\begin{align*}
X_0(l|l) &= \bar{x}(l|l), & W_0^{(m)} &= \lambda/(n + \lambda), \\
X_i(l|l) &= \bar{x}(l|l) + (\sqrt{(n + \lambda)P(l|l)})_i, & i = 1, \ldots, n, & W_0^{(c)} &= \lambda/(n + \lambda) + (1 - \alpha^2 + \beta), \\
X_{i+n}(l|l) &= \bar{x}(l|l) - (\sqrt{(n + \lambda)P(l|l)})_i, & i = n + 1, \ldots, 2n, & W_i^{(m)} &= W_i^{(c)} = 1/\{2(n + \lambda)\}, \\
& & i = 1, \ldots, 2n, & \end{align*}
\]

where \(\kappa \in \mathbb{N}\), \((\sqrt{(n + \kappa)P(l|l)})_i\) is the \(i\)th row or column of the matrix square root of \((n + \kappa)P(l|l)\), and \(W_i\) is the weight associated with the \(i\)th point. \(\lambda = \alpha^2(n + \kappa) - n\) is a scaling parameter and \(\eta = \sqrt{n + \lambda}\). \(\alpha\) is a positive scaling parameter which can be made arbitrarily small to minimize the higher order effects (e.g., \(1e - 2 \leq \alpha \leq 1\)). \(\kappa\) is a secondary scaling parameter which is usually set to either 0 or \(3 - n\). \(\beta\) is an extra degree of freedom scalar parameter used to incorporate extra prior knowledge of the distribution of \(x\) (for Gaussian distributions, \(\beta = 2\) is optimal).

Precise implementation of the SUF-based channel estimator in CDMA environments is given by the following chart:

Step 1. The sigma point is calculated as

\[
X(l) = \begin{bmatrix}
\bar{x}(l|l) \\
\bar{x}(l|l) + \eta \sqrt{P(l|l)} + Q \\
\bar{x}(l|l) - \eta \sqrt{P(l|l)} + Q
\end{bmatrix}.
\]

Step 2. The SUF time updates as follows:

(1) The transformed set is given by instantiating each point through the process model

\[
X_i(l + 1|l) = F X_i(l|l).
\]

(2) The predicted mean is computed as

\[
\bar{x}(l + 1|l) = \sum_{i=0}^{2n} W_i^{(m)} X_i(l + 1|l).
\]

(3) The predicted covariance is computed as

\[
P(l + 1|l) = \sum_{i=0}^{2n} W_i^{(c)} \left[ \left( X_i(l + 1|l) - \bar{x}(l + 1|l) \right) \left( X_i(l + 1|l) - \bar{x}(l + 1|l) \right)^T \right].
\]

(4) Instantiate each of the prediction points through the observation model

\[
Z(l + 1|l) = h(X(l + 1|l)).
\]

(5) The predicted observation is calculated by

\[
\bar{z}(l + 1|l) = \sum_{i=0}^{n+1} W_i Z_i(l + 1|l).
\]
Step 3. The SUF measurement updates as follows:

1. The innovation covariance is given by

   \[ P_{zz}(l+1) = 2n \sum_{i=0}^{2n} W_i(c) [Z_i(l+1|l) - \bar{z}(l+1|l)] [Z_i(l+1|l) - \bar{z}(l+1|l)]^T. \]

2. Since the observation noise is additive and independent, the innovation covariance is

   \[ P_{v\nu}(l+1) = P_{zz}(l+1) + \sigma_n^2. \]

The cross-covariance matrix of \( x \) and \( z \), is determined by

\[ P_{xz}(l+1) = 2n \sum_{i=0}^{2n} W_i(c) [X_i(l+1|l) - \bar{x}(l+1|l)] [Z_i(l+1|l) - \bar{z}(l+1|l)]^T. \]

The Kalman gain matrix can be found according to

\[ K(l+1) = P_{xz}(l+1)/P_{v\nu}(l+1). \]

The update mean is calculated

\[ \bar{x}(l+1) = \bar{x}(l+1|l) + K(l+1)v(l+1), \quad v(l+1) = z(l+1) - \bar{z}(l+1|l). \]

The update error covariance is also provided by

\[ P(l+1) = P(l+1|l) - K(l+1)P_{zz}(l+1)K(l+1)^T. \]

The SUF is not required to calculate the Jacobian, and the prediction stage only consists of standard linear algebra operations (e.g., matrix square root calculation).

3.3. Square-root unscented filter for channel tracking

The most computationally expensive operation in the SUF is to calculate the new set of sigma points at each time update. This requires the computation of a matrix square-root of the state covariance matrix, \( P \in \mathbb{R}^{n \times n} \), given by \( \text{SS}^T = P \). Efficient implementation using a Cholesky factorization, in general, presents \( O(n^3/6) \) computational complexity [16], and the operation of the square-root of \( P \), an integral part of the SUF, requires computation of full covariance matrix \( P \) be recursively updated. In the SRUF implementation, however, \( S \) will be propagated directly, avoiding the requirement of re-factorizing at each time step. In general, the algorithm will still present \( O(n^3) \) complexity, but with improved numerical properties [20] and guaranteed positive semi-definiteness of the state covariance.

The SRUF makes use of three powerful linear algebra techniques: QR decomposition (in Matlab notation denoted as qr), Cholesky factor updating (cholupdate), and efficient least squares (‘/’ operator) [15]. Taking three linear algebra techniques into consideration, the complete SRUF-based channel estimator for CDMA systems is given by:

Step 1. The sigma point is calculated as

\[ X(l) = [\tilde{x}(l|l) \quad \tilde{x}(l|l) + \eta S(l|l) \quad \tilde{x}(l|l) - \eta S(l|l)]. \]

Step 2. The SRUF time updates as follows:

1. The transformed set is given by instantiating each point through the process model

   \[ X_i(l+1|l) = FX_i(l|l). \]

2. The predicted mean is computed as

   \[ \tilde{x}(l+1|l) = 2n \sum_{i=0}^{2n} W_i^{(m)} X_i(l+1|l). \]
(3) The predicted Cholesky factor is computed as
\[
S(l+1|l) = \text{qr}\left\{ \sqrt{W^{(c)}_l} \left( X(l+1|l) - x(l+1|l) \right) \right\},
\]
\[
S(l+1|l) = \text{cholupdate}\{ S(l+1|l), X(l+1|l) - x(l+1|l), W^{(c)}_0 \}.
\]

(4) Instantiate each of the prediction points through the observation model
\[
Z(l+1|l) = h(X(l+1|l)).
\]

(5) The predicted observation is calculated by
\[
\bar{z}(l+1|l) = \sum_{i=0}^{2n} W^{(m)}_i Z_i(l+1|l).
\]

Step 3. The SRUF measurement updates as follows:

(1) The innovation Cholesky factor is given by
\[
S_Z(l+1) = \text{qr}\left\{ \sqrt{W^{(c)}_l} \left( Z(l+1|l) - z(l+1|l) \right) \right\},
\]
\[
S_Z(l+1) = \text{cholupdate}\{ S_Z(l+1), Z(l+1|l) - z(l+1|l), W^{(c)}_0 \}.
\]

(2) The cross-covariance matrix of \( x \) and \( z \) is determined by
\[
P_{xz}(l+1) = \sum_{i=0}^{2n} W^{(c)}_i \left[ X_i(l+1|l) - \bar{x}(l+1|l) \right] \left[ Z_i(l+1|l) - \bar{z}(l+1|l) \right]^T.
\]

(3) The Kalman gain matrix can be found according to
\[
K(l+1) = \left( P_{xz}(l+1) / S_\xi(l+1) \right) / S_\xi(l+1).
\]

(4) The update mean is calculated
\[
\tilde{x}(l+1) = \bar{x}(l+1|l) + K(l+1)v(l+1), \quad v(l+1) = z(l+1) - \bar{z}(l+1|l).
\]

(5) The update Cholesky factor is also provided by
\[
U(l+1) = K(l+1) S_\xi(l+1), \quad S(l+1) = \text{cholupdate}\{ S(l), U, -1 \}.
\]

These two steps (Step 2(3)) replace the time update of \( P \) in the SUF. The same two-step approach (Step 3(1)) is applied to the calculation of the Cholesky factor of measurement update equations. In contrast to the way in which the Kalman gain is calculated in the standard SUF, two nested inverse solutions are now used for the following expansion of \( K(S_\xi S_\xi^T) = P_{xz} \) (Step 3(2)). Since \( S_\xi \) is square and triangular, efficient “back-substitutions” can be used to solve for \( K \) directly without the requirement of a matrix inversion. Finally, the posterior measurement update of the Cholesky factor of the state covariance is calculated (Step 3(5)).

3.4. Particle filter for channel tracking

Under the Bayesian framework, an emergent technique for obtaining the posterior probability density function is known as particle filtering. Based on sequential importance sampling, the PF [14] might be viewed as an extension of the sequential Monte Carlo methodology. The use of the Monte Carlo method in computing the marginalized minimum mean square error (MMMMSE) estimate (7) requires generation of random samples \( \{ \tilde{x}^{(l)} \}_{l=1}^N \) from the posterior distribution \( p(x|\tilde{z}) \), where \( \tilde{z}^l = \{ z(0), z(1), \ldots, z(l) \} \). Then (7) can be approximated by a set of \( N \) samples with associated weights, denoted by \( \{ x^{(l)}, w^{(l)} \}_{l=1}^N \):

\[
p(x|\tilde{z}) \approx \sum_{i=1}^N w^{(i)}_l \delta(x - x^{(i)}_l),
\]

where \( \delta(\bullet) \) is the Dirac delta function.
From (10), the conditional mean state estimate $\hat{x}_l = E[x_l|z_l]$ and the corresponding error covariance $\bar{P}_l = E[(x_l - \hat{x}_l)(x_l - \hat{x}_l)^T]$ can be calculated:

$$
\hat{x}_l = \sum_{i=1}^N w_i^{(l)} \bar{x}_i^{(l)}, \quad \bar{P}_l = \sum_{i=1}^N w_i^{(l)}(\bar{x}_i^{(l)} - \hat{x}_l)(\bar{x}_i^{(l)} - \hat{x}_l)^T.
$$

At the end of each recursion, the particles are resampled to ensure they occur with the same probability as the weights.


4.1. Cramer–Rao lower bound

CRLB is a lower bound on the covariance matrix of any unbiased estimator [21]. Suppose $\hat{x}$ is an unbiased estimator of a vector of deterministic unknown parameters $x$ (i.e., $\hat{x} = E[x]$), then the estimator’s covariance matrix satisfies

$$
J^{-1} \leq E\{(\hat{x} - x)(\hat{x} - x)^T\},
$$

where $J$ is the $2KM \times 2KM$ Fisher information matrix given by

$$
J = E\left\{\left[\frac{\partial}{\partial x} \ln A(z')\right]\left[\frac{\partial}{\partial x} \ln A(z')\right]^T\right\}.
$$

$A(z')$ stands for the log-likelihood function of the observed data $z'$ with respect to $x$. In this case, the observed data are the received samples $z(l)$ for $l = 1, 2, \ldots, L$. Hence, $z' = [z(1), \ldots, z(L)]^T$. Using (4), the expression for $z(l)$ can be written in a more compact form as

$$
z(l) = c^T(l)D(l)a(l) + n(l),
$$

where

$$
c = \begin{bmatrix} c_{11}(l) \\ \vdots \\ c_{KM}(l) \end{bmatrix}, \quad D(l) = \begin{bmatrix} d_{11}(l) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & d_{KM}(l) \end{bmatrix}, \quad a(l) = \begin{bmatrix} \tilde{a}_{11}(lT_s - m_k(l)T_b - \tau_{11}(l)) \\ \vdots \\ \tilde{a}_{KM}(lT_s - m_k(l)T_b - \tau_{KM}(l)) \end{bmatrix}.
$$

The CRLB presented here will still be a valid bound on the performance of estimators based on the model (4). Since the noise is a Gaussian random variable [6], the log-likelihood function of $z'$ given $x$ is

$$
\ln A(z') = \text{const} - \frac{1}{\sigma^2} \sum_{l=1}^{L} [z(l) - c^T(l)D(l)a(l)]^2.
$$

Since the channel coefficients $c$ of $x$ are complex valued, to calculate the fisher information matrix (FIM), the unknown parameters are parameterized as real values. Thus, we re-define the unknown parameters as follows:

$$
\tau = [\tau_1, \ldots, \tau_{KM}]^T, \quad c = [c_{11}, \ldots, c_{KM}]^T = \alpha + j\beta, \quad \theta = [\alpha^T, (\beta)^T, (\tau)^T],
$$

where $\alpha = \Re\{c\}$ and $\beta = \Im\{c\}$ denote the real and imaginary parts of complex channel coefficients ($c$), respectively. $\theta$ is the set of $3KM$ channel parameters, and $x$ is $2KM$-dimensional vector. Thus, all channel parameters ($\theta$) are real. Now, in order to calculate the FIM, $J \in R^{3KM \times 3KM}$, the partial derivative with respect to the parameter vector $\theta$ is required.

$$
\frac{\partial}{\partial \theta} \ln A(z) = \begin{bmatrix} \frac{\partial}{\partial \alpha} \ln A(z) \\ \frac{\partial}{\partial \beta} \ln A(z) \\ \frac{\partial}{\partial \tau} \ln A(z) \end{bmatrix}.
$$

For each individual term $\alpha_i$ of the vector $\alpha$, the derivatives of $A(z')$ with respect to $\alpha_i$, $\beta_i$, and $\tau_i$ are given by
\[
\frac{\partial}{\partial \alpha_i} \ln A(z') = \frac{1}{\sigma_n^2} \sum_{l=1}^{L} \left[ \left[ (r(l) - c(l) \mathbf{D}(l) \mathbf{a}(l)) \ast d_{i,m_k}(l) a(1T_s - m_k(l)T_b - \tau_i(l)) \right] \right],
\]

(17)

\[
\frac{\partial}{\partial \beta_i} \ln A(z') = \frac{1}{\sigma_n^2} \sum_{l=1}^{L} \left[ \left[ (r(l) - c(l) \mathbf{D}(l) \mathbf{a}(l)) \ast d_{i,m_k}(l) a(1T_s - m_k(l)T_b - \tau_i(l)) \right] \right],
\]

(18)

\[
\frac{\partial}{\partial \tau_i} \ln A(z') = \frac{1}{\sigma_n^2} \sum_{l=1}^{L} \left[ \left[ (r(l) - c(l) \mathbf{D}(l) \mathbf{a}(l)) \ast d_{i,m_k}(l) \frac{\partial}{\partial \tau_i} a(1T_s - m_k(l)T_b - \tau_i(l)) \right] \right].
\]

(19)

From the definition (11) and using (16), the FIM \( \mathbf{J} \) can be divided into three \( KM \times KM \) FIM submatrices, as follows:

\[
\mathbf{J} = \begin{bmatrix}
\mathbf{J}_{\alpha \alpha} & \mathbf{J}_{\alpha \beta} & \mathbf{J}_{\alpha \tau} \\
\mathbf{J}_{\beta \alpha} & \mathbf{J}_{\beta \beta} & \mathbf{J}_{\beta \tau} \\
\mathbf{J}_{\tau \alpha} & \mathbf{J}_{\tau \beta} & \mathbf{J}_{\tau \tau}
\end{bmatrix},
\]

(20)

where the matrices \( \mathbf{J}_{\alpha \alpha}, \mathbf{J}_{\alpha \beta}, \mathbf{J}_{\beta \alpha}, \mathbf{J}_{\beta \beta}, \mathbf{J}_{\beta \tau}, \mathbf{J}_{\tau \tau} \) are defined as

\[
\mathbf{J}_{\alpha \alpha} = E \left\{ \left[ \frac{\partial}{\partial \alpha} \ln A(z') \right] \left[ \frac{\partial}{\partial \alpha} \ln A(z') \right]^T \right\}, \quad \mathbf{J}_{\alpha \beta} = E \left\{ \left[ \frac{\partial}{\partial \alpha} \ln A(z') \right] \left[ \frac{\partial}{\partial \beta} \ln A(z') \right]^T \right\}, \quad \mathbf{J}_{\alpha \tau} = E \left\{ \left[ \frac{\partial}{\partial \alpha} \ln A(z') \right] \left[ \frac{\partial}{\partial \tau} \ln A(z') \right]^T \right\},
\]

\[
\mathbf{J}_{\beta \alpha} = E \left\{ \left[ \frac{\partial}{\partial \beta} \ln A(z') \right] \left[ \frac{\partial}{\partial \alpha} \ln A(z') \right]^T \right\}, \quad \mathbf{J}_{\beta \beta} = E \left\{ \left[ \frac{\partial}{\partial \beta} \ln A(z') \right] \left[ \frac{\partial}{\partial \beta} \ln A(z') \right]^T \right\}, \quad \mathbf{J}_{\beta \tau} = E \left\{ \left[ \frac{\partial}{\partial \beta} \ln A(z') \right] \left[ \frac{\partial}{\partial \tau} \ln A(z') \right]^T \right\},
\]

\[
\mathbf{J}_{\tau \alpha} = E \left\{ \left[ \frac{\partial}{\partial \tau} \ln A(z') \right] \left[ \frac{\partial}{\partial \alpha} \ln A(z') \right]^T \right\}, \quad \mathbf{J}_{\tau \beta} = E \left\{ \left[ \frac{\partial}{\partial \tau} \ln A(z') \right] \left[ \frac{\partial}{\partial \beta} \ln A(z') \right]^T \right\}, \quad \mathbf{J}_{\tau \tau} = E \left\{ \left[ \frac{\partial}{\partial \tau} \ln A(z') \right] \left[ \frac{\partial}{\partial \tau} \ln A(z') \right]^T \right\},
\]

and \( \mathbf{J}_{\beta \alpha} = \mathbf{J}_{\alpha \beta}^T, \mathbf{J}_{\tau \alpha} = \mathbf{J}_{\alpha \tau}^T, \mathbf{J}_{\tau \beta} = \mathbf{J}_{\beta \tau}^T \).

Consequently, the CRLBs for the estimator of the channel coefficients and delays is given by

\[
\mathbf{J}^{-1} = \begin{bmatrix}
\text{CRLB}(\alpha) & \Pi_{\alpha \beta} & \Pi_{\alpha \tau} \\
\Pi_{\beta \alpha}^T & \text{CRLB}(\beta) & \Pi_{\beta \tau} \\
\Pi_{\alpha \tau}^T & \Pi_{\beta \tau}^T & \text{CRLB}(\tau)
\end{bmatrix},
\]

(21)

where CRLB(\( \alpha \)) and CRLB(\( \beta \)) are the Cramer–Rao lower bound for the real and imaginary parts of channel coefficient (c), respectively. CRLB(\( \tau \)) stands for the Cramer–Rao lower bound of time delays. It is desirable that the performance of the developed estimator approaches the CRLB. The matrix IT is of no interest since it does not influence the CRLB. The CRLB for channel amplitude is defined as CRLB(\( \epsilon \)) \( \simeq \sqrt{\text{CRLB}(\alpha)^2 + \text{CRLB}(\beta)^2} \).

### 4.2. Computational complexity comparison

The computational complexity of the Kalman filter, particle filter approach and the proposed algorithms such as SUF and SRUF is evaluated and compared. The computational complexity is evaluated in terms of the Big O (viz., \( O(n) \)) notation and only the main computational steps are considered. Let \( L \) (\( L = KM \)) denote the dimension of the state vector. \( K \) stands for the number of users, and \( M \) denotes the number of propagation paths. \( N \) represents the number of particles (\( x \in C^{L \times 1} \), \( F \in R^{L \times L} \), \( P \in C^{L \times L} \)). We derive the computational complexity of the channel estimators due to their main computational steps.

From Section 3, EKF requires approximately \( O(L^3) \) floating point operations (since the matrix times matrix multiplication \( FP_{xx} \) in the most time consuming step). The scaled unscented filter presents a computational complexity approximately on the order of \( O(L^2(2L + 1)) \approx O(L^3) \), mainly induced by the matrix times matrix multiplication \( (FX)_{i=0}^{2L} \). Notice also that the scaled root unscented filter requires approximately \( O(L^2(2L + 1)) \approx O(L^3) \) operations due to the matrix times matrix multiplication \( (FX)_{i=0}^{2L} \), whereas the particle filter requires approximately \( O(NL^2) \) floating point operations (flops) due to the matrix times vector multiplication and sampling step \( (FX_i)_{i=1}^N \).
The computational complexity of the EKF, PF, SUF, and SRUF algorithms is presented in Table 1. This indicates that the particle filter is approximately 25 times slower than EKF, SUF, and SRUF in an application with $L = 4$ and $N = 100$. Whereas the complexity of EKF is similar to that of SUF and SRUF in an application with $L = 4$, as will be seen in the next section, their MSE performances exhibit marked differences. Note also that as the dimensionality ($N$) increases, PF presents a higher complexity, and consequently a slower speed.

5. Numerical analysis

The performances of the SUF and SRUF are now examined for yielding parameter estimates to multiuser detectors. The SUF and SRUF-based estimators are compared with an EKF [6,7] and PF-based estimator. The multipath profile structures, which can be modeled by the 3-tap delay line of Joint Technical Committee (JTC), are chosen in Table 2 [22].

The model is deviated slightly by assuming a classical Doppler spectrum for all taps. The tap gains for the in-phase and quadrature components of the channel coefficients for each user were implemented by using eight oscillators in Jakes’ fading simulator [23] and normalized so that the average power is unity. For the state model, the augmented state transition matrix of (3) was chosen to be $F = 0.999I$. In addition, the process noise covariance matrix was $Q = 0.001I$. The users’ PN spreading codes are chosen from the set of Gold codes of length 31. The signal-to-noise ratio (SNR) at the receiver of the weaker user is 5 dB and the near-far ratio is 20 dB. The oversampling factor (sample/chip) is 2.

The three parameters ($\alpha, \beta$, and $\kappa$) of the SRUF estimator are assumed to be $\alpha = 1$, $\beta = 2$, and $\kappa = 0$, because the additive noise is Gaussian.

Fig. 2 presents the estimation error for the channel coefficients and time delays with Doppler frequency $fdTs = 0.02$ with the imperfect power controlled using the EKF and the SRUF, respectively. As the figures indicate, both estimators are able to accurately track the time-varying channel amplitudes and time delays of the weaker user.

The ability of the estimator to track time-varying parameters (first arrived multipath) is presented in Figs. 3 and 4, where the time variants of the channel delays can be expressed by the model (3). The weaker user and the stronger user are assumed to present the normalized Doppler frequencies $fdTs = 0.05$ and 0.10, respectively. Such a great Doppler frequency is not likely to occur in practical situations. However, the proposed estimator is able to converge and track the parameters in such harsh scenarios. As indicated in the figure, the estimator and tracker is able to accurately track the time-varying channel amplitudes and time delay of each user, even for fast fading rates. In addition, although the average near-far ratio is 20 dB, the instantaneous ratio varies drastically due to the channel fading, since there is no power control. As presented in Fig. 5, the instantaneous near-far ratio with respect to the second user varies from $-20$ to 25 dB. In spite of the varying powers, the tracker is still able to demonstrate excellent performance.

In order to further quantify the performance of the estimator, the MSE of the proposed estimator is presented. $L = 500$ samples (observations times) were simulated. $N_s = 100$ different realizations of the proposed SRUF and EKF were generated. The comparison is in terms of the mean of the squared error (SE), computed as follows:
Fig. 2. Parameter estimation errors for channel amplitudes and time delay.

Fig. 3. Time-varying channel amplitude tracking.

\[ SE(l) = \sum_{i=1}^{L} [x(l) - \hat{x}_s(l)]^2, \quad m(SE) = \frac{1}{N_s} \sum_{s=1}^{N_s} SE(s), \]

where \( \hat{x}_s(l) \) is computed using the \( s \)th realization of the recursive filters.
The performance of the SRUF estimator compared to the EKF, SUF, PF, and CRLB. Fig. 6 compares the SRUF mean-square error for the channel parameter estimates to the corresponding CRLB and EKF. Since the nonlinear estimator is not guaranteed to be efficient, the mean-square estimation error is above the CRLB. The figures indicate that there is a slight increase in the MSE as the near-far ratio increases. A near-far resistant estimator is expected
to be unaffected by the near-far ratio. However, the EKF estimator is rapidly increasing the MSE, above a near-far ratio of 20 dB, demonstrating that larger error occurs. The near-far ratio means that the increased power of the second user is treated as increased noise compared to the case of equal power users. In this case, the EKF estimator is not near-far resistant, but the SRUF-based estimator is almost near-far resistant. Thus, it is seen that the channel estimates obtained using the SUF, SRUF, and PF suffer much less from increasing the near-far ratio than the EKF. It is expected that the SRUF estimator with second-order accuracy is more near-far resistant and has stronger noise power immunity than the EKF estimator with first-order accuracy. The MSE-difference between the SRUF and SUF is very small but the SRUF has increased numerical stability over the SUF, because the SRUF has no square root form (see Step 1 in Section 3.3).

Fig. 7 shows the MSE performances of the proposed multiuser receiver in frequency selective fading for normalized Doppler frequencies from \( f_d T_s \approx 0.01 \) to 0.1. The SUF, SRUF, and PF-based channel estimators present a good performance. Even though the channel is fast fading (the normalized Doppler frequency is high \( f_d T_s \approx 0.1 \)), the proposed channel estimators exhibit a good performance. Thus, the simulations demonstrate the superiority of the SUF and SRUF-based multiuser receiver approaches over the EKF-based multiuser receiver. The PF-based channel estimator performance is the best, but its complexity is too expensive compared to that of SUF and SRUF.

The bit error rate (BER) performance of the proposed receiver with the EKF, PF, SUF, and SRUF-based channel estimator is shown in Fig. 8 with normalized Doppler frequency \( f_d T_s \approx 0.02 \). The SUF and SRUF-based multiuser receivers are clearly better than the EKF-based multiuser receiver and perform close to the perfect channel-based multiuser receiver, too. But the PF presents too expensive computational complexity compared to the SUF and SRUF. Thus, this result illustrates that the SUF and SRUF-based channel estimators are more efficient than the EKF and PF-based channel estimator.

The computation time required by the above-mentioned algorithms is presented in Fig. 9, for simulations implemented on an Intel Pentium IV-2.4 GHz processor using MATLAB. The computation time for PF is much greater than that of EKF, SUF, and SRUF. More importantly, the SUF, and SRUF present a similar computation time as the EKF, even though their MSEs present more improved performance. This result is almost similar with the analytical result presented in Section 4.2.
6. Conclusions

This paper proposed a novel adaptive multiuser detector structure. The proposed receiver structure and robust recursive filters are implemented to operate both as blind joint channel estimator and data detector. The SUF and
SRUF have better numerical properties and guarantees positive semi-definiteness of the underlying state covariance relative to the EKF, and provide improved performance compared to EKF. In this paper, channel estimators based on the SUF and SRUF are presented and shown to be capable of estimating the channel coefficients and time delays in MAI for a near-far ratio of 40 dB. The SUF and SRUF have been shown to present improved performance over the EKF. The SUF and SRUF can provide a better alternative to nonlinear filtering than the EKF since they exhibit superior numerical conditioning. Computer simulations also demonstrate that the proposed schemes provide a more viable means than EKF for tracking time-varying amplitudes and delays in CDMA systems.

References


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