On the Optimal Solution for BER Performance Improvement in Dual-Hop OFDM Relay Systems

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Abstract—In this paper bit error rate (BER) performance of DPSK modulated dual-hop OFDM based relay systems implementing ordered subcarrier mapping (SCM) at relay station (R) is analyzed. We propose a new solution, where in the OFDM amplify-and-forward (AF) relay system with fixed gain (FG) at the R station, one or more subcarriers having the lowest signal-to-noise ratios (SNRs) on both hops are first omitted, and then the SCM is performed. Analytical and simulation results have shown that in this way significant BER performance improvements are attained in the regions of medium and high SNRs on both hops. Then, we compare BER performances of the most common OFDM relay systems with SCM, i.e. the systems applying decode-and-forward (DF), AF variable gain (AF VG) relaying, and AF FG relaying, where the same solution based on omitting the worst subcarriers on both hops is assumed. Through these comparisons, an optimal solution for BER performance improvement in the given channels’ conditions, is defined.

Keywords—OFDM relay system; BER; subcarrier mapping; AF FG; AF VG; DF; optimal solution

I. INTRODUCTION

Relay systems as a solution for coverage extension and performance improvement of wireless communications systems have attracted a great research interest in the last decade. Particular attention is paid to them in mobile cellular systems, where a relay station (R) takes part in communication process between the base station (source – S) and end-user (D – destination) in downlink communication scenarios, where direct link between S and D cannot be established. Such a scenario, denoted as dual-hop scenario, is adopted in both standards for IMT-Advanced systems, LTE-Advanced and WirelessMAN-Advanced systems, [1]. According to these standards, R station in the next generation mobile cellular systems will be OFDM (Orthogonal Frequency Division Multiplex) based and will perform decode-and-forward (DF) relaying. DF relaying assumes that R station fully decodes the signal received from S, and then again re-encodes it before forwarding toward D. Another considered solution for OFDM based R stations in mobile cellular systems is amplify-and-forward (AF) relaying, where the signal received from S is only amplified by R and forwarded to D, [1]-[3]. This kind of relaying introduces smaller delay than DF, and is easier to implement. In such a system, the signal received by R may be amplified by fixed gain (FG), where the gain is constant regardless of the variations of S-R channel, or by variable gain (VG), where the gain factor depends on the state of S-R channel. AF VG relaying method requires S-R channel state information (CSI). OFDM based AF relay systems have been subject of intensive research interest in recent years, and it is expected that this kind of solution should become a part of some future specifications of mobile cellular systems, [3].

Advantages of OFDM as a physical layer solution for relay systems are well recognized and used in a solution for capacity enhancement and/or bit error rate (BER) performance improvement through ordered subcarrier mapping (SCM) at R station. [3]-[9]. Namely, it is proven in [4] that ergodic capacity in dual-hop OFDM relay systems is maximized if subcarriers from the first hop (S-R link) are increasingly ordered in accordance to their instantaneous signal-to-noise ratios (SNRs) at R station and then are mapped to appropriate subcarriers from the second hop (R-D link), which are also increasingly ordered with respect to their instantaneous SNRs. This kind of SCM scheme is denoted as best-to-best SCM (BTB SCM) scheme. However, when BER is performance of interest, authors have shown in [5], using majorization theory of inequalities, that in OFDM AF dual-hop relay systems BTB SCM improves performance only in the region of small values of average SNRs on both hops. For medium and higher values of average SNRs on hops, the so called best-to-worst SCM (BTW SCM) scheme should be implemented, where increasingly ordered subcarriers from the first hop are mapped at R station to decreasingly ordered subcarriers on the second hop. Analytical evaluation of BER performances of OFDM AF FG and OFDM AF VG relay systems given in [6] and [7] have approved this conclusions. Further BER performance improvement in OFDM relay systems with SCM can be achieved if one or more subcarriers having the lowest SNRs (worst subcarriers) on both hops are omitted, before SCM is performed, like it is shown in [7] and [8] for OFDM AF VG relay systems with BTB SCM, and for OFDM DF relay systems with BTB SCM, respectively.

In this paper we analytically examine BER performance of DPSK (Differentially Phase Shift Keying) OFDM AF FG relay systems with both BTB SCM and BTW SCM, where one
or more the worst subcarriers from both hops are not used. DPSK scheme is chosen for analytical convenience, and as the BER performances of the other DPSK modulated OFDM based relay systems with SCM are available in the literature for comparison. The influence of omitting the worst subcarriers on both hops in this system has not been examined yet, and it is especially interesting to see the level of performance improvement with such an solution in case of BTW SCM implementation. In order to identify the optimal solution for BER improvement performance in dual-hop OFDM relay systems, the obtained results are compared with corresponding BER performances of OFDM AF FG and OFDM DF relay systems with BTB SCM. Rayleigh fading statistics is assumed on both hops, for all the considered systems.

The paper is organized as follows. Section II describes the analyzed system’s models. BER performance analysis of OFDM AF FG relay systems with SCM is given in Section III. The obtained analytical and simulation results are presented in Section IV, while Section V gives some concluding remarks.

II. SYSTEM MODELS

We analyze dual-hop OFDM based relay systems with ordered subcarrier mapping at the R station, in a scenario where the complete communication is performed through the R station, i.e., there is no direct communication between S and D terminal. Three different relaying strategies are considered: AF FG, AF VG and DF. For OFDM AF FG systems, two ordered SCM schemes, BTB SCM and BTW SCM, known for enabling BER performance improvement in different channel conditions, are examined. It is assumed that all communicating terminals are equipped with a single antenna. Orthogonality between the S-R and R-D channels is achieved by dividing communication process into two time slots, i.e., R operates in a half-duplex mode. It is assumed that R has perfect channel knowledge of both S-R and R-D links, what is important from the point of choosing the appropriate SCM. In order to perform signal demodulation it is necessary that D knows the permutation function performed at R.

Fig. 1 presents block scheme of the OFDM AF FG relay station with SCM. After OFDM demodulation, performed through FFT (Fast Fourier Transformation), SCM is implemented according to known CSI information on both S-R and R-D links. $H_{1,i}$ and $H_{2,i}$ are the $i$th subcarrier channel transfer functions on the S-R link and R-D link, respectively. $M$ is total number of subcarriers. IFFT (Inverse Fast Fourier Transform) block for OFDM modulation follows the SCM mapping, and before the transmission, the signal is amplified with a fixed gain $G$. Block scheme of the OFDM AF VG relay station with SCM is given in Fig. 2. Here, after OFDM demodulation, there is an amplifying block in each of subcarrier branches. The gain introduced in the $i$th subcarrier branch is equal to $G_i=1/H_{1,i}$. Such a solution requires constant estimation of the S-R channel and makes OFDM AF VG relay system with SCM more complex for realization than it is the OFDM AF FG relay system.

Fig. 3. Block scheme of the OFDM DF relay station with SCM

Relay station in the OFDM DF relay system with SCM is presented in Fig. 3. In this system, the R station performs complete decoding of the received signal, before SCM is performed. After the SCM block, signal is re-encoded and OFDM modulated. Such an approach enables separation of S-R and R-D links, which means that different symbol mapping schemes may be implemented on S-R and R-D links, depending on channels conditions. This brings performance benefits, but at the cost of more complex realization compared to OFDM AF relay systems with SCM.

A scenario with independent and identically distributed (i.i.d.) Rayleigh fading among the subcarriers on S-R and R-D channels is assumed in all the systems considered. Probability density function (PDF) and cumulative distribution function (CDF) of the SNR in each S-R subcarrier channel are given by $f_{SR}(x)=\lambda_{SR}\exp(-\lambda_{SR}x)$ and $F_{SR}(x)=1-\exp(-\lambda_{SR}x)$, while the corresponding PDF and CDF of the SNR in each R-D subcarrier channel are given by $f_{RD}(x)=\lambda_{RD}\exp(-\lambda_{RD}x)$ and $F_{RD}(x)=1-\exp(-\lambda_{RD}x)$, respectively. $\lambda_{SR}=1/\eta_{SR}$ and $\lambda_{RD}=1/\eta_{RD}$ denote the inverse of the average SNR on S-R and R-D links.

Further on, BER performance for DPSK modulated OFDM AF FG relay system with SCM is determined.
III. BER PERFORMANCE ANALYSIS

The post-FFT signal on the $i$-th subcarrier, received at the OFDM relay station, is given by:

$$Y_{i,k} = X_i H_{i,k} + N_{i,k}, \quad 1 \leq i \leq M,$$  \hspace{1cm} (1)

$X_i$ is data symbol sent by $S$ on the $i$-th subcarrier. $N_{i,k}$ represents additive white Gaussian noise on the $i$-th subcarrier with variance $\mathbb{E}(N_{i,k}^2) \approx N_0$, with $\mathbb{E}(\cdot)$ denoting the expectation operator. Assuming that the SCM function $\nu(i)$ performs mapping of the $i$-th subcarrier from the first hop to the $k$-th subcarrier on the second hop, the frequency domain signal at $D$ of OFDM AF FG relay system with SCM can be written as:

$$Y_{D,k} = GH_{2,k} Y_{R,i(0)} + N_{2,k}$$

$$= GH_{2,k} H_1 X_i + GH_{2,k} N_{1,i} + N_{2,k}, \quad 1 \leq k \leq M,$$  \hspace{1cm} (2)

where $N_{2,k}$ is the additive white Gaussian noise at $D$, on the $k$-th subcarrier, with variance $\mathbb{E}(N_{2,k}^2) \approx N_0$. Using (2) and the described gain, the end-to-end SNR on the $k$-th subcarrier can be presented as:

$$\gamma_{k,end} = \frac{\gamma_{s,k} \gamma_{RD}}{\gamma_{RD} + \rho}$$

(3)

where $\gamma_{s,k} = \varepsilon_s |H_{i,k}|^2 / (N_0)$ and $\gamma_{RD} = \varepsilon_R |H_{2,k}|^2 / (N_0)$ denote instantaneous SNRs on the $i$-th subcarrier of the first hop and $k$-th subcarrier of the second hop, respectively. $\varepsilon_s$ and $\varepsilon_R$ represent average symbol energies per subcarrier transmitted by $S$ and by $R$, respectively. $\rho$ is the coefficient given through $\rho = \varepsilon_R / (G^2 N_0)$. In order to evaluate BER performance of DPSK modulated OFDM AF FG relay systems with SCM, moment generating function (MGF) of the SNR for the $k$-th subcarrier on each hop has to be known. Thus, we use MGF of the SNR for the $k$-th subcarrier in the system implementing BTB SCM, derived in [6]:

$$M_{\gamma_{k,end}}(s) = \frac{1}{\bar{\gamma}_{SR}} \sum_{i=1}^{M} \sum_{k=1}^{M} \alpha_i \alpha_k \left[ \frac{1}{\beta_i} + \frac{\rho A_{i,j}}{T_j(s)} \left[ \frac{\rho}{\bar{\gamma}_{RD}} - \frac{\rho A_{i,j}}{\beta_j T_j(s)} \right] e^{\rho \delta_i E_i} \right]$$

(4)

where $E_i(\cdot)$ represents the exponential integral function defined in [10, (5.1.1)]. The coefficients $\alpha_i$ and $\beta_i$ are given through:

$$\alpha_i = (-1)^i M \frac{(M-1)}{k-1} \frac{(k-1)}{i} \quad \text{and} \quad \beta_i = i + M - k + 1.$$  \hspace{1cm} (5)

In (5), $\cdot$ denotes binomial coefficients. The coefficients $A_{i,j}$ and $T_j(s)$ are introduced in (4) for the more clear presentation of this relation, and they are equal to:

$$A_{i,j} = \beta_i \beta_j / \bar{\gamma}_{SR} \bar{\gamma}_{RD}, \quad \text{and} \quad T_j(s) = s + \beta_j / \bar{\gamma}_{SR}.$$  \hspace{1cm} (6)

Using the same approach, MGF of the SNR for the $k$-th subcarrier at $D$ in the case of the BTW SCM scheme implemented at $R$ is derived as, [6]:

$$M_{\gamma_{k,end}}(s) = \frac{1}{\bar{\gamma}_{SR}} \sum_{i=1}^{M} \sum_{k=1}^{M} \alpha_i \alpha_k \left[ \frac{1}{\beta_i} + \frac{\rho B_{i,j}}{T_j(s)} \left[ \frac{\rho}{\bar{\gamma}_{RD}} - \frac{\rho B_{i,j}}{\beta_j T_j(s)} \right] e^{\rho \bar{\gamma}_{RD} E_i} \right]$$

(7)

In (7), the coefficients $\delta_i$ and $e_i$ are equal to:

$$\delta_i = (-1)^i M \frac{(M-1)}{k-1} \frac{(k-1)}{i} \quad \text{and} \quad e_i = i + k,$$  \hspace{1cm} (8)

while $B_{i,j}$ can be written as:

$$B_{i,j} = \beta_i \beta_j / \bar{\gamma}_{SR} \bar{\gamma}_{RD}.$$  \hspace{1cm} (9)

Using the MGF based approach for BER performance analysis, BER for the $k$-th subcarrier at $D$ of the assumed DPSK modulated system is obtained through, [11]:

$$P_{b,k} = 0.5 \cdot M_{\gamma_{k,end}}(1),$$  \hspace{1cm} (10)

while BER for the complete OFDM AF FG relay system with SCM is derived through averaging (10) over all $M$ subcarriers:

$$P_b = \frac{1}{M} \sum_{k=1}^{M} P_{b,k}.$$  \hspace{1cm} (11)

By substituting (4) or (7) in (10) and then in relation (11), BER performance of the DPSK modulated OFDM AF FG relay system implementing BTB SCM or BTW SCM is obtained.

It has been mentioned that the OFDM AF FG relay system implementing BTW SCM outperforms the system with BTB SCM in terms of achievable BER performance in the regions of medium and high values of average SNRs on both hops. This motivated us to examine the level of BER performance improvement achieved in the case where one or more subcarriers with the lowest SNRs on both hops are not used, and to examine the mutual performance ratio for this two SCM schemes in such a case.

BER performance of the OFDM AF FG relay system with BTB SCM, where $m$ subcarriers with the lowest SNRs from both hops are omitted, is easy to obtain analytically from (11), if the summation starts from $k=m$ instead of $k=1$, and averaging is done over $M$-m subcarriers, i.e.:

$$P_b = \frac{1}{M-m} \sum_{k=m}^{M} P_{b,k}.$$  \hspace{1cm} (12)

In order to analytically evaluate BER performance of the OFDM AF FG relay system with BTW SCM, without the worst $m$ subcarriers from both hops, let us first assume that the subcarriers from the first hop are increasingly ordered in accordance to their SNR, and subcarriers from the second hop...
are decreasingly ordered with respect to their SNRs. If the worst \( m \) subcarriers from each of the two hops are omitted, then it means that the \((m+1)\)st worst subcarrier from the first hop is mapped to the best subcarrier on the second hop, thus giving the expression for the end-to-end SNR on the \( k \)-th subcarrier at \( D \) in the form:

\[
\gamma_{k, \text{end}} = \frac{\gamma_{k,M,SR}\gamma_{1,RD}}{\gamma_{1,RD} + \rho}, \quad k = 1, \ldots, M - m
\]  

(13)

From (13), it is obvious that we need order statistics for the subcarrier from the first hop having the \((k + m)\)th lowest SNR. By modifying the approach given in [6], PDF of the SNR for this subcarrier from the S-R link is obtained as:

\[
f_{k}^{w}_{M,SR}(x) = \sum_{i=0}^{k-1,m} \lambda_{SR} e^{-\beta_{i}x},
\]  

(14)

where

\[
\alpha_{i} = (-1)^{i \gamma} \left( \begin{array}{c} M - 1 \\ k - 1 + m \\ i 
\end{array} \right) \quad \text{and} \quad \beta_{i} = i + M - k + m.
\]  

(15)

Using the same approach as the one presented in [6], we derive MGF of the SNR for the \( k \)-th subcarrier at \( D \) in the OFDM AF FG relay system with BTW SCM, where \( m \) subcarriers from both hops are not used:

\[
\mathcal{M}_{\gamma_{k, \text{end}}}(s) = \frac{1}{\rho T_{SR}} \sum_{j=0}^{M-1} \mathcal{M}_{\gamma_{k,j,SR}}(s) + e^{-\beta_{j}T_{SR}}
\]  

(16)

\[
+ e^{-\beta_{j}T_{SR}} E_{i} \left( \frac{\rho B_{j}}{T_{SR}} \right) \frac{1}{\rho - \rho B_{j}}, \quad k = 1, \ldots, M - m
\]

with

\[
B_{j} = \beta_{j} e_{i} |\gamma_{SR} \gamma_{RD}|.
\]  

(17)

Substituting (16) in (11) enables BER performance evaluation of the system considered.

IV. RESULTS

The subsequent analytical and simulation results assume perfectly synchronized dual-hop OFDM based relay systems with implemented SCM. As in the real case scenario, realization of the SCM scheme on a subcarrier basis would require large signaling overhead, adjacent subcarriers should be grouped in chunks, and SCM can be realized on chunks basis, like it is proposed in [9]. Thus, \( M \) subcarriers of the OFDM relay system may actually present \( M \) chunks with uncorrelated fading.

It is assumed that noise variances at \( R \) and \( D \) are equal, i.e. \( \mathcal{N}_{01}=\mathcal{N}_{02} \), as well as the average symbol energies transmitted by \( S \) and by \( R \), \( \epsilon_{S} = \epsilon_{R} \). In the OFDM AF FG relay system the so called semi-blind scenario is considered, where \( R \) uses knowledge on channel state information about the S-R link to calculate the gain \( G \):

\[
G^{2} = \mathbf{E}\left[ \epsilon_{S} \left| \mathbf{H}_{k}^{1} \right| \epsilon_{S} + \mathbf{N}_{02} \right],
\]  

(18)

yielding to:

\[
G^{2} = \epsilon_{S} \mathbf{E}\left[ \epsilon_{S} \left| \mathbf{H}_{k}^{1} \right| \right] \epsilon_{S} \mathbf{E}\left[ \frac{1}{\mathbf{N}_{02}} \right].
\]  

(19)

Simulation results are obtained through Monte Carlo simulations of the part of the OFDM relay system that belongs to frequency domain, what is possible and accurate approach, as we have assumed perfect synchronization among the communication terminals. The subcarrier channel transfer functions on both hops are generated as independent Gaussian complex random variables with zero mean.

Fig. 2 gives analytical and simulation results for the DPSK modulated OFDM AF FG relay system implementing both BTB SCM and BTW SCM, where \( k = 1 \) worst subcarrier on each hop is omitted before SCM is implemented (w/o \( k = 1 \)). A scenario with equal average SNR values on both hops is considered. Systems with \( M = 32 \) and \( M = 64 \) subcarriers (chunks) are analyzed. For the sake of comparison, BER performances of the ordinary (w/o SCM) OFDM AF FG relay system is also presented, as well as of the OFDM AF FG relay system with SCM.

Total matching between the simulation and analytical results in Fig. 2 confirms the accuracy of the undertaken analytical approach. The given BER results show that omitting subcarriers with the lowest SNRs on both hops preserves the mutual relation in terms of BER performance between OFDM AF FG relay systems implementing BTW SCM and BTB SCM. Namely, in the region of small values of average SNRs on hops, relay system implementing BTB SCM w/o \( k = 1 \) achieves very small advantage compared to corresponding system with BTW SCM, while in the region of medium and high values of the average SNR on hops, relay system with BTW SCM w/o \( k = 1 \) attains significantly better BER performance. Thus for example, if comparing systems with \( M = 64 \) subcarriers (chunks) w/o \( k = 1 \), it can be seen that, for the BER value of 10\(^{-2}\), the system with BTW SCM achieves SNR gain of approximately 10dB compared to the corresponding system with BTB SCM, slightly lower than 10dB SNR gain compared to the ordinary AF FG relay system, and about 7dB SNR gain when comparing to the AF FG relay system with BTW SCM which uses all \( M \) subcarriers. Even better BER performances are attained in the OFDM AF FG relay system with BTW SCM w/o \( k = 1 \), having less subcarriers (\( M = 32 \)).

Such a significant BER performance improvement, obtained by omitting just one subcarrier with the lowest SNR on each hop of the OFDM AF FG relay system with BTW SCM, motivated us to examine if this solution can be considered as the optimal one for the dual-hop OFDM based relay systems with SCM, i.e. to compare its BER performance with the performance of corresponding OFDM based DF and AF VG relay systems with SCM. We used analytical results obtained in [7] and [8] for the competing OFDM relay systems, as well as the conclusion derived in the latter one,
that the BTB SCM scheme presents optimal solution for BER performance improvement in OFDM systems with DF relaying for all SNR regions of interest.

In order to obtain more comprehensive insight into possible optimal solution for BER performance improvement in dual-hop OFDM relay systems with SCM, we analyzed scenario with good channel conditions on the S-R link, as it corresponds to downlink communication between the base station (S) and the relay station (R), where the position of R is carefully chosen. BER performances in such a scenario, presented as a function of the average SNR on the R-D link, for two values of the average SNR on the S-R link, $\gamma_{SR}=15$dB and $\gamma_{SR}=20$dB, are shown in Fig. 6. In the former case, the system with $M=128$ subcarriers without $k=6$ worst subcarriers from both hops is analyzed, while in the latter case it is taken that $M=256$ and $k=8$ worst subcarriers from each of the two hops are omitted.

Results given in Fig. 6 provide new insight into the solution for BER performance improvement in dual-hop OFDM relay systems with SCM. It can be seen that the OFDM DF relay system with BTB SCM slightly prevails in BER performances, not only for very small $\gamma_{SR}$ values, but also in the region of very high values of $\gamma_{RD}$, for the both values of the analyzed average SNR on the S-R link. It turns out that the optimal solution strongly depends on the channel conditions on both hops.

In the real case scenario, for the greatest range of possible values of average SNRs on hops, OFDM AF FG implementing BTW SCM where the worst subcarriers from each of the two hops are omitted, attains the best BER performance. The SNR gain achieved in this region of SNR values may be very significant. Thus for example, for $\gamma_{SR}=20$dB and $\gamma_{RD}=10$dB, the SNR gain that this system attains compared to the corresponding OFDM DF relay system with BTB SCM is almost 6dB, while for the case when $\gamma_{SR}=20$dB and $\gamma_{RD}=15$dB, this SNR gain is almost 5dB. On the other side, when the channel conditions on both hops (or just on the R-D link) are very bad, and in cases when $\gamma_{RD}$ is 2-3dB higher than $\gamma_{SR}$, the OFDM DF relay system with BTB SCM not using the worst subcarriers, represents the optimal solution for BER performance improvement in dual-hop OFDM based relay systems.
V. CONCLUSIONS

In this paper we firstly examined, analytically and through simulations, BER performances of dual-hop OFDM AF FG relay system with SCM, in the case where one or more subcarriers with the lowest SNRs on both hops are not used. It has been shown that significant BER performance improvement is attained with this solution, especially in the region of medium and high values of average SNRs on hops, where the BTW SCM scheme prevails in terms of BER performances.

Having this in mind, we wanted to determine if such solution can be generally considered as the optimal one for BER performance improvement in dual-hop OFDM based relay systems. Thus, we compared its BER performance with BER performances of dual-hop OFDM DF and OFDM AF VG relay systems with SCM, where also the worst subcarriers on both hops are omitted. The obtained results have shown that the optimal solution depends on the channel conditions on both hops. In the region of low values of average SNRs on hops, as well as in the cases where the average SNR on the R-D link is 2-3 dB higher than the average SNR on the S-R link, the OFDM DF relay system with BTB SCM and without the worst subcarriers on both hops attains the best BER performances. Otherwise, the OFDM AF FG relay system with BTW SCM, where the worst subcarriers from the both hops are omitted, represents the optimal solution which enables significant BER performance improvements. Besides, this system is the simplest for implementation.

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