Interference management for multiple multicasts with joint distributed source/channel/network coding

Nicola Cordeschi*, Valentina Polli, Enzo Baccarelli

Abstract—This paper focuses on the QoS-constrained jointly optimal adaptive distributed source coding, channel coding, network coding and power control for Co-Channel Interference (CCI)-limited wireless multiple class multicast networks, such as, for example, Wireless Sensor Networks (WSNs). The goal is to allocate the available system-wide resources by jointly performing Loss-Less Distributed Source Coding (LLDSC) and Intra-Session Network Coding (ISNC), while leveraging channel coding and power control for CCI-mitigation. Due to the presence of CCI, the resulting cross-layer optimization problem is inherently nonconvex. Hence, we develop a distributed, iterative and asynchronous algorithm for the optimal adaptive QoS management of the available bandwidth/power/flow resources. Actual performance and adaptive capability of the proposed resource management algorithm in the presence of: i) abrupt changes of the statistics of the source flows; ii) failures of the interior network nodes; and, iii) fast fading, are numerically tested.

Index Terms—Distributed systems, Wireless Sensor Networks (WSNs), Adaptive QoS resource management, CCI-affected multiple class multicast, Loss-Less Distributed Source Coding (LLDSC), Intra-Session Network Coding (ISNC).

I. INTRODUCTION

Joint distributed source coding and intra-session network coding allow optimal capitalization of the available system-wide resources by extending the functionality of the source and interior network nodes from generating/storing/forwarding of packets to performing of arbitrary algebraic operations on the source streams transported by the network [1]. By referring to the Internet-based wireless Differentiated Service (DiffServ) networking architecture for the QoS support of (possibly, heterogeneous) multiple multicast sessions, this contribution focuses on the joint optimization of LLDSC, channel coding, ISNC and power control, when the transport network is affected by (possibly, time-varying) CCI, so that the resulting cross-layer optimization problem is nonconvex. Overall, the main contributions of this paper may be so summarized: i) we develop a cross-layer distributed framework for the QoS joint optimization of the source rates at the Application/Transport layers, when multiple constraints induced by the ISNC at the Network layer, the CCI control at the MAC layer, the power control and channel coding at the Physical layer are simultaneously present. The resulting problem constitutes the so called Multisource Multicast Primary Optimization Problem (MMPOP); ii) we present a loss-less two-level decomposition of the MMPOP into two sub-problems, namely, the Source-Network Coding Problem (SNCP) and the Efficient Resource Allocation Problem (ERAP). Coupling between these sub-problems is provided by the corresponding (generally, nonconvex) Multisource Multicast Capacity Region (MMCR). Afterwards, we provide a set of formal conditions that guarantee that, by solving the SNCP on a convex outer bound of the actual MMCR, we obtain the exact solution of the (nonconvex) MMPOP; iii) finally, we develop a distributed, scalable and adaptive algorithm for the implementation of the proposed optimal resource management policy, that requires limited signaling among adjacent network nodes.

A. Related Work

The (aforementioned) two-level decomposition of the MMPOP leverages some formal results recently proved in [6] about the convex bounding of the capacity region of CCI-affected networks.

Regarding related contributions more focused on the LLDSC, [1] shows that, under suitable feasibility conditions, random linear network coding suffices to attain both the networking capacity (as dictated by the max-flow min-cut bound of [2]) and the Slepian-Wolf source-rate region of [9]. Being triggered by this optimality result, some (more recent) contributions focus on the practical design of joint distributed source and network encoders for the loss-less transport of data over multi-terminal networks [10], [14]. The related problem of the optimal resource management in multicast networks is the focus of [16] and [19]. Specifically, by exploiting the CCI-free nature of the considered multicast scenarios, [16] and [19] develop distributed resource allocation algorithms that optimize routing and coding by allowing the sinks to adjust the corresponding source rates. However, the application scenarios considered in [16] and [19] refer to fading and CCI-free networks. More recently, by referring to lossy data compression in networks with correlated sources and assigned link capacities, [20] develops an adaptive resource allocation algorithm that maximizes an aggregate utility function which is defined in terms of distortion levels. Interestingly, the algorithm in [20] does not require coordination among the involved source nodes, and assumes CCI-free single class multicasting with fixed (i.e., known in advance) link capacities, so that the tackled problem is convex by design.

The rest of this paper is organized as follows. After the network modeling of Section II, Section III introduces the afforded MMPOP and Section IV deals with its (closed-form) solution. Section V presents a general procedure for the analytical characterization of tight convex outer bounds of the (generally nonconvex) capacity region of the underlying MMPOP. Afterwards, Section VI presents an iterative algorithm for the distributed and adaptive implementation.

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of the MMPO solution of Section IV in randomly time-variant networking scenarios. Numerical performance tests and performance comparisons are the focus of Section VII, while some final remarks are pointed out in the conclusive Section VIII.

Passing to consider the adopted notation, $A \triangleq [a(v, l), v = 1, \ldots, V; l = 1, \ldots, L]$ indicates a $(V \times L)$ matrix with the $(v, l)$-th entry equal to $a(v, l)$, whereas $b \triangleq [b_1, \ldots, b_L]^T$ represents a $(L \times 1)$ column vector. Furthermore, $\overline{r}_l \in \mathbb{R}^V$ is the $l$-th unit column vector of $\mathbb{R}^V$, while $\overline{O}_F$ and $\mathbb{1}_L$ are the $(L \times 1)$ vectors with all zero and all unit entries, respectively. $\log(\cdot)$ is the natural logarithm, $f^{-1}(y)$ is the inverse of the scalar function $y \triangleq f(\cdot)$ and $A \triangleq |A|$ is the cardinality of set $A$. Finally, $H(X|Y)$ is the entropy (in bit/s) of the r.v. $X$ conditioned to the value of the r.v. $Y$, and $\delta(k, l)$ is the Kronecker’s delta.

II. MULTIPLE CLASS MULTICAST OVER CCI-AFFECTED NETWORKS

Due to the composition of individual transmission links, the topology of fading and CCI-affected (possibly, mobile) wireless networks may be time-varying, such as, for example, WSNs [4, Chaps.15,18]. In this section, we consider static networking scenarios, while, in Section VI, we focus on time-varying networks. The considered network is described by a directed graph $G \triangleq (V, L)$, where $V$ is the set of nodes and $L$ is the set of the feasible links, with cardinality $V \triangleq |V|$ and $L \triangleq |L|$, respectively. Spatially correlated source nodes generate discrete-time quantized sequences of independent (in the time domain) identically distributed (i.i.d) symbols. In agreement with the LLSDC paradigm [8], source nodes encode their flows without resorting to inter-source message passing. In the considered network, a directed point-to-point link $l$ from the transmit node $t(l)$ to the receive node $r(l)$ is feasible when the gain $g_{t}(l), r(l)$ of the corresponding physical channel is positive. Let $A \triangleq [a(v, l)]$ be the $(V \times L)$ node-link incidence matrix [3, p.683] that describes the feasible topology of the network graph $G$, and let $A_{s, t} \triangleq [a_s(v, l)] \triangleq \max \{A, O_v x_l\}$ be the corresponding multicast source matrix. In our framework, $F \geq 1$ variable-rate multicast sessions, each one identified by the corresponding source-set/flow/sink-set triplet: $(S_i \subseteq V, f_i \in \mathbb{R}_+^V, D_i \subseteq V)$, $i = 1, \ldots, F$, diseminate their flows over multiple network paths. Therefore, $F$ indicates the number of the active multicast sessions, $D_i$ and $S_i$ are the sink and source sets of the $i$-th session, while $D \triangleq \bigcup_{i=1}^{F} D_i$ and $S \triangleq \bigcup_{i=1}^{F} S_i$ denote the overall multicast sink and source sets, respectively. Since a multi-flow source node may be always modeled as multiple colocated (possibly, correlated) single-flow source nodes, in the following, we directly assume the presence of (possibly correlated) single-flow source nodes, so that the sets: $\{S_i, i = 1, \ldots, F\}$ do not overlap. However, each sink node may still receive multiple flows, that may also belong to different sessions. Hence, in our framework, the sink-sets $\{D_i, i = 1, \ldots, F\}$ may overlap and statistical dependence among all source flows is allowed. Then, as in [1], the flow $f(s)$ (bit/s) generated by the source node $s \in S_i$ gives rise to an $(L \times 1)$ link-flow vector $x^s \in (\mathbb{R}_+^V)^L$, $s \in S_i$, whose $l$-th entry, $x^s_l$, indicates the portion of $f(s)$ carried by the $l$-th link. Hence, the $l$-th link conveys a total flow of $x^s_l = \sum_{s \in S_i} x^s_l$. According to this notation, the overall flow $f_i$ of the $i$-th session and the corresponding link-flow vector $\overline{x}_i$ can be expressed as: $f_i \triangleq \sum_{s \in S_i} f(s)$ and $\overline{x}_i \triangleq \sum_{s \in S_i} x^s$, respectively. Furthermore, as in [7], INSC is considered as a valuable means to improve network efficiency, so that the following relationship holds: $x^s_l = max_{s=1,\ldots,L}(\{x^s_l\})$, $s \in S_i$, where $x^s_l$ is the part of $x^s$, $s \in S_i$, intended for the $j$-th sink node $d^j \in D_i, j = 1, \ldots, D_i$, of session $i$. In agreement with the DiffServ paradigm, we also assume that each session belongs to a different service class and each service class demands for specific QoS guarantees and priority levels. Hence, without loss of generality, we label the active multicast sessions with increasing integer-valued IDentity numbers (IDs), that correspond to not increasing priority levels. Since the flow of the $i$-th session is served at each interior node in agreement with its priority level, the corresponding per-link delay function: $\Delta_i(C, x_i, \ldots, x_F)$ adopted to measure the average queue-plus-transmission delay may jointly depend on the session-ID $i$, the available link-capacity $C$ and all total traffic flows $\{x_1, \ldots, x_F\}$ actually conveyed by the considered link. Hence, after noting that: $x_i \triangleq \sum_{s \in S_i} \{max_{j=1,\ldots,L}(\{x^s_l\})\}, i = 1, \ldots, F$, as in [12, Chap.5], we assume that the following basic properties are retained by each per-link delay function $\Delta_i(\cdot)$: i) for any assigned set of variables $\{C, x_1, \ldots, x_F\}$, it is not decreasing in the session-ID $i$; ii) for any assigned $i$ and $\{x_1, \ldots, x_F\}$, it is strictly decreasing in $C$; iii) for any assigned $i$ and $C$, it is not decreasing in $\{x_1, \ldots, x_F\}$; and, finally, iv) for any assigned $i$, $\Delta_i(\cdot)$ is jointly convex in the $F + 1$ variables $(C, x_1, \ldots, x_F)$. Besides fading, several topological and network-depending parameters (such as, the cross-correlation coefficients of the utilized access codes, the beamforming coefficients, etc.) may affect the link gains. Hence, we denote by $G \triangleq [g(k, l)]$ the $(L \times L)$ matrix that gathers all the (nonnegative) gains between transmit-receive nodes, i.e., $g(k, l) \triangleq g(l(k), r(l)), k, l = 1, \ldots, L$. Thus, for each link $l \in L$ with transmit power $P(l)$ (measured in Watt (W)), we can express the corresponding SINR($l$) measured at the corresponding receive node $r(l)$ as in

$$\text{SINR}(l) \triangleq \frac{\Gamma(l) g(l, l) P(l)}{\sum_{k=1, k \neq l}^{L} g(k, l) P(k) + N(l)}$$

where $\Gamma(l) > 0$ is the SINR-gap that accounts for the desired target BER, while the denominator in (1) is the receive noise $N(l)$ (W) plus CCI power. The resulting capacity $C(l)$ (bit/s) is modeled as a SINR function $\Psi_l(\text{SINR}(l))$ nonnegative, continuous and strictly increasing for $\text{SINR}(l)$, with $\Psi_l(0) = 0$.

Finally, we map each target $BER^\ast$ value into a corresponding maximum gap value $\Gamma_{\max}^\ast$, so that the following set of gap-constraints: $\Gamma(l) \leq \Gamma_{\max}^\ast, l = 1, \ldots, L$, captures the BER-depending QoS levels to be guaranteed by the Physical layer. Overall, all the mentioned per-link parameters may be gathered into the following $(L \times 1)$ column vectors: $\overline{x}_i^s$ (total link flow vector), $x^s_l, s \in S$ (sub-session flow vector), SINR
(SINR vector), \( \bar{\Gamma} \) (SINR-gap vector), \( \bar{C} \) (capacity vector) and \( \bar{P} \) (power vector). Table I recaps the introduced taxonomy.

III. THE CROSS-LAYER OPTIMIZATION PROBLEM

Let \( \{f_1, \ldots, f_P \} \) (bit/s) be the set of vectors collecting the per-session multicast flows of all source nodes \( \{s \in S \} \). Thus, goal of the MMPOP is to compute the set of variables: \( \{f_1, \ldots, f_P, \bar{x}_s, \ldots, \bar{x}_p, \bar{P}, \bar{\Gamma}, \bar{C} \} \) which minimizes a given system-wide cost function \( \Phi(\cdot) \), i.e.,

\[
\Phi(f_1, \ldots, f_P, \bar{x}_s, \ldots, \bar{x}_p, \bar{P}, \bar{\Gamma}, \bar{C}) \leq 0,
\]

(2)

under the Slepian-Wolf constraint on the feasible source rates \( \{f_1, \ldots, f_P \} \), and three suitable sets of per-session cross-layer constraints. Specifically, the first subset of constraints is formally stated as follows:

\[
A_{x_s}^T f(s) (e_s - e_d) = 0,
\]

(3.1)

\[
x_s^T(l) - Div(i) f(s) \leq 0,
\]

(3.2)

We note that, in addition to the usual flow conservation law in (3.1), \( x_s^T(l) \) has to comply with the constraint in (3.2). Specifically, the \( i \)-th path-diversity factor \( Div(i) \in [0,1] \) in (3.2) controls the minimum number of distinct paths to be employed by the source node \( s \in S \) to each sink \( d_j \in D_i \), and \( Div(i) < 1 \) guarantees that each \( d_j \in D_i \) is connected by multiple different paths to the corresponding source node \( s \in S_i \). Passing to consider the second set of constraints, it is formally defined as follows:

\[
H(X_0|S \setminus X_0) - \sum_{s \in X_0} f(s) \leq 0, \forall X_0 \subseteq S \triangleq \bigcup_{i=1}^{F} S_i,
\]

(4.1)

\[
\sum_{i=1}^{L} \Delta_i \left( C(l), x_1(l), \ldots, x_p(l) \right) - \nabla_x(i) \leq 0,
\]

(4.2)

\[
\bar{x}_i \in \left( \mathbb{R}_{>0}^S \right)^L, \bar{x}_p \in \left( \mathbb{R}_{>0}^S \right)^L.
\]

(4.3)

In this set of constraints, the QoS requirement is dictated by (4.2), that bounds the maximum allowed per-session delay \( \nabla_x(i) \). The (global) Slepian-Wolf constraint in (4.1) on the overall set of the feasible source rates guarantees lossless recovery at the sinks of all (possibly, spatially correlated) source flows [9]. Finally, at the MAC and Physical layers, the following constraints are assumed to operate:

\[
g(l, l) - G_{\text{max}}(l) \leq 0, \quad l = 1, \ldots, L,
\]

(5.1)

\[
- G_{\text{min}}(k, l) + g(k, l) \geq 0, \quad l = 1, \ldots, L, \quad k \neq l,
\]

(5.2)

\[
\sum_{l=1}^{L} a_s(v, l) P(l) - P_{\text{max}}(v) \leq 0, \quad v \notin D,
\]

(5.3)

\[
\Gamma(l) - \Gamma_{\text{max}}(l) \leq 0, \quad l = 1, \ldots, L,
\]

(5.4)

\[
g(k, l), P(l), \Gamma(l) \geq 0, \quad l = 1, \ldots, L.
\]

(5.5)

Specifically, we upper limit in (5.1) the achievable gains of the feasible network links and lower limit in (5.2) the minimum allowed CCI coefficients. The former constraint accounts, for example, for the maximum allowed transmit/receive antenna gains, while positive \( G_{\text{min}} \)'s typically occur in resource limited networks, where the communication resources cannot be allotted to the active links in a fully orthogonal way [7, Chapter 5]. Finally, at the Physical layer, (5.4) and (5.3) enforce a maximum allowed per-link BER and a maximum per-node power budget, respectively. Optimizing the link gains \( \{g(k, l), k \leq l\} \) and the gap coefficients \( \{\Gamma(l)\} \) under per-node power constraints constitutes, indeed, a form of channel coding [11]. Formally speaking, we require that the objective \( \Phi(\cdot) \) function in (2) is a real-valued, jointly convex function of the link capacities \( \bar{C} \), session flows \( \{\bar{f}_i\} \) and total link flows \( \{\bar{x}_i\} \). In practice, the objective function may be used to enforce source-rate control, network operator's goals (e.g., load-balancing and fairness) or their appropriate trade-offs [15]. Just as an example, according to [15], an objective function of practical interest may be the following one:

\[
\Phi(\cdot) = \theta \left[ \sum_{i=1}^{L} x_T(l) D(x_T(l)) \right] - (1 - \theta) \left[ \sum_{i=1}^{F} \sum_{s \in S_i} U_\alpha(f(s)) \right],
\]

(6)

where \( \theta \in [0,1] \) is a tunable weight factor, \( x_T(l) \) is the total traffic flow conveyed by the \( l \)-th link, and [15]

\[
U_\alpha(f) \equiv \begin{cases} \log(f), & \alpha = 1 \\ (1 - \alpha)^{-1} f^{-1}, & 0 < \alpha < 1 \end{cases}
\]

is the so-called \( \alpha \)-fair utility function adopted to measure the utility acquired by the \( i \)-th session.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning/Role</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S )</td>
<td>set of source nodes with ( s \in (1, \ldots, F) ) and ( f \in \left[ 0, \bar{f} \right] )</td>
</tr>
<tr>
<td>( D )</td>
<td>set of sink nodes, with ( d_j \in (1, \ldots, L) ) and ( d \in \left[ 0, \bar{d} \right] )</td>
</tr>
<tr>
<td>( \bar{f} )</td>
<td>flow-rate of node ( s ), ( \bar{d} )</td>
</tr>
<tr>
<td>( \bar{r}_j(l) )</td>
<td>portion of ( \bar{x}_j(l) ), conveyed by the ( l )-th link, and intended for the sink node ( d_j \in D_i )</td>
</tr>
<tr>
<td>( \bar{r}_j(l) )</td>
<td>portion of ( \bar{x}_j(l) ), conveyed by the ( l )-th link, i.e., ( \bar{x}_j(l) \leq \bar{r}_j(l) )</td>
</tr>
<tr>
<td>( f )</td>
<td>traffic flow of source ( i ), i.e., ( f = \sum_{s \in S_i} f(s) )</td>
</tr>
<tr>
<td>( \bar{r}(l) )</td>
<td>flow of source ( i ) conveyed by link ( l ), i.e., ( \bar{x}<em>i(l) = \sum</em>{s \in S_i} f(s) )</td>
</tr>
<tr>
<td>( \bar{r}(l) )</td>
<td>total traffic flow conveyed by link ( l ), i.e., ( \bar{r}(l) = \sum_{i=1}^{L} \bar{r}_j(l) )</td>
</tr>
<tr>
<td>( \bar{P} )</td>
<td>(( F \times L )) matrix of the link gains</td>
</tr>
<tr>
<td>( \bar{P} )</td>
<td>( \bar{P} ) (( F \times L )) node-power vector</td>
</tr>
<tr>
<td>( \bar{P} )</td>
<td>( \bar{P} ) (( F \times 1 )) link-capacity vector</td>
</tr>
<tr>
<td>( \bar{C} )</td>
<td>actual ( L )-dimensional capacity region</td>
</tr>
<tr>
<td>( \bar{U} )</td>
<td>feasible ( L )-dimensional vector bound on ( \bar{C} )</td>
</tr>
</tbody>
</table>

IV. THE SOLVING APPROACH

The stated MMPOP is generally a nonconvex cross-layer optimization problem, due to the nonconvexity of the SINR-induced relationship in (1) between powers and link capacities. As a consequence, no analytical expressions are, up to date, available for computing the optimal solution \( \{f_1^*, \ldots, f_P^*, \bar{x}_1^*, \ldots, \bar{x}_p^*, \bar{P}^*, \bar{\Gamma}, \bar{C}^* \} \) of the nonconvex MMPOP (see [20] and references therein). However, by leveraging the coupling role played by the link-capacities, we (shortly) present an optimal two-level solving approach, where: i) the upper-level tackles the Source-Network Coding Problem (SNCP), and generates the optimal link capacity vector \( \bar{C}^* \); and, ii) the lower-level solves the corresponding Efficient Resource Allocation Problem (ERAP), that aims at supporting the requested link capacity target vector \( \bar{C} \). Specifically, let \( \Pi \triangleq \{ (\bar{P}, \bar{\Gamma}, \bar{G}) : (5.1)-(5.5) \) are simultaneously met \} be the region of the \( (L^2 + 2L) \)-dimensional Euclidean space.
comprising all the triplets $\{\vec{P}, \vec{r}, G\}$ that meet the MAC and
Physical layers’ constraints. Furthermore, let $\bar{S} \triangleq \{\text{SINR} \triangleq \text{SINR}(1), \ldots, \text{SINR}(L)\}$ be the resulting $L$-dimensional set of feasible SINR vectors. Hence, the resulting Multicast Multisource Capacity Region (MMCR) is formally defined as $C \triangleq \{\bar{C} \in \mathbb{R}_+^L : \exists \text{SINR} \in \bar{S} : C(l) \leq \Psi_l(\text{SINR}(l)), \forall l\}$, so that the corresponding SNCP is the following optimization problem:

$$\begin{align*}
\min_{\vec{f}, \vec{r}, \bar{x}, \bar{C}} & \quad \Phi(\vec{f}, \vec{r}, \bar{x}, \bar{C}) \\
\text{s.t.:} & \quad \text{MMPOP constraints in (3.1)-(4.3)}, \\
& \quad \bar{C} \in C,
\end{align*}$$

(8.1)

while the ERAP is, in turn, defined as in

$$\begin{align*}
\min_{\vec{P}, G, \vec{r}} & \quad \varphi(\vec{P}, G), \\
\text{s.t.:} & \quad \text{MMPOP constraints in (5.1)-(5.5)},
\end{align*}$$

(9.1)

$$\text{SINR}^*(l)/\text{SINR}(l) - 1 \leq 0, \quad l = 1, \ldots, L.$$  

(9.3)

In (9.3), $\text{SINR}^*(l) \triangleq \Psi_l^{-1}(C^*(l))$, and $C^*(l)$ is the target capacity value of link $l$, that is obtained by solving the SNCP in (8). Furthermore, since the objective function $\Phi(\cdot)$ in (2) depends only implicitly on the MAC/Physical resources, that is, through the link-capacity vector $\bar{C}$, it may happen that multiple $\vec{P}^*$’s and $G^*$’s matrices lead to the same optimal capacity vector $\bar{C}^*$. Hence, the task of the objective function $\varphi(\vec{P}, G)$ in (9.1) is to enforce efficient resource allocation by picking up the most resource efficient solution over the set $\{\vec{P}^*, G^*\}$ of the (possible, multiple) ones. So, the resulting ERAP retains the basic structural properties detailed in the following Proposition 1 (see [6] for the proof).

**Proposition 1.** When $\varphi(\vec{P}, G)$ in (9.1) is posynomial [17, p.712] in $\{\vec{P}, G\}$, the ERAP becomes an instance of geometric programming, and, therefore, it is solvable by convex optimization.

As formally proved by the following Proposition 2, the SNCP and ERAP are coupled problems, and their interaction is ruled out by the optimal capacity vector $\bar{C}^*$ and the corresponding MMCR $C$ (see [6] for the proof).

**Proposition 2.** Let $C$ be the aforementioned MMCR. Then, the MMPOP in (2)-(5) admits the same solution of the cascade of the SNC-plus-ERA problems in (8) and (9).

The proposed two-level decomposition retains the following formal properties, that may allow the computation of the exact solution of a nonconvex MMPOP by solving the corresponding convex SNC-plus-ERA problem (see [6] for the proof).

**Proposition 3.** Let us consider a convex outer-bound $C_0$ of the MMCR $C$, i.e., $\{C \subseteq C_0\}$, and let $\bar{C}$ be the link-capacity vector obtained by solving the $C_0$-relaxed SNCP\(^1\). Then, the following properties hold:

\(^1\)The $C_0$-relaxed SNCP is still defined by (8.1)-(8.3), but with the region $C$ in (8.3) replaced by the considered convex outer bound $C_0$.

1. when the $C_0$-relaxed SNCP is infeasible, then the MMPOP is also infeasible;
2. when the $C_0$-relaxed SNCP is feasible and the ERAP is infeasible (i.e., $\bar{C} \notin C$), then no conclusion can be drawn about the feasibility/infeasibility of the MMPOP;
3. when the $C_0$-relaxed SNCP and the ERAP are both feasible (i.e., $\bar{C} \in C$), then the MMPOP is feasible and its link-capacity solution $\bar{C}^*$ coincides with $\bar{C}$.

As explicative comments, we point out that, since the actual (possibly, nonconvex) MMCR $C$ of the MMPOP is present in (8.3), the cascade of the SNC-plus-ERA problem is equivalent to the MMPOP (see Proposition 2). However, when $C$ in (8.3) is replaced by a convex outer bound $C_0$, the $C_0$-relaxed SNC-plus-ERA problem is equivalent to the MMPOP when Case 1 or Case 3 of Proposition 3 occur, while the equivalence is no longer guaranteed when Case 2 of Proposition 3 happens.

Lastly, we note that, from an application point of view, the $C_0$-relaxed SNCP meets Case 3 of Proposition 3 when it admits (at least one) capacity vector solution $\bar{C}^*$ that falls into the considered outer bound $C_0$.

**V. Characterization of convex outer bounds**

In principle, multiple convex outer bounds $C_0$’s of an (assigned) $C$ may be devised, so that we are interested in looking for the simplest-to-characterize convex outer bound that is capable to guarantee the occurrence of Case 3 of Proposition 3. This goal may be attained by resorting to the polyhedral outer bound $C_0$ of the actual capacity region $C$ that is derived in the Appendix A of [21]. This outer bound is analytically described by the following set of $L^2$ linear inequalities:

$$0 \leq C(k) \leq C_M(k), \quad k = 1, \ldots, L,$$

(10.1)

$$C(k) - \zeta_{kl}C(l) - \xi_{kl} \leq 0, \quad k \neq l, \quad k, l = 1, \ldots, L,$$

(10.2)

where the following dummy positions hold (see the Appendix A of [21]):

$$C_M(k) \triangleq \Psi_k(\Gamma_{\max}(k)G_{\max}(k)P_{\max}(t(k))/N(k)), \quad k \neq l,$$

(11.1)

$$\zeta_{kl} \triangleq \frac{(C_M(k) - \Psi_{k,l}(C_M(l)))}{\Psi_{k,l}(C_M(k) - C_M(l))}, \quad k \neq l.$$  

(11.2)

$$\xi_{kl} \triangleq \frac{\Psi_{k,l}(C_M(l) - C_M(l))}{\zeta_{kl}}, \quad k \neq l.$$  

(11.3)

Specifically, $C_M(k)$ in (11.1) is the maximum capacity allowed the $k$-th link, while $\Psi_{k,l}(C_M(l))$ and $\Psi_{k,l}^{-1}(C_M(k))$ are the values of the function $\Psi_{k,l}(x)$ and its inverse, respectively, evaluated at $x \equiv C_M(l)$ and $y \equiv C_M(k)$, respectively. In principle, the function $\Psi_{k,l}(\cdot)$ may be any convex upper bound of the composite function: $\Psi_k(\Psi_l^{-1}(x))$. Interestingly, when all $G_{\min}$’s vanish, we have (see (A.4) of [21] and the related text): $\Psi_{k,l}(C_M(l)) = C_M(k), \quad k \neq l$, so that (see (11.2)): $\xi_{kl} = 0$, and, then, the set of constraints in (10.2) reduces to the ones in (10.1). As a consequence, for vanishing $G_{\min}$’s, the resulting outer bound $C_0$ becomes

$$C_0 = \{\bar{C} : 0 \leq C(k) \leq C_M(k), \quad k = 1, \ldots, L\},$$

(12)
that coincides (by definition) with the actual capacity region $C$ for the CCI-free case.

VI. THE OPTIMAL RESOURCE MANAGEMENT ALGORITHM

In this Section VI, we develop an adaptive (e.g., self-configuring) and distributed algorithmic implementation of the proposed solution of the SNC-plus-ERA problem.

A. Adaptive implementation of the solution of the relaxed SNCP

Being a convex optimization problem meeting the Slater’s conditions [17, Chapter 5], the $C_0$-relaxed SNCP may be solved through an application of the corresponding Karush-Kuhn-Tucker (KKT) optimality conditions [17, Chapter 4]. For (scalar) gradients in (14) are detailed in the final Appendix. For (suitable) set of per-node scalar gradient-based updatings.

The feasible point that satisfies the following algebraic (vector) analytical form:

$$
L_i(f_i, x_i) = \Phi_i + \sum_{i=1}^{L_i} \sum_{j=1}^{D_i} \sum_{l=1}^{V} \mu_{ijl} \times
$$

$$
+ \sum_{l=1}^{F} \sum_{j=1}^{D_i} \sum_{l=1}^{V} \mu_{2ijl} (x_i^T - f(s)D_i(v))
$$

$$
+ \sum_{x_0 \in S} H(x_0) - \sum_{s \in \mathcal{S}} f(s)
$$

$$
+ \sum_{i=1}^{L_i} \left( \sum_{l=1}^{V} \mu_{3i} (C(l) - \nabla^2_l) + \sum_{l=1}^{V} \mu_{4i} (C(l) - \xi_{lk} \delta(l) - \xi_{lk})
$$

where $\mu_i \triangleq [\mu_{i1}, \ldots, \mu_{iF}]^T$ is the (column) vector of all Lagrangian multipliers of the constraints in (8.2) and (10). Since strong duality and Lagrangian min-max equality hold [17, Chapter 6], the solution of the constrained $C_0$-relaxed SNCP is the feasible point that satisfies the following algebraic (vector) equation: $L_i(f_i, x_i) = 0$. In order to be capable to effectively cope with the (possibly, abrupt and unpredictable) time-fluctuations of the source entropies in (4.1), this solution can be iteratively computed through a (suitable) set of per-node scalar gradient-based updatings. Specifically, the $k$-th scalar iterate assumes the following analytical form:

$$
\eta(k+1) = \eta(k) - \omega(k) \nabla L_i(f_i, x_i) + \sum_{l=1}^{V} \mu_{3il} (C(l) - \xi_{lk} \delta(l) - \xi_{lk})
$$

where $\omega(k) \nabla L_i(f_i, x_i) = \sum_{i=1}^{L_i} \sum_{j=1}^{D_i} \sum_{l=1}^{V} \mu_{ijl} \times

$$
B. Adaptive implementation of the solution of the ERAP

Let $\widetilde{C} = [\widetilde{C}(1) \ldots \widetilde{C}(L)]^T$ be the solution of the $C_0$-relaxed SNCP that, under Case 3 of Proposition 3, coincides with the optimal one, $\tilde{C}^*$, of the MMPOP. The ERAP in (9) may be recast in a convex form through the following log-transformation of the involved variables: $z_l \triangleq \log(P(l))$, $y_l \triangleq \log(G(l))$, $W(k, l) \triangleq [w_{k,l} \triangleq \log(k, l)]$, and, then, by leveraging the posynomial structure of the objective function in (9.1). So doing, the corresponding Lagrangian function of the ERAP in (9), when $\varphi(\widetilde{P}, \widetilde{G}) = \sum_{l=1}^{L} P(l)$, reads as in:

$$
\sum_{l=1}^{L} \lambda_{1l} (\widetilde{SINR}(l) - \sum_{i=1}^{F} \sum_{l=1}^{V} \mu_{2i} (x_i^T - f(s)D_i(v)))
$$

$$
+ \sum_{x_0 \in S} H(x_0) - \sum_{s \in \mathcal{S}} f(s)
$$

$$
+ \sum_{i=1}^{L_i} \left( \sum_{l=1}^{V} \mu_{3i} (C(l) - \xi_{lk} \delta(l) - \xi_{lk})
$$

$$
\sum_{i=1}^{L_i} \sum_{j=1}^{D_i} \sum_{l=1}^{V} \mu_{2ijl} (x_i^T - f(s)D_i(v))
$$

$$
+ \sum_{x_0 \in S} \left( H(x_0) - \sum_{s \in \mathcal{S}} f(s) \right)
$$

$$
+ \sum_{i=1}^{L_i} \left( \sum_{l=1}^{V} \mu_{3i} (C(l) - \xi_{lk} \delta(l) - \xi_{lk})
$$

$$
+ \sum_{i=1}^{L_i} \left( \sum_{l=1}^{V} \mu_{3i} (C(l) - \xi_{lk} \delta(l) - \xi_{lk})
$$

In the above expressions, the scalar: $I_l \triangleq \sum_{k=1}^{L_i} \sum_{l=1}^{V} e^{x_k + w_{kl}} + N(l)$ plays the role of aggregate CCI-plus-noise power af-
fecting the \(l\)-th link and it may be directly measured by the corresponding receive node \(r(l)\), while the scalar
\[
D_l = \lambda_l \Psi_1^{-1}(C(l)) e^{-z_l - y_l - w_{l1}},
\]
is the corresponding (real-valued) SINR target value.

C. Time-varying networks and adaptive resource allocation

In wireless networks, link gains and/or inter-source correlations may experience unpredictable time-variations over a communication session, mainly due to node mobility, fading, node failure and traffic congestion [4, Chap.18]. In our framework, these fluctuations are reflected into fluctuations of the \(G_{\text{max}}\)’s in (5.1) and the conditional entropies at the l.h.s. of (4.1). In order to model these time-variations, in the sequel, we assume that: i) the time-axis is partitioned into time-slots of assigned duration \(T_o\) (s); ii) the inter-source correlations, link gains and network topology do not vary over each time-slot, but they may vary on a per-slot basis in a (possibly) fully unpredictable way; and, iii) the iteration index \(k\) in (14)-(16) runs faster than the slot-time \(T_o\), and \(T_f\) (s) (with \(T_f < T_o\)) is the duration of each \(k\)-indexed iteration (i.e., \(T_f = (T_o/200)\) in the simulated settings of Section VII). Hence, an effective means for tracking unpredictable network fluctuations in a distributed and adaptive way is provided by the gradient-descendent algorithm developed in [18] for the adaptive updating of (scalar) stepsize sequences. In our framework, the updating algorithm for \(a_{u(k)}^{(l)}\) reads as [18, Equation (2.4)]:
\[
a_{u(k+1)}^{(l)} = \max \left\{ 0; \min \left\{ a_{\text{max}}; a_{u(k)} - \alpha \nabla_u L_v^{(k)} V_u^{(k)} \right\} \right\},
\]
where \(a_{\text{max}}\) and \(\alpha\) are positive scalar constants, while the scalar \(V_u^{(k)}\) is iteratively updated as in [18, Equation (2.5)]:
\[
V_u^{(k+1)} = \left( 1 - a_{u(k)} \right) V_u^{(k)} - \nabla_u L_v^{(k)}.
\]
After replacing \(a_{u(k)}^{(l)}\) and \(\nabla_u L_v^{(k)}\) by \(\omega_{l_i}^{(l)}\) and \(\nabla_l L_v^{(l)}\) respectively, we obtain the corresponding expressions for the updating of \(\{\omega_{l_i}^{(l)}\}\) in (14).

D. Implementation aspects and complexity issues

Practical implementation of Slepian-Wolf (SW) source encoders requires that the set of the entropies in (4.1) are estimated at the sink nodes and communicated back to the source nodes during the set-up phase of the encoding process (see, for example, [8, Chapter 6], [14] and [20] for details on the practical implementation of SW encoders).

Regarding the signalling overhead of the presented resource allocation algorithm, we note that the distributed implementation in (16), (17) of the solution of the ERAP requires inter-node message-passing. However, since \(I_t\) in (17.1)-(17.3) may be directly measured by the corresponding receive node \(r(l)\), only \(D_l\) in (18) is the link information to be flooded by the receive node \(r(l)\) to its interfering nodes, i.e., the transmit nodes \(\{t(k)\}\) for which \(g(k,l) > 0\), \(k \neq l\). Hence, in the worst case, the resulting network-wide aggregate signalling overhead scales as the product \(O(LN_{\text{max}}^t)\), where \(L\) is the number of network links, and \(N_{\text{max}}^t\) is the maximum number of transmit nodes that induce CCI at a single receive node.

Formally speaking, the signalling overhead sustained by link \(l\) may be accounted for by replacing the \(l\)-th capacity, \(C(l)\), by:
\[
(1 - y(l))C(l)
\]
in the (previously reported) MMPOP’s formulation, where \(y(l) \in [0,1]\) is the (dimension-less) fraction of \(C(l)\) consumed for the signaling task. Regarding the evaluation of \(y(l)\), let \(b \geq 1\) and \(T\) be the (integer-valued) number of bits of the binary representation of each SINR value \(D_l\) in (18) and the average number of iterations (in multiple of the previously defined iteration interval \(T_f\)) requested by (14), (16) for the convergence to the steady-state. Therefore, under the (conservative) assumption that each node runs the iterations in (14), (16) at the beginning of each time-slot (e.g., at rate \((1/T_0)\) (s\(^{-1}\))), the product: \(y(l)C(l)\) equates: \(bT N_{\text{max}}^t / T_o\) (bit/s), that, by definition, is the (absolute) value of the per-link average signaling flow.

In order to account for the effects of the (possible) asynchronous implementation of the per-node iterations in (14), (16), in the carried out simulations of Section VII, the transmission delay affecting the signaling flow conveyed by each simulated link has been modeled as a random variable evenly distributed over the interval \([0,10]\), where the limit 10 is the maximum simulated per-link signaling delay in multiple of the (aforementioned) iteration-time \(T_f\). So doing, we have numerically ascertained that the per-link delay-jitters affect the behavior of the proposed resource management algorithm in the transient-state by stretching the resulting average convergence interval of the iterations in (14), (16) of about 4% over the perfectly synchronous case (see Figs.3.4).

VII. NUMERICAL TESTS AND PERFORMANCE COMPARISONS

In the carried out numerical tests, we adopt the (usual) Shannon-Hartley’s logarithmic formula:
\[
C(l) = B \log_2(1 + \text{SINR}(l))
\]
for measuring the capacity of the \(l\)-th link, with bandwidth \(B = 1\) (MHz). The corresponding polyhedral outer bound of (10) on the resulting capacity region is implemented in all the carried out numerical tests. Queueing delay has been analytically modelled as in [12, Section 3]. Furthermore, unless otherwise stated, the following link-power summation: \(\sum P(l)\) is used as cost-function in (9.1). Table II reports the simulation parameters. Interfering links are considered to be those sharing a common receive node and the \(G_{\text{min}}\) entry of Table II indicates their minimum value. A layered Benchmark Scheme (BS) has been also implemented and its performance has been compared with the one of the proposed algorithm of Section VI. Specifically, the considered BS implements a system, where: i) the minimum-cost multicasting algorithm with LLDSC and ISNC of [16, Section IV] works at the Network layer; ii) CSMA/CA with RTS/CTS-based link reservation is implemented at the MAC layer; and, iii) no power control is performed at the Physical layer (i.e., all the active transmit nodes operate at \(P_{\text{max}}\); see (5.3)). Specifically, according to...

<p>| TABLE II |</p>
<table>
<thead>
<tr>
<th>MAI SIMULATION PARAMETERS</th>
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<tbody>
<tr>
<td>(N = 0.01\text{mW})</td>
</tr>
<tr>
<td>(G_{\text{min}} = 10^{-2})</td>
</tr>
<tr>
<td>(H(S_1) = H(S_2) = 2\text{Mb/s})</td>
</tr>
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the IEEE802.11a standard [3, Section 8.6], [7, Section 11.1],
the implemented RTS/CTS-aided CSMA/CA scheme operates
at a slot time of 9 (μs), with SIFS/DIFS intervals set to 16
(μs) and 25 (μs) respectively.

Let us consider the butterfly-shaped network topology of
Fig.1, where two sessions belonging to the same QoS class
are active, and let us analyze the power consumption of
the proposed and the benchmark schemes when the conditional
entropy $H(S_1|S_2)$ varies over the range $[0, H(S_1)]$ and CCI
is present (i.e., $G_{min} = 0.01$). The plots of Fig.2 show
that the proposed scheme guarantees better performance, even
when the sources are statistically independent. Specifically, the
power gain of the proposed scheme over the benchmark one
passes from 66% at $H(S_1|S_2) = 0$ to 80% at $H(S_1|S_1) =
H(S_1)$ (see Fig.3).

![Fig. 1. Butterfly network with two active multicast sessions that share
the same QoS requirements and destination sets.](image)

![Fig. 2. Steady-state total power consumption of the proposed and benchmark
schemes for several values of $H(S_1|S_2)$.](image)

Convergence behavior and resilience capability to node-
failures of the proposed scheme may be appreciated through an
examination of the time-plots of Fig.3. They report the
time-evolutions of the total flow received by the sink $d_2$ of Fig.1
at $H(S_1|S_1) = H(S_1)$ and $H(S_1|S_2) = 0$, under the per-
link delay-jitter phenomena previously described in Section
VI-D. Good convergence to the optimal MMPOP solutions
(marked by the horizontal lines in Fig.3) is achieved within
80 – 90 iteration cycles. Quick reactivity with respect to the
node failure (which occurs at $k = 200$) is supported by the
fact that the optimum is approached within 20 iterations (see
the plots of Fig.3 for $k > 200$).

Finally, the solid plot of Fig.4 supports the conclusion that
the proposed algorithm of Section VI is also capable to quickly
react to abrupt changes of the inter-source correlation and track
fading-induced fluctuation of the link gains. Specifically, this
plot reports the time-behavior of the flow at the sink node

$$d_1$$ of Fig.1, when, at $k = 200$, $H(S_1|S_2)$ drops from $H(S_1)$
to zero and, in addition, the maximum gains of the links 8, 14 and 15 of Fig.1 begin to change (at a rate of 10% per
slot) from the initial values of Table II to: $G_{max}(8) = 1.2,$
$G_{max}(14) = 0.8$ and $G_{max}(15) = 0.5$, respectively. The
dotted plot of Fig.4 reports the corresponding flow behavior
when the resource allocation remains statically set to the one
computed at $k < 200$. The behavior of the solid plot of Fig.4
confirms that the proposed adaptive resource management
algorithm is capable to successfully counterbalance all the
introduced time-variations.

![Fig. 4. Flow time-evolution at $d_2$ in Fig.1 for the application scenario of
Section VII in the presence of time-variations.](image)

VIII. CONCLUSION

In this contribution, we presented a distributed, adaptive and
asynchronous resource management algorithm for the joint
lossless Source, Channel and Intra-Session Network Coding
in power-limited CCI-affected multiple multicast wireless
networks. The carried out performance tests confirm that the
optimal joint allocation of source, network and transmission
resources may lead to noticeable power and bandwidth sav-
ings, especially when the source streams are correlated and the
QoS requirements on the allowed end-to-end network delays
are strict. This conclusion is also confirmed by the additional
numerical tests reported in [21].

APPENDIX

Let $\Phi(.)$ be given in (6), and let $\Delta_i^{(c)}(.)$, $\Delta_i^{(sa)}(.)$, $n \leq i$, indi-
cate the partial derivatives of the $i$-th delay function
$\Delta_i(\cdot)$ in (4.2) performed with respect to $C$ and $x_n$, $n = 1, \ldots, F$, respectively. Since the non-differentiability of the maximum function may affect the differentiability of $\Phi_i(\cdot)$ in (6) and that of the SNCP’s constraints in (8.2), as in [11, Equations (6),(7)], we resort to the following upper bound:

$$x^*(\Delta) \triangleq \max_{j=1, \ldots, D_i} \{ x_i^*(\Delta) \} \subseteq \left[ \sum_{j=1}^{D_i} \left( x_i^*(\Delta) \right)^m \right]^{1/m} \text{,}$$

that converges to the actual maximum for large $m$. So doing, the following expressions for the scalar gradients of the Lagrangian function in (13) with respect to the involved primal variables can be derived:

$$\nabla f(x) \tilde{L} = (\theta-1)(f(s))^{-\alpha} + \sum_{j=1}^{D_i} \left( \sum_{v=1}^{V} \mu_{1isjv} (\delta(v, d_j) - \delta(v, s)) \right) - \sum_{l=1}^{L} \mu_{2isjl} D_{iv}(i) \right) - \mu_{3x_0} I_{[s \in X_0]} \right) \right), (A.1)$$

$$\nabla x_i^*(\Delta) \tilde{L} = \mu_{2isjl} D_{iv}(i) \right) - \mu_{3x_0} I_{[s \in X_0]} \right) \right) \right), \left( A.3 \right)$$

where $I_{[s \in X_0]}$ in (A.1) is unit for $s \in X_0$, while it vanishes for $s \notin X_0$. Furthermore, since the function $\tilde{L}(\cdot)$ in (13) is linear with respect to the scalar components of $\mu_{\cdot\cdot}$, each (scalar) gradient: $\nabla_{\mu_{\cdot\cdot}} \tilde{L}$ equates the corresponding associated scalar constraint in (13).

REFERENCES


