Abstract—In this paper the level of bit error rate (BER) performance enhancement achieved by subcarrier mapping (SCM) at the relay (R) station in dual-hop OFDM non-regenerative Variable Gain (VG) relay system, is examined. Closed form BER expressions are derived for the assumed M-QAM (M-ary Quadrature Amplitude Modulation) modulated system, in the case of Rayleigh fading channels on both hops, when two SCM schemes denoted as Best-to-Best SCM (BTB SCM) and Best-to-Worst SCM (BTW SCM) are implemented. The obtained results show that, for all the signal-to-noise ratio values of interest, the analyzed relay system with BTW SCM achieves better BER performance, or at least the BER performance equal to the case when BTW SCM scheme is implemented. Having in mind that BTB SCM scheme also maximizes the system capacity, it is shown that it can be considered as the optimal SCM scheme for the OFDM non-regenerative VG relay system.

I. INTRODUCTION

The expansion of wireless communication systems over the last decade was enabled through implementation of appropriate technical solutions, which have succeeded to meet customer’s demands. In order to achieve the required high data rates and quality of service level, the next generation of cellular systems, IMT-Advanced systems, will incorporate, among other advanced technical solutions, OFDM (Orthogonal Frequency Division Multiplexing) based relay (R) stations for the system capacity enhancement and coverage area extension, [1]. Regenerative R stations, that decode the received signal and again re-encode it before forwarding toward destination (D) terminal, are already included in the accepted standards for IMT-Advanced systems, while non-regenerative R stations, that amplify and forward (AF) the received signal, are expected to become a part of the next IMT-Advanced specifications. R station implementing AF technique may amplify the received signal with the fixed gain (FG), or with the variable gain (VG), depending on its possibility to estimate the channel between the source of information (S) and R. Non-regenerative relay systems are less complex than regenerative systems, and they introduce shorter latency. Analyses presented in this paper consider the OFDM non-regenerative (AF) relay system applying VG at the R station.

Although the standards for both IMT-Advanced systems, LTE-Advanced and WirelessMAN-Advanced, have been accepted, there is still ongoing intensive research activity on the OFDM based relay systems, all with the goal to further improve their performance or to optimize their functions. One of the methods for capacity improvement and/or bit error rate (BER) minimization is achieved by mapping subcarriers from the S-R link (first hop) to appropriate subcarriers on the R-D link (second hop), in accordance with their instantaneous signal-to-noise ratios (SNRs), [2]-[7]. The concept of subcarrier mapping (SCM) at the R station was introduced in [2], and shortly afterwards was discussed in [3]. In these papers it was proved that the capacity of dual-hop OFDM based relay system can be maximized if the subcarriers from the first hop are ordered according to their instantaneous SNRs, and then mapped to corresponding subcarriers on the second hop, which are also ordered in accordance with their instantaneous SNRs. This SCM scheme is denoted as Best-to-Best SCM (BTB SCM). However, when BER performance of the OFDM AF based relay systems is considered, it was shown that the BTB SCM scheme minimizes the BER results only in the region of small SNRs, [5]. On the other side, for the medium and high SNR values, the SCM scheme denoted as Best-to-Worst SCM (BTW SCM) is proposed for implementation in order to minimize BER performance. BTW SCM scheme assumes that the subcarriers from the first hop, which are increasingly ordered according to their SNRs, are mapped to the decreasingly ordered subcarriers on the second hop. The BER performance analysis in [4] for DPSK (Differentially Phase Shift Keying) modulated OFDM FG AF relay system with SCM, performed through derivation of the closed form BER expressions, has approved the conclusions conducted in [5]. In this paper we examine the level of BER performance improvement of coherently modulated dual-hop OFDM VG AF relay system, achieved through SCM at the R station. Using the probability density function (PDF) based approach, BER expressions, which are generally applicable to any \textit{M}-QAM (\textit{M}-ary Quadrature Amplitude Modulation)
modulated OFDM VG AF relay system, with both BTB SCM and BTW SCM, are derived.

The paper is organized as follows: Section II describes the system model of the analyzed OFDM VG AF relay system with SCM. The BER performance derivation is given in Section III, while Section IV presents the numerical results. Section V concludes the paper.

II. SYSTEM MODEL

We consider dual-hop OFDM VG non-regenerative relay system, with three communication terminals, S, R and D, each equipped with single antenna. The scenario without direct link between S and D is assumed, where R operates in half-duplex mode. The R station has FFT (Fast Fourier Transformation) block, performing OFDM demodulation. After being demodulated, the signals in parallel branches are amplified in such a way that the influence of the subcarrier channels on the mapping block follows after the amplification of each subcarrier, and then the signal is again OFDM modulated through IFFT (Inverse Fast Fourier Transformation) block. In order to perform signal demodulation at the system receiving end, it is necessary that D knows the permutation scheme performed at R.

![Figure 1. Non-regenerative OFDM VG relay system with SCM](Image)

The signal received at the R station on the i-th subcarrier, after OFDM demodulation can be presented as:

\[ Y_{R,i} = X_i H_{R,i} + N_{R,i}, \quad 1 \leq i \leq N, \]  

(1)

where \( N \) is total number of subcarriers and \( X_i \) is data symbol sent by source on the i-th subcarrier, \( N_{R,i} \) represents additive white Gaussian noise on the i-th subcarrier with variance \( \mathbb{E}(N_{R,i}^2) = N_i \). \( \mathbb{E}(\cdot) \) denotes the expectation operator. Assuming that the SCM function \( \upsilon(\cdot) \) performs mapping of the i-th subcarrier from the first hop to the k-th subcarrier on the second hop, the frequency domain signal at D can be written as:

\[ Y_{D,k} = G_H H_{S,D} Y_{R,k} + N_{D,k} \]

\[ = G_H H_{S,D} X_i H_{R,k} + G_H H_{S,D} N_{R,k} + N_{D,k}, \quad 1 \leq k \leq N, \]  

(2)

with \( H_{S,D} \) denoting the transfer function of the k-th subcarrier on the second hop. \( N_{D,k} \) is the additive white Gaussian noise at the destination on the k-th subcarrier, having variance \( \mathbb{E}(N_{D,k}^2) = A_{D0} \). The fadings in the S-R and R-D channels are assumed to be independent and identically distributed (i.i.d.) among the subcarriers, with Rayleigh fading distribution.

III. BER PERFORMANCE ANALYSIS

In order to derive BER expressions for the analyzed relay system, the PDF of the SNR at D has to be defined.

A. PDF of SNR

For the OFDM VG AF relay system with SCM, in the scenario with the Rayleigh fading on both hops, the relation for PDF of the SNR at D derived in [6] is used. Thus, for the BTB SCM scheme, the PDF of the SNR for the k-th subcarrier at D is obtained as:

\[ f_{Y_{D,k}}(x) = 2 \sum_{i=1}^{N-k} \frac{K_i(\sqrt{\frac{x}{\gamma_{R,i} \gamma_{D,i}}})}{\sqrt{\gamma_{R,i} \gamma_{D,i}}} \frac{\beta_i}{\sqrt{\gamma_{R,i} \gamma_{D,i}}} \left( \sqrt{\frac{x}{\gamma_{R,i} \gamma_{D,i}}} - 1 \right) \]  

(3)

where \( K_i(\cdot) \) and \( K_i(\cdot) \) are zero and first order modified Bessel functions of the second kind defined in [8, eqs. (9.6.21), (9.6.22)], \( \gamma_{R,i} \) and \( \gamma_{D,i} \) are average SNRs on the first and second hop, respectively. The coefficients \( \alpha_i \) and \( \beta_i \) have the values:

\[ \alpha_i = (-1)^i N \begin{pmatrix} N-1 \\ k-1 \\ i \end{pmatrix} \quad \text{and} \quad \beta_i = i + N - k + 1, \]  

(4)

with \( (\cdot) \) representing the binomial coefficient.

The PDF of SNR for the k-th subcarrier at D, in the case of BTW SCM scheme is obtained in a similar form, [6]:

\[ f_{Y_{D,k}}(x) = 2 \sum_{i=1}^{N-k} \frac{K_i(\sqrt{\frac{x}{\gamma_{R,i} \gamma_{D,i}}})}{\sqrt{\gamma_{R,i} \gamma_{D,i}}} \frac{\beta_i}{\sqrt{\gamma_{R,i} \gamma_{D,i}}} \left( \sqrt{\frac{x}{\gamma_{R,i} \gamma_{D,i}}} - 1 \right) \]  

(5)

with the coefficient \( \delta_i \) given as:

\[ \delta_i = (-1)^i N \begin{pmatrix} N-1 \\ k-1 \\ i \end{pmatrix} \]  

(6)
B. BER of the OFDM VG AF relay system with SCP

Using the PDF based approach for BER performance analysis, BER for the \(k\)-th subcarrier at the destination is defined with:

\[
P_{\text{BER}}^{k} = \int P_{\gamma} f_{\gamma,\text{mod}}(\gamma) d\gamma,
\]

where \( f_{\gamma,\text{mod}}(\gamma) \) is the modulation dependent BER which is for M-QAM equal to:

\[
P_{\gamma} = \sum_{n=1}^{\infty} S_{n} Q\left(\sqrt{n} \gamma\right).
\]

\(Q(\cdot)\) is the Gaussian Q function, that can be expressed in terms of the complementary error function as \(Q(x) = 0.5erfc(x/\sqrt{2})\). The coefficients \(S_{n}\) and \(T_{n}\) in (8) depend on the type of the M-QAM modulation. For example, in the case of 4-QAM: \(P_{\gamma} = Q(\sqrt{3} \gamma)\) and for 16-QAM:

\[
P_{\gamma} = 0.75 \cdot Q\left(\sqrt{0.27} \gamma\right) + 0.5 \cdot Q\left(\sqrt{1.87} \gamma\right) - 0.25 \cdot Q\left(\sqrt{3.07} \gamma\right). [9].
\]

Having PDFs of SNR function for BTB SCM and BTW SCM expressed in terms of the modified Bessel functions of the second kind, prevalence finding closed form solution of the integral given in (7). Thus, we use the approximation of integral given in (7). Having PDFs of SNR function for BTB SCM and BTW SCM expressed in terms of the modified Bessel functions of the second kind, prevalence finding closed form solution of the integral given in (7). Using the integral given in (7) and the closed form solutions of integrals \(I_{a,b}\) in [10 Eq. (6.621.3)], the closed form solutions of integrals \(I_{a,b}\) in (8) are obtained in the form:

\[
I_{a,b} = \frac{a \cdot (\gamma_{n,j}(0))^{b}}{(\gamma_{n,j}(0)+2 \beta_{j})^{a}} \cdot F\left(d,b+1,\frac{5}{2}; \frac{I_{n,j}(c)-2 \beta_{j}}{2 \beta_{j}}\right),
\]

where the introduced coefficients \(B_{j}\) and \(L_{m,j}\) are equal to \(B_{j} = \beta_{j} / |\gamma_{m,j}|\) and \(L_{m,j}(x) = x \cdot T_{m,j} + \beta_{j} / |\gamma_{m,j}| + \epsilon / |\gamma_{m,j}|\). The coefficients \(a,b,c,d\) for \(I_{a,b}\) have the same values as the corresponding coefficients for \(I_{a,b}\).

The overall BER of the OFDM VG AF relay system with SCM is obtained through averaging \(k\)-th subcarrier BER value over all \(N\) subcarriers.

IV. RESULTS

We considered the scenario with perfectly synchronized OFDM VG AF relay system implementing SCM at R. The OFDM system with \(N=32\) subcarriers is modeled. It is assumed that the noise variances at R and D are the same (\(N_{01} = N_{02}\)). We present the BER performance of 4-QAM modulated analyzed system, which does not affect the generality of the performed analysis and conclusions.

Simulation results are obtained through Monte Carlo simulations, where only the frequency domain part of the analyzed system is taken into consideration, as the system is assumed to be perfectly synchronized. The subscriber transfer functions on the first and second hop are generated as independent complex Gaussian random variables with zero mean and variance 1/2, meaning that the average subcarrier power is equal to 1. Fifty OFDM symbols are transmitted through each channel realization.

BER graphs of the OFDM VG AF relay system with and without SCM are given in Fig. 2. The scenario with equal average SNRs on both hops is assumed.

As it can be seen through the comparison of the analytical and simulation results, the analytically obtained BER graphs present very tight approximations of the real BER values, which confirms the accuracy of the undertaken approach. The given graphs show that the BTB SCM scheme achieves the best BER performance for the SNR values per hop up to 15dB, while for the higher SNR values per hop, the system with BTW SCM scheme has slightly better BER results. For example, BTB SCM scheme has SNR gain which is more than, or equal to, 1dB compared to the system without SCM, for the SNR values per hop up to 10dB. This is particularly important, as it shows that the BER performance improvement is achieved even in the case when the channel conditions on
both hops are very bad. One more significant reason in favor of BTB SCM scheme implementation is in the fact that BTB SCM scheme maximizes the capacity in OFDM based relay systems, [3], [4]. On the other side, analyzing the high SNRs region, it is obvious that the difference between the BER performance of the system with BTW SCM scheme and the system without SCM scheme is almost negligible.

It should be recalled that the BER performance of the assumed system with BTW SCM scheme for the most SNRs of interest.

On the other side, it has been demonstrated that the BTW SCM scheme achieves the best BER performance in the region of small and medium SNRs, while, at the same time, it maximizes the system capacity for all the SNR values.

On the other hand, the performance of the system with BTB SCM scheme and BTW SCM, thus proving again that BTW SCM scheme should not be considered as an option for performance improvement of the OFDM VG AF relay systems.

V. CONCLUSIONS

In this paper we analyzed the possibility for BER performance improvement of OFDM VG non-regenerative (AF) relay system, through implementation of BTB SCM and BTW SCM. Thus, the closed-form BER expressions for the case of M-QAM modulations are derived. The obtained results have confirmed that the SCM technique should be implemented as the solution for the performance improvement of OFDM VG AF relay systems, and its contribution is particularly significant in the region of small SNRs on both hops. The presented BER results have also shown that the optimal SCM scheme for the considered relay system is BTB SCM scheme, which achieves the best BER performance in the region of small and medium SNRs, while, at the same time, it maximizes the system capacity for all the SNR values.

On the other side, it has been demonstrated that the BTW SCM scheme achieves the best BER performance in the region of high SNR values, but the level of performance improvement compared to the system without SCM is negligible. Thus, it can be concluded that a kind of hybrid OFDM VG AF relay system should be proposed for implementation, in order to achieve optimal BER performance. Such a hybrid system should enable a switch from BTB SCM scheme, to the system without SCM, in the region of medium SNR values.

REFERENCES