Space Division Competitive Access for Infrastructured Wireless Mesh Networks

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Abstract—This paper focuses on the competitively optimal power-control, signal-shaping and interference mitigation for wireless mesh networks composed by Multiple-Antenna noncooperative transmit terminals and a base station affected by spatially colored Multi-Access Interference (MAI). The target is the competitive maximization of the information throughput of the uplink of each link active over the network. For this purpose, the MAI-impaired network is modeled as a noncooperative strategic game. Specifically, the main contribution of this paper may be summarized. First, we consider a power-control, signal-shaping and interference mitigation algorithms allowing the implementation of asynchronous Space-Division Multiple Access (SDMA) strategies able to guarantee the competitive maximization of the users’ rate under both Quality of Service (QoS) guaranteed and QoS contracted access policies. Second, we give evidence that the developed SDMA outperforms (in terms of aggregate throughput) the conventional orthogonal ones, especially in operating scenarios affected by strong MAI. Third, we consider the multiple access capacity that is the maximum number of users able to be connected with a requested level of QoS in terms of rate.

Index Terms—Multiple Antennas, Game Theory, MAI, SDMA, Power-Allocation, Competitive Optimality, Wireless Mesh Networks.

I. INTRODUCTION AND GOALS

The implementation of Multi-Antenna systems are uniformly recognized as one of the challenge for future communications. They allow to achieve high bit rate, present very reliable performance in terms of error rate without increasing power and can extend coverage of hot spot cells. This challenge may aid the new frontiers of communications as for networks that are planned to be setup in rural areas or where Fiber To The Building (FTTB), Power Line Communications (PLCs) and xDSL systems result to be too expensive. One of the possible solutions consists in Wireless Mesh Networks (WMNs) [1] that are considered as possible candidate to implement WiMAX and to allow the end-user the Internet connection so to solve the problem of digital divide.

A. State of Art

Due to the expected capability to guarantee “always-on” radio access, WMNs are emerging as the main candidate for supporting next-generation high-throughput Personal Communication Services (PCSs) [2]. To accomplish the resulting increasing demand of access throughput advanced by the network users, the spatial dimension provided by Multi-Antenna terminals gives rise to an additional network resource that may be also effectively exploited [2]. In this context, a key question concerns the evaluation of the ultimate set of rates deliverable to the networking users under different QoS requirements and system constraints (such as, power limits, network topology, number of allowed transmit/receive antennas and so on). For the case of cellular (e.g., centralized) networks equipped with base stations, this question has been tackled by separately considering the Multiple-Access (MAC) uplink and the Broadcast (BC) downlink [3,4]. Specifically, SDMA achieved via Multiple-Antenna transceivers is today recognized a primary mean for increasing the capacity of wireless multiple (e.g., centralized) networks [5]. In fact, SDMA at the physical layer may be combined with (contention-based or reservation-based) multiple access protocols operating at the MAC layer, so to enable intensive frequency/time/code channel reuse by multiple spatially separable cellular users [5,6]. A SDMA-based smart WLAN system for applications with centralized Access Points has been recently proposed in [7] for cooperation with the IEEE 802.11 MAC protocols. Currently, several companies as Iospan wireless, Metawave, Navini and Arraycom are developing SDMA products for cellular system that support multiple access schemes. The “ad-hoc” scenario is more recent and then less well understood. In this case, the additional difficulty arises from the fact that an “ad-hoc” network is an example of interference channel [8]. Since this last is, indeed, the superposition of multiple MAC and BC channels [8], barring some particular cases [8] the corresponding set of achievable users’ throughput is unknown, and it resists to closed-form analytical evaluation even in the simpler case of single-antenna terminals [8]. To complete the picture, the landmark paper [9] leads to the somewhat disappointing conclusion that, when the number of active users increases, an “ad-hoc” network becomes MAI limited, so that the achievable transmission throughput per user asymptotically vanishes.

B. Proposed Contributions

In any case, power-control, spatial signal-shaping and interference mitigation are central issues for the optimized design of MAI-limited networks. In fact, the information throughput (measured in bits/slot) conveyed by each link depends not only on the power-allocation and signal-shaping performed by each transmitter, but also on the power-allocations, signal-shaping of all other transmitters active over the network and interference mitigation at receive side. Thus, the optimized design of the overall network involves a performance tradeoff among all active transmitters. Such tradeoff is the subject of the present work. Specifically, the power-control, signal-shaping and interference mitigation algorithms we propose aim to maximize the information throughput conveyed by each link active over the network and are based on the modelling of uplink of a
WMN as a noncooperative strategic Game. The Game-The-ory point of view has been adopted in several recent contributions dealing with the power-control problem for wireless networks [10,11,12,13]. However, all these works focus on scenarios characterized by single-antenna terminals and then they fully neglect the spatial-dimension of the system. On the contrary, in emerging WMNs built up by Multi-Antenna transceivers, the spatial-dimension of the overall system is crucial and it must be explicitly taken into account in order to optimized the network throughput. Formally stated, the main result of this contribution is that, under suitable conditions, the Game has a Nash Equilibrium (NE) under both QoS-guaranteed and Contracted QoS access policies. This result leads to iterative power-control, signal-shaping and interference mitigation algorithms, able to achieve the equilibrium point in an asynchronous way. Specifically, we want anticipate some key results of this work.

- The presented approach is optimal in a competitive sense and, then, it strikes an optimized balance between maximizing each user own rate and minimizing the induced interference effects.
- The proposed algorithms allow to implement Guaranteed and Contracted QoS access policies and then they may account for multiple QoS classes.
- When the QoS requirements (measured in terms of requested throughput) advanced by the users are no sustainable by the network, then the proposed power-control, signal-shaping and interference mitigation algorithms move the working point of the network (expressed in terms of delivered throughput) to the nearest one sustainable by the system.
- Several numerical results support the conclusion that the proposed distributed algorithms outperform conventional orthogonal ones in terms of conveyed rate, specially in networking scenarios affected by strong MAI.

Finally, by considering the performance of the developed power-control and signal-shaping algorithms, in the last part of the paper we consider the multiple-access capacity by showing how the spatial dimension is able to give the possibility to access to a large number of users.

C. Organization of the work

The remainder of this work is organized as follows. After the system modeling of Sect.II, Sect.III deals with the evaluation of the conveyed information throughput in networking environments affected by MAI. In Sect.IV the optimized power-allocation and interference mitigation are presented. Thus, after shortly reviewing in Sect.V of Game Theory essentials, in Sect.VI we propose a Game for Access. Actual performance of the proposed power-allocation and signal-shaping Game in terms of conveyed network throughput are tested in Sect.VI. Capital letters indicate matrices, lower-case underlined symbols denote vectors, while characters overlined by arrow denote block-matrices and block-vectors. Apexes *, †, ‡ mean conjugation, transposition and conjugate-transposition respectively, while lower-case letters will be used for scalar quantities. In addition, det [A] and Tr[a] indicates the (block) vector obtained by the ordered stacking of the columns of A. Finally, is the m-dimensional zero-vector, lg denotes natural logarithm and δ(m, n) is the Kronecker delta.

II. THE SYSTEM MODELING

The application scenario we consider models a wireless mesh network (WMN) [1,14] where a (large) number of transmit-receive nodes simultaneously attempt to access the medium over a limited-size cell by using an access scheme based on space-division so giving arise to multiple access interference (MAI). The (complex base-band equivalent) point-to-point radio channel linking a transmit node Tx to the receive node Rx (base-station) is sketched in Fig.1. Simply stated, it is composed by a transmit unit equipped with antennas communicating to a receive unit equipped with antennas via a Multiple-Input Multiple-Output (MIMO) radio channel impaired by both slow-variant flat Rayleigh fading and additive MAI induced by adjacent transmit nodes active over the same hot-spot cell. The path gains may be considered mutually uncorrelated. Furthermore, the path gains may be assumed time-invariant over signalling periods, after which they change to new statistically independent values held for another signalling periods, and so on. The resulting "block-fading" model well represents the main features of several frequency-hopping or packet based interleaved 4G systems, where each transmitted packet is detected independently on any other [17]. About the MAI affecting the uplink,

1This scheme may represent low/medium mobility mesh routers accessing to base station or mesh clients accessing to mesh router.

2The assumption of flat fading is met when RF bandwidth is the coherence bandwidth of the channel.
its statistics mainly depend on the network topology [17], and in the application scenario here considered it is reasonable to assume these last constant over (at least) an overall packet [17]. However, since both path gains \{h_{ij}\} and MAI statistics may change from a packet to another, we assume that transmitters and receiver in Fig.1 are not aware of them at the beginning of each transmitted packet. Hence, we assume that the coded and modulated streams radiated by the transmit antennas are split into packets composed by \(T \geq 1\) slots, where the first \(T_L \geq 0\) slots are used by the receiver for learning the MAI statistics (see [17]), the second \(T_r \geq 0\) slots are employed for estimating the path gains \(\{h_{ij}\}\) of the forward MIMO channel (see [17]), and the last \(T_{pay} \triangleq T - T_r - T_L\) slots convey payload data (see [17]). Obviously, this packet structure induces rate reduction since \(T_{pay} < T\).

A. The Payload Phase

As in [11], we suppose to get estimation of interference covariance matrix \(K_d\) and channel coefficients matrix \(H\) and by basing on these last and actual packet \(\mathcal{P}\) to be transmitted, the transmit node of Fig.1 suitable shapes the signal streams \(\{\phi_i(n) \in \mathbb{C}^1, T_L + T_r + 1 \leq n \leq T\},\ 1 \leq i \leq t,\) to be radiated during the payload phase. The corresponding (sampled) signals \(\{y_i(n) \in \mathbb{C}^1, T_L + T_r + 1 \leq n \leq T\},\ 1 \leq j \leq r,\) measured at the outputs of the receive antennas may be modelled as [15,17]

\[
y_j(n) = \sqrt{\frac{a}{t}} \sum_{i=1}^{t} h_{ji} \phi_i(n) + d_j(n), T_L + T_r + 1 \leq n \leq T, \tag{1}
\]

where the sequences \(d_j(n) \triangleq v_j(n) + w_j(n),\ 1 \leq j \leq r,\) account for the overall disturbances (e.g., MAI plus thermal noise) experienced during the payload phase while \(a = L^{-z}\) takes care for the path loss over a distance of \(L\) meters. Therefore, after assuming that the transmitted streams meet the (usual) power constraint [15]

\[
\frac{1}{t} \sum_{j=1}^{t} E[||\phi_i(n)||^2] \leq P, T_L + T_r + 1 \leq n \leq T, \tag{2}
\]

the resulting SINR \(\gamma_j\) measured at the output of the \(j\)-th receive antenna during the payload phase equates (see eqs.1, 2)

\[
\gamma_j = \frac{a P}{(N_0 + c_{jj})}, 1 \leq j \leq r. \tag{3}
\]

Furthermore, from (1) we also deduce that the \((r \times 1)\) column vector \(\mathbf{y}(n) \triangleq [y_1(n) \ldots y_r(n)]^T\) collecting the outputs of the \(r\) receive antennas over the \(n\)-th payload slot is linked to the \((t \times 1)\) column vector \(\mathbf{\phi}(n) \triangleq [\phi_1(n) \ldots \phi_t(n)]^T\) of the corresponding signals radiated by the transmit node as in

\[
\mathbf{y}(n) = \sqrt{\frac{a}{t}} \mathbf{H}^t \mathbf{\phi}(n) + \mathbf{d}(n), T_L + T_r + 1 \leq n \leq T, \tag{4}
\]

where \(\mathbf{d}(n) \triangleq [d_1(n) \ldots d_r(n)]^T, T_L + T_r + 1 \leq n \leq T\) is the temporally white Gaussian sequence of disturbances with spatial covariance matrix still given by \(K_d\). Furthermore, directly from (2), it follows that the \((t \times t)\) spatial covariance matrix \(\mathbf{R}_\Phi \triangleq E[\mathbf{\phi}(n) \mathbf{\phi}(n)^H]\) of the \(t\)-dimensional signal vector radiated during each slot must meet the following power constraint:

\[
\text{Tra}[\mathbf{R}_\Phi] \equiv E[\mathbf{\phi}(n) \mathbf{\phi}(n)^H] \leq t P, T_L + T_r + 1 \leq n \leq T. \tag{5}
\]

Finally, after stacking the \(T_{pay}\) observed vectors in (4) into the corresponding \((T_{pay}t \times 1)\) block vector \(\mathbf{Y} \equiv [\mathbf{y}^T(T_L + T_r + 1) \ldots \mathbf{y}^T(1)]^T\), we may compact the \(T_{pay}\) relationships (4) into the following one:

\[
\mathbf{Y} = \sqrt{\frac{a}{t}} (T_{pay} \otimes \mathbf{H})^t \mathbf{\Phi} + \mathbf{d}, \tag{6}
\]

where the (block) covariance matrix of the corresponding disturbance (block) vector in (6) \(\mathbf{d} \triangleq [\mathbf{d}^T(T_L + T_r + 1) \ldots \mathbf{d}^T(1)]^T\) equates

\[
E[\mathbf{d} \mathbf{d}^H] = I_{T_{pay}} \otimes K_d, \tag{7}
\]

while the squared norm of the block vector \(\mathbf{\Phi} \triangleq [\mathbf{\phi}^T(T_L + T_r + 1) \ldots \mathbf{\phi}^T(1)]^T\) of the random signals transmitted during the payload phase is constrained as in (see (2))

\[
E[\mathbf{\Phi} \mathbf{\Phi}^H] \leq T_{pay} t P. \tag{8}
\]

III. PROBLEM SETUP

By considering the access of \(n^*\) users to the medium, that is, the requests advanced by multiple users to communicate with a base-station (or a mesh router), we have to take into account for a parameter allowing both to choose access strategy and to evaluate the resulting performance [8]. The parameter is the Shannon Capacity, e.g., the transmission rate that assures the existence of a code able to lower the error decoding probability to zero [19]. In order to maximize the transmission throughput (e.g., rate) we have to select the "best" power allocation able to achieve the supremum in the following expression

\[
\mathcal{R}(\mathbf{H}) \triangleq \sup_{\mathbf{\Phi}} E[\mathbf{\Phi}^H \mathbf{\Phi}] \leq T_{pay} P \frac{1}{T_{pay}} I(\mathbf{Y}; \mathbf{\Phi} \mathbf{H}), \text{ (nats/slot)} \tag{9}
\]

where \(I(\mathbf{Y}; \mathbf{\Phi} \mathbf{H})\) is the information throughput conditioned on the actual estimated version of \(\mathbf{H}\). Furthermore, the dependence on \(\mathbf{H}\) let the function \(\mathcal{R}(\mathbf{H})\) a random variable to be averaged on \(\mathbf{H}\), that presents a probability density function (pdf) described by the following relationship [17]

\[
p(\mathbf{H}) = \left(\frac{1}{\pi(1 - \sigma^2)}\right)^t \exp \left\{-\frac{1}{(1 - \sigma^2)} \text{Tra}[\mathbf{H}\mathbf{H}^H]\right\}, \tag{10}
\]

where \(\sigma^2\) is the channel estimation error variance. So, the averaged version of the rate can be represented as

\[
\mathcal{R} = E[\mathcal{R}(\mathbf{H})] = \int \mathcal{R}(\mathbf{H}) p(\mathbf{H}) d\mathbf{H}, \text{ (nats/slot)}. \tag{11}
\]
About the expression reported in eq.(9), this last depends on the allocation strategy we want to pursue. Now, we can resort to two typical approaches pursued in the literature in order to allocate power over multiple antennas [20] so to maximize rate. The first case requires the existence of a feedback link able to feed to the transmitter information about channel and interference or, under the realistic hypothesis of power allocation algorithm performed at the receiver, only the power levels to be employed in the allocation. The expression for the information throughput is given by [17]

\[ I(Y; \phi | H) = T_{pay} \log \det \left( I_r + \frac{a}{t} K_d^{-1/2} H^T R_\phi H K_d^{-1/2} + a \sigma_e^2 P K_d^{-1} \right) \]

\[ - \log \det \left( I_r + \frac{a \sigma_e^2 T_{pay}}{t} (K_d^{-1})^* \otimes R_\phi \right). \]  

where, the matrix \( R_\phi \) is the output of an algorithm that, in the simple case of perfect channel state information (PCSI) at the transmitter, becomes the standard waterfilling problem solution [20].

In the second case, we consider no channel state information (NCSI) at the transmitter, so no feedback channel is required since, as known from [20], the allocation maximizing the information throughput \( I(Y; \phi | H) \) that is given by is achieved simply by allocating the same power level over the antennas so \( R_\phi = \frac{P}{t} I_r \). Furthermore, in both cases an additional module can be added at the receiver side and this consists in a spatial signal processor able to mitigate the effect of interference and, consequently, to enhance rate level. We can anticipate that, in the case of NCSI, the interference mitigation module operates only according to the power level while, when imperfect channel state information (ICSI) is considered, the shaping influences the performance.

IV. OPTIMIZED POWER-ALLOCATION

In order to achieve the supremum in (9), we must proceed to carry out the power-constrained maximization of the conditional throughput. For this purpose, let us indicate as

\[ K_d = U_d A_d U_d^\dagger, \]  

the Singular Value Decomposition (SVD) of the MAI spatial covariance matrix \( K_d \), where

\[ A_d \triangleq \text{diag}\{ \mu_1, ..., \mu_r \}, \]

is the corresponding \((r \times r)\) diagonal matrix of the magnitude-ordered singular values of \( K_d \). Thus, after introducing the \((t \times r)\) matrix

\[ A \triangleq H^T K_d^{-1/2} U_d, \]

accounting for the combined effects of the imperfect channel estimate \( H \) and spatial MAI \( K_d \), let us denote as

\[ A = U_A D_A V_A^\dagger, \]

the corresponding SVD, where \( U_A \) and \( V_A \) are unitary matrices, while

\[ D_A \triangleq \begin{bmatrix} \text{diag}\{k_1, ..., k_s\} & 0_{s \times t-r-s} \\ 0_{t-r-s \times s} & 0_{t-r-s \times t-r-s} \end{bmatrix}, \]  

is the \((t \times r)\) diagonal matrix collecting the \( s \) \((s \geq \min\{r, t\})\) magnitude-ordered singular-values \( k_1 \geq k_2 \geq ... \geq k_s > 0 \) of the matrix \( A \). Finally, for future convenience, let us also introduce the following dummy positions

\[ \alpha_m \triangleq \frac{\mu_m k_m^2}{(\mu_m + P \sigma_e^2)^2}, 1 \leq m \leq s; \quad \beta_l \triangleq \frac{\sigma^2 T_{pay}}{t \mu_l}, 1 \leq l \leq r. \]  

Thus, it can be proven [17] that the application of the Kuhn-Tucker conditions [21] allows us to evaluate the optimized transmit powers \( \{ P^*(m), 1 \leq m \leq t \} \) achieving the constrained supremum in (9) as in [22].

After the optimization procedure, carried out by implementing the algorithm reported in [22], we can evaluate the rate as function of \( H \), according to the following expression

\[ R(\hat{H}) = \sum_{m=1}^{r} \Lambda \left( 1 + \frac{\sigma^2 P^*(m)}{\mu_m} \right) + \sum_{m=1}^{s} \left( \log(1 + \alpha_m P^*(m)) \right) - \frac{1}{T_{pay}} \sum_{m=1}^{r} \log \left( 1 + \beta_l P^*(m) \right). \]  

where the covariance matrix of spatial-shaping can be considered given by

\[ R_\phi(\text{opt}) = U_A \text{diag}\{ P^*(1), ... P^*(s), 0_{t-r-s} \} U_A^\dagger. \]

A. Interference Mitigation

The approach for interference suppression/mitigation is based on an estimation and successive subtraction from the received sequence of an estimated version of MAI. The rate, in this case, is quite similar to that in (12) since the only difference consists in considering the matrix \( K_{d,r} \), that is the residual interference covariance matrix, in place of \( K_d \).

By taking into account a linear estimator for this problem (that is Gaussian), it can be observed that this last is efficient since, as known from estimation theory [23], it approaches the Cramer Rao Bound (CRB). Hence, the general expression for the estimator is

\[ \hat{D} = YA, \]  

where \( A \) is the \( r \times r \) matrix obtained from the following equation derived from the Orthogonal Projection Lemma

\[ E\{ (\hat{D} - D)^\dagger Y \} = 0_{r \times r}. \]

By substituting the expression eq.(21) in eq.(22) and by solving with respect to \( A \) we obtain

\[ A^\dagger E\{ Y^\dagger Y \} = E\{ D^\dagger Y \}, \]

that leads to

\[ A = \left( E\{ D^\dagger Y \} (E\{ Y^\dagger Y \})^{-1} \right)^\dagger. \]
The above expression is not explicit so in order to give details we express the term $E[D^tY]$ as
$$E[D^tY] = K_d,$$ (25)

since the terms $D$ and $\sqrt{a}\Phi H$ are statically independent. In addition the term $E[Y^trY]^{-1}$ can be rewritten as
$$E[Y^trY] = E\left\{ (\sqrt{aH} + D)^t (\sqrt{aH} + D) \right\} =
= aE[H^t\Phi^t\Phi H] + K_d =
= aE[H^tR_{\Phi}H] + K_d,$$ (26)

where the term $\Phi^t\Phi$ is $P^{(t)}I$, when NCSI and uniform allocation are considered3. So the expression for $A$ is given by
$$A = \left( K_d \left( aE[H^tR_{\Phi}H] + K_d \right)^{-1} \right)^t,$$ (27)

so the estimated version of $D$ is given by
$$\hat{D} = Y \left( K_d \left( aE[H^tR_{\Phi}H] + K_d \right)^{-1} \right)^t.$$ (28)

In order to evaluate the residual interference covariance matrix we have to evaluate
$$K_{de} = E\{D^tD\} = K_d + K_d - 2Re\{E[D^tY]\}A,$$ (29)

where the term $K_d$ is given (and not reported cause of tedious algebra) by
$$K_d = K_d \left\{ aE[H^tR_{\Phi}H] + K_d \right\}^{-1} K_d^t,$$ (30)

so the eq.(29) can be rewritten as
$$K_{de} = K_d \left( I - 2 \left\{ aE[H^tR_{\Phi}H] + K_d \right\}^{-1} \right)^t + \left\{ aE[H^tR_{\Phi}H] + K_d \right\}^{-1} K_d^t.$$ (31)

Let us consider now what are the “limit” conditions for interference. When the term $aE[H^tR_{\Phi}H]$ becomes negligible with respect to matrix $K_d$ the matrix $K_{de}$ approaches the white noise one, that is the interference free case. On the other hand, if we consider high level of term $aE[H^tR_{\Phi}H]$, we obtain that no interference is mitigated, since $K_{de} = K_d$.

V. GAMES THEORY ESSENTIALS

In order to model the dynamic behavior of the WMN composed by multiple mutually interfering no cooperating transmitters, we resort to the formal tool of the Game Theory [24]. We recall that a noncooperative and strategic game $G \triangleq \langle N, \mathcal{A}, \{u_g\} \rangle$ has three components [24,25]: a finite set $N \triangleq \{1, 2, \ldots, n^*\}$ of players, a set $A_g$, $g \in N$ of possible actions for each player and a set of utility functions. Specifically, after denoting as $\mathcal{A} \triangleq A_1 \times A_2 \times \ldots \times A_n$, the space of action profiles [25], let us indicate as $u_g : \mathcal{A} \rightarrow \mathbb{R}$ the $g$-th player’s utility function. Thus, after indicating by $a \in \mathcal{A}$ an action profile, by $a_g \in A_g$ the players action in a and by $a_{-g}$ the actions in a of the other $(n^* - 1)$ players, we can say that $u_g(a) \equiv u_g(a_g, a_{-g})$ maps each action profile a into a real number [25]. In particular, in a strategic noncooperative game each player chooses a suitable action $a^*_g$ from his action set $A_g$ so to maximize its utility function, according to the following game rule [13]:
$$a^*_g \equiv \max_{a_g \in A_g} u_g(a_g, a_{-g}).$$ (32)

Therefore, since there is no cooperation among the players, it is important to ensure the dynamic stability of the overall game. A concept which relates to this issue is the so-called Nash Equilibrium (NE). Simply stated, a Nash Equilibrium is an action profile $a^*$ at which no player may gain by unilaterally deviating [24,25]. So, a NE is a stable operating point of the Game, because no player has any profit to change his strategy [24]. More formally, a NE is an action profile $a^*$ such that for all $a_g \in A_g$ the following inequality is satisfied [24,25]:
$$u_g(a^*_g, a_{-g}) \geq u_g(a_g, a_{-g}), \forall g \in N, \forall a_g \in A_g.$$ (33)

VI. THE PROPOSED ACCESS GAME

Let us focus now on the WMN composed by $n^*$ mutually interfering transmit Multi-Antenna units trying to communicate with a base station. The ultimate task of the $g$-th transmitter is to maximize the information throughput $R(g), g = 1, \ldots, n^*$, sustained by the corresponding link $T_{gx} \rightarrow R_g$ via suitable power-allocation and shaping of the signals radiated by $T_{gx}$ and interference cancellation at receive side. Since the signals radiated by $g$-th transmitter induces MAI on the base-station and we assume the transmitters not exchanging information (e.g., the transmitters do not cooperate), we may model the interaction between transmit nodes active over the network as a noncooperative strategic game [24,25]. Specifically, in the considered networking scenario the players’ set $N$ is composed by the $n^*$ transmitters, while the set of actions $A_g$ available to the $g$-th player is the set of all the covariance matrices $\{R_{\Phi}(g)\}$ meeting the power constraint (22), so we can pose
$$A_g \triangleq \{R_{\Phi}(g) : 0 \leq \text{Tr}[R_{\Phi}(g)] \leq t_g P_g\}, \quad g = 1, \ldots, n^*.$$ (34)

This means that the generic action $a_g$ of $T_{gx}$ consists in the transmission of a Gaussian distributed payload sequence with covariance matrix $R_{\Phi}(g)$. Furthermore, the utility function $u_g(.)$ for the $g$-th uplink pair is the conditional throughput conveyed by the $g$-th uplink, so that we can write (see eq.(123))
$$u_g(a) \triangleq u_g(R^{(1)}_{\Phi}, \ldots, R^{(g)}_{\Phi}, \ldots, R^{(n^*)}_{\Phi}) \equiv \frac{1}{t_{pay}} I(\tilde{Y}^{(g)}_{\Phi} : \phi_{\Phi} | H_g)$$

4The notation $u_g(a_g, a_{-g})$ emphasizes that the $g$-th player controls only own action $a_g$, but his achieved utility depends also on the actions $a_{-g}$ taken by all other players [24,25].
\[ \log \det \left( I_n + \frac{a}{t_g} (K_{dx}^{(g)})^{-1/2} H_y^T R_{\phi}^{(g)} (K_{dx}^{(g)})^{-1/2} + \sigma^2 g P(g) (K_{dx}^{(g)})^{-1} \right) \]

\[ - \frac{1}{T_{pay}} \log \det \left( I_{t_g} + \frac{a \sigma^2 g}{t_g} T_{pay} ((K_{dx}^{(g)})^{-1})^* \otimes R(g) \right), \tag{35} \]

where the \( g \)-th MAI covariance matrix \( K_{dx}^{(g)} \) depends on the spatial covariance matrices \( \{ R_{\phi}^{(i)}, i \neq g \} \) of the signals radiated by the interfering transmitters and cancellation. About the rule of the game, each player (e.g., transmitter \( T_{xg} \)) chooses the action \( R_{\phi}^{(g)}* \) maximizing the throughput conveyed by own link, so we can write

\[ R_{\phi}^{(g)}* \equiv \max_{R_{\phi}^{(g)} \in A_g} \left\{ \frac{1}{T_{pay}} I \left( \overline{y}^{(g)}, \overline{g}^{(g)} \mid H_y \right) \right\}, \tag{36} \]

being \( g = 1, \ldots, n^* \).

**Remark - Interference mitigation and Nash Equilibrium**

In the game sketched above, we apparently do not consider the allocation problem related to interference cancellation. As it results clear from (31) the residual interference (after mitigation) to be considered for allocation depends on the spatial shaping employed at the transmitter. So, after the first performed allocation, the receiver proceeds to mitigate the effect of MAI and then, through a feedback link, to send to the transmitter information about \( H \) and \( K_{dx} \), or simply on the powers to be employed at the antennas. So the procedure is iterative and stops when the allocation matrix \( R_{\phi} \) does not change considerably from an iteration to another [22].

An important issue deals with the considerations about the existence of a NE and its uniqueness. Before entering in depth, let us consider the rate function. As shown in [27], when the interference mitigation is considered, the rate function is not concave in the available power since, as from (31), the transmitted power influences the residual interference level. Hence, by assuming low power regime, we can affirm that the maximum is unique for the rate function.

Now, let us pay attention to three different situations for signal to interference ratio (SIR) levels. When the SIR level is high, as stated before, the interference is negligible so, according to [22], the existence and uniqueness can be proved. On the other hand, when low SIR is considered, the interference mitigator is able to perform very reliable cancellation so we arrive at the same condition of high SIR. Last, in the medium SIR case, the uniqueness of NE cannot be assured formally but practical considerations about networking scenarios allow us to consider "most probable" situation the low SIR one. In fact, by resorting to the low power regime assumption and under the hypothesis of uniform nodes distribution over a cell area, we can consider the SIR as given by \( SI R = 1/n^* \) that, for high \( n^* \) value, let the user fall in the low SIR scenario and a unique NE is guaranteed under the hypotheses reported in [22].

**VII. COMPETITIVE ACCESS AND SYSTEM PERFORMANCE**

In this Section we present the performance for the optimized power-allocation, signal-shaping and interference mitigation mainly under QoS-guaranteed and contracted-QoS policies for network composed by a base station and \( n^* \) transmitting nodes. Before proceeding, some remarks about the considered QoS policies are in order. We consider the QoS from an information throughput point of view. Thus, we consider guaranteed user’s QoS, and where not possible, we resort to the concept of contracted QoS defined according to predefined multiple QoS classes. Since these throughput classes are set according to the multiple QoS requirements that the MAC layer requests from the physical layer, the procedure may be considered an instance of resource allocation algorithm working according to the cross-layer principle. Specifically, the approach we consider attempts to achieve the target throughput classes dictated by the MAC layer and, if these classes are not achievable due to the MAI, the algorithm attempts to achieve the next lower QoS classes by decreasing the throughput requested by the users.

To test the effectiveness of the proposed approach, several numerical tests have been carried out.

**A. The Achievable Throughput Regions**

The set of simultaneous throughput achieved by the \( n^* \) links \( T_{xy} \rightarrow R_x, g = 1, \ldots, n^* \) active over the ad-hoc network may be described by resorting to the concept of achievable rate region [15,21]. Roughly speaking, for a given statistical description of the network links and a set of constraints on the network input statistics (power, distribution, etc.), the corresponding achievable rate region by the overall network is the closure of all rates \( n^* \)-ples \( (R_1, \ldots, R_{n^*}) \) that can be simultaneously sustained by the communication channels \( T_{xy} \rightarrow R_x, g = 1, \ldots, n^* \), active over the network [14,25]. Barring some partial contributions, till now no closed-form analytical formulas are available for the computation of the achievable rate region of an interference network [9,13]. In Fig.2 the two-dimensional rate regions for different transmit/receive approaches, when a simple network composed by two users accessing the medium is considered (for sake of representation), are shown. The system parameters we refer to are characterized by the users equipped with \( t_1 = t_2 = 4 \) transmit antennas while the base station is equipped with \( r = 4 \) receive ones. The considered Signal-to-Noise Ratio (SNR) is 20dB. By considering the region for different shaping approaches, we label as \( A \) the region characterized by uniform allocation without any form of spatial shaping and interference cancellation. As it results clear, it presents for user 1 a rate equal to 20 bits/slot when the user 2 does not transmit and the maximum network rate (by considering the two users simultaneous transmission) is achieved for \( R_1 = R_2 = 11 \) bits/slot that give a total network rate of 22 bits/slot. After, the region labeled as \( B \) (that contains the \( A \) region) refers to the uniform allocation as in the previous case with the additional feature of interference mitigation. Although no differences are considered when one of the two users does not transmit (no interference to be suppressed), the maximum level for the two users scenario is achieved for \( R_1 = R_2 = 16.5 \) that leads to a sum rate of 33 bits/slot. Furthermore, when only signal-shaping is considered the \( C \) region

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\(^5\)From this point of view, the Best Effort strategy is a particular case of the contracted QoS one, where the number of QoS classes approaches infinity.
represents the achievable rate pair \((R_1, R_2)\). This last presents a rate for user 1 that is 25 bit/slot (when user 2 does not transmit) since, through the signal shaping, we achieve higher value of rate [22] with respect to uniform allocation. Hence, the maximum rate for the two users is achieved for \(R_1 = R_2 = 21\) that gives a sum rate of 42 bits/slot. Obviously region \(C\) contains

region \(B\). By dealing with \(D\) region, this is the locus that collects the \((R_1, R_2)\) points achievable by the considered system when signal-shaping at the transmitter and interference mitigation at the receiver (base station) are considered. The \(D\) region contains \(C\), and it presents the same values on the axes (case of no interference) of \(C\) region since the interference cancellation does not produce effects when interference is absent. The maximum, for the two user case, is achieved for \(R_1 = R = 23\) that gives a total rate of 46 bits/slot. This value is close to the ideal case, represented by the \(E\) region that represent the case of continuous transmission and perfect interference cancellation. However, it does not represent the case of an orthogonal access as TDMA as will be appear clear in the following.

**B. Access Game-vs-Orthogonal Access**

The proposed approach is able to outperform conventional orthogonal access as, for example, Time Division Multiple Access (TDMA) that requires that each user transmit once in a frame. The above conclusion is also supported by the \(T\) region Fig.2 that reports the throughput achieved by the standard TDMA for the same networking squared scenarios previously considered. In fact, an examination\(^6\) of Fig.2 shows that, although the TDMA is a technique assuring orthogonal (e.g., collision free) multiple access, nevertheless the corresponding throughput are below than those achieved by running the proposed Access Game, specially when the MAI effects are substantial. Overall, the Access Game-vs-TDMA comparison of Fig.2 supports for the superiority of the competitively optimal access strategies over orthogonal ones, at least in networking scenarios where the spatial-dimension of the system may be efficiently exploited to perform MAI suppression both at the transmitter and the receiver. In addition the proposed scheme can accept asynchronism between all the transmitters (giving arise to collisions), then this means that the base station requires not a network synchronization (required for orthogonal, non colliding, access) but only to know the arrival times (distances) of the \(n^*\) users so to perform channel and interference estimation. This gain is counterbalanced by a drawback that consists into installing an interference cancellation module at the receiver for each node to serve.

**C. Multiple Access Game Capacity**

Since the pursued approach seems to offer to the users high rates, it can be interesting to evaluate, for an assigned (e.g., required) level of QoS, how many users are able to access the medium with the constrained quality level when different values of transmit or receive antennas are considered. In particular, in Fig.3 we evaluate the maximum number of users able to access when \(t = 4\) transmit antennas are considered for each transmitter and \(r\) ranging from 1 to 8 are considered at the base-station, with different QoS levels that are 10, 15, 20 bits/slot. Two are the main aspects to underline. First, the system is able to allow access to a number of users that increases if the required QoS level decreases. Second, the three plots show a different behavior (number of user accessing for additional receive antennas) if we consider \(r < t\) or \(r > t\) and this can be justified by taking into account for the capacity increasing that depends on \(\min(r, t)\) [20] so when \(r < t\) the rate is influenced by \(r\) while if \(r > t\) the rate is influenced by \(t = 4\). In addition, by increasing \(r\) we obtain better performance by dealing with interference mitigation that, from a rate region point of view, means that we are approaching the ideal behavior (\(E\) region) as we were in interference-free scenario. In Fig.4 a different situation is studied. We consider, for the same QoS levels of Fig.3, the number of users able to access when \(r = 8\) and \(t\) ranges from 1 to 8. Since in this case we do not analyze the performance for \(t > r\) we can affirm that the increasing number of users able to access with the required QoS increases since the

\(\text{Fig. 2. Rate regions for different transmit/receive approaches (} t = r = 4).\)

\(\text{Fig. 3. Number of users accessing the net under QoS-guaranteed policy for variable } r \text{ values.}\)
signal-shaping (not performed when \( t = 1 \)) is able to increase rate and, at the same time, since the rate goes as \( \min(r, t) \), \( t \) is the parameter that influences the achievable rate. It is also important to note that since the base station is assumed to be equipped with \( r = 8 \) receive antennas this assures good level of interference cancellation. By comparing Fig.3 and Fig.4 it is possible to appreciate how, by requiring 20 bits/slot when \( r = 8 \), if the users are equipped with \( t = 4 \) transmit antennas, by fact, the number of users able to access is equal to 6, while if \( t = 8 \) the number is 19. This means that, by considering a \( T_{coh} = T = 1 \text{ ms} \) and packets of length 1000 slots (slot time 1μs), with \( T_{pay} \approx 900 \text{ slots/sec} \) we have a Throughput of \( \Sigma = RT_{pay} = 18 \text{ Mb/s} \) (with a bandwidth of 1 MHz) giving an aggregate throughput of 108 Mb/s for \( t = 6 \) and 242 Mb/s for \( t = 8 \). Obviously \( t = 8 \) and \( t = 6 \) present interesting results but, in practice, it is reasonable to believe that \( t = 2 \) by asking for more the \( r = 8 \) antennas at base station. As example, with \( t = 2 \) and \( r = 8 \), QoS level of 10Mb/s, we have an aggregate throughput of 140 Mb/s with a single user rate of 11.7Mb/s. Last, in Fig.5 we detail the system behavior similar to Fig.3, while we consider (according to the approach pursued in [22]) the possibility for the user to “contract” a QoS level that is the 90% of the required rate. In this case it is possible to appreciate how the number of users accessing the medium is higher than that of the corresponding QoS level and the gain decreases when \( r \) increases since the rate increases and cancellation is performed in a more reliable way so, from a rate region point of view, we are approaching better the corresponding \( \Sigma \) region.

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