Application of Perturbation Methods to the Analysis of Low Frequency Inter-Area Oscillations

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Abstract: This paper presents some advances on the development of perturbation methods for the analysis of inter-area oscillations in complex power systems. A nonlinear model of the power system that includes the representation of second-order effects is proposed. Based on this representation, the normal forms of vector fields are used to analyse non-linear modal interactions under stressed operating conditions. The procedures developed are tested on a practical 46-machine, 191-bus test system.

Keywords: Normal forms, power system dynamics, eigenanalysis

I. INTRODUCTION

Undamped or sustained system oscillations have been experienced in many power systems, involving areas or groups of generators swinging against each other [1,2]. As recognised by many researchers, the very nature of the inter-area phenomenon is not well understood and may include nonlinear interaction of the system oscillation modes, specially under high stress conditions [3-5].

Over the last few years, a number of analytical methods for the analysis and control of inter-area oscillations have been developed. References [1] and [2] summarise some of the techniques currently available for their study. Foremost, among the existing techniques, perturbation methods can be used to increase the fundamental understanding of inter-area oscillations and to aid in the development of control strategies to mitigate their impact on system dynamic performance. The physical idea behind these methods is to approximate the non-linear power system model by a low order representation that allows for a closed form solution to the original non-linear model [6-8].

In recent years there has been an increased interest for research into the application of perturbation techniques to the study of system oscillations. Tamura et al. first studied the non-linear modal interaction phenomenon under the subject of auto-parametric resonance [9,10]. More recently, normal forms of vector fields have been used to determine the interacting modes of oscillations and to identify the nature and characteristics of the inter-area mode phenomenon [3,10]. Further developments in this methodology show that control modes may have a significant interaction with electromechanical modes, specifically inter-area modes [11,12]. Special efforts are being devoted to identify and quantify the effects of this interaction in stressed power networks. The inclusion of these methodologies to include more detailed system characteristics, however, remains an open area of research.

This paper presents some advances on the development of perturbation methods for the analysis of inter-area oscillations in complex power systems. Emphasis is being placed on the analysis of weakly interconnected power systems characterised by several poorly damped inter-area modes.

A non-linear model of the power system that includes the representation of second-order effects is proposed. Based on this representation, the normal forms of vector fields are used to analyse non-linear modal interactions under stressed operating conditions.

The proposed methodology is tested on a practical 46-machine, 191-bus test power system. Study results include the analysis of nonlinear effects of two critical inter-area modes on system dynamic performance. The results obtained are compared with conventional eigen-analysis and the step-by-step simulation of the non-linear power system representation.

II. NONLINEAR POWER SYSTEM MODEL

A. Basic Concepts

The dynamic behaviour of the power system can be described by the following non-linear state equation:

\[ \mathbf{x} = f(\mathbf{x}) \]  

where \( \mathbf{x} \) is an n-dimensional vector, and \( f(\mathbf{x}) \) is an autonomous real n-dimensional vector field. Assuming \( f \) is continuous and can be expanded, the Taylor or power series expansion of the m-th component about an initial operating condition is:

\[ \Delta f_m(\mathbf{x}) = \sum_{k=1}^{n} \frac{\partial f_m}{\partial x_k} \Delta x_k + \]  

\[ \frac{1}{2} \sum_{k,j=1}^{n} \frac{\partial^2 f_m}{\partial x_k \partial x_j} \Delta x_k \Delta x_j + \text{h.o.t.}, \quad m = 1, \ldots, n \]  

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B. First Order Approximations

A first order state representation is obtained by expanding (1) about an equilibrium point and neglecting high-order terms. The solution of the linear approximation with initial conditions $x(0)=x^0$ is

$$x_m(t) = \sum_{i=1}^{n} e^{\lambda_i t} u_m v_i^T x^0 = \sum_{i=1}^{n} P_{mi}(t)$$

where $\lambda_i$ ($i=1,...,n$) are the system eigenvalues and $u_i; v_i$ are the right and left eigenvectors associated with mode $i$; $P_{mi}(t)$ represents the participation of the $i$-th mode in building the state response of the $m$-th state. Eq. (3) expresses the time deviation of the $m$-th state as a linear combination of system modes. At $t=0$, $x_m^0=1$, $P_{mi}(0)=u_m v_i^T$ indicating the initial contribution of the $i$-th mode to the $m$-th state. Participation factors are used in this research to assist in machine grouping and the computation of first order modal generation and power flow [13].

C. Higher Order Approximations

A higher order state representation can be obtained from the Taylor expansion of the system model. Expanding (1) about the post-disturbance SEP and retaining second order effects, the state representation becomes:

$$x = Ax + \left[\begin{array}{c} x^T H^1 x \\ x^T H^2 x \\ \vdots \\ x^T H^n x \end{array} \right] + h.o.t.$$ (4)

where $H^k$ represents $k$-th the Hessian matrix of second derivatives. In order to remove the coupling due to the linear terms in (4), let the transformation $x=Uy$ be introduced, where $U$ is the matrix of right eigenvectors associated with the state matrix $A$. Assuming that the system eigenvalues $\lambda_1, \lambda_2, ..., \lambda_n$ are different, it is obtained that

$$y_m = \lambda_m y_m + \sum_{p,q=1}^{n} C_{pqmn} y_p y_q + \sum_{p,q=1}^{n} C_{pqmn} y_p y_q v_r + ..., m = 1,2,...,n$$ (5)

Here, the terms $C_{pqmn}$ and $C_{pqmn}$ represent the effect of non linear interaction (second and third order effects) of mode $m$. Eqn. (5) can be properly analysed by any of the existent perturbation methods, such as the Normal Form Method or the method of Averaging [6,14].

III. PERTURBATION ANALYSIS

A. The Method of Normal Forms

The physical idea behind the method of normal forms is to construct a sequence of transformations which successively remove the non-linear terms in (5). starting from second order terms. Let to this end, the system in (5), be represented in the Jordan form as:

$$y = Jy + \sum_{r=2}^{n} Y_r(y) + h.o.t.$$ (6)

where $J$ is a diagonal matrix with entries $\lambda_1, \lambda_2, ..., \lambda_n$, and $Y_r(y)$ is a real vector-valued function whose components are homogeneous polynomials of degree $r$ in (6). Second and higher-order terms can be removed by applying the non-linear transformation [6]

$$y = z + h_1(z)$$ (7)

into (6) where $h_1(z)$ belongs to the space $H_0$, which is spanned by the vector-valued monomials

$$x^m e_i = x_1^m x_2^m ... x_n^m e_i$$ (8)

The transformation themselves are obtained from the non-linear relationship:

$$L_n h_1(z) = Y_n(z)$$ (9)

where $L_n$ is the Lie bracket. Ref. [7,8] describe the main computations involved in this process. It should be noted that although the terms $h_1(z)$ which describe the transformation, are non-linear function of $z$, they are found by solving a sequence of linear problems given by [7]:

$$h_{2m} = \begin{pmatrix} Y_{2m} \\ \lambda_k + \lambda_j - \lambda_m \end{pmatrix}$$ (10)

If no resonance conditions are found, the system can be represented in terms of the normal form as $z = Jz$.

B. Approximate Time-Domain Solutions

Time-domain solutions in the original co-ordinate system can be obtained by using the normal-form transformation in (7). Hence,

$$y_m(t) = \sum_{r=1}^{n} \lambda_m e^{\lambda_m t} + \sum_{k=1}^{n} \sum_{j=1}^{n} h_{2k} e^{\lambda_k t} e^{\lambda_j t}$$ (11)

where the second terms in (11) represents the effect of the mode pair $(\lambda_k, \lambda_j)$ on the mode $\lambda_m$ [9]. This process can be extended to obtain timedomain solutions for $x(t)$ using the original variable transformation $x=Uy$ as:

$$X_m(t) = \sum_{j=1}^{n} U_{mj} e^{\lambda_j t} + \sum_{j=1}^{n} \sum_{k=1}^{n} h_{2k} e^{\lambda_k t} e^{\lambda_j t}$$ (12)

Once, closed form solutions are obtained, the contribution of each generator, transmission path and load to the oscillation flow can be obtained by using the procedure proposed in section IV.
C. Analytical Indices for Detecting Dominant Modal Interaction

The approach proposed in [3] is adopted to detect dominant modal interaction. Here, a useful interaction index is obtained by comparing the linear term given for the solution of the linear part in (6) with the solution found by the method of normal forms in (11). This approach quantifies the effect of higher order terms given in the closed form solution in (11). Hence, the difference between the approximate solution given for the method normal forms and the linear solution in (6) yields:

\[
(y_0 - z_j^0) e^{\lambda_j t} + \sum_{k=1}^{n} \sum_{m=1}^{n} h_{2km} z_k^0 z_m^0 e^{(\lambda_k + \lambda_m) t}
\]

(13)

Further, considering only large terms \(h_{2km} z_k^0 z_m^0\) and assuming that other terms are small, a useful interaction index is obtained from:

\[
\text{Index1} = \left| y_0 - z_j^0 + \max (h_{2km} z_k^0 z_m^0) \right|
\]

(14)

In this context, a second index that quantifies the effect of higher order terms can be defined as:

\[
\text{Index2} = \max \left| h_{2km} z_k^0 z_m^0 \right|
\]

(15)

Computation of non-linear interaction indices and machine grouping is done as follows:

1. Sort the \( y \) given by \( y_0 = V x_0 \). Select large entries within an arbitrary maximum value of \( y \).
2. Calculate the index1 for the variables retained in step 1.
3. Calculate the Index2 for the largest values found in index1. If Index2 is larger than 1 is obtained a strong non-linear modal interaction, given by the combinations of modes. If the index is smaller than 1, it indicates that the linear mode is dominant.
4. Examine the pattern of oscillation associated with the modes found by the large entries in index1.
5. Obtain the linear and non-linear participation factors [16] from the interaction modes identified by the Index1.

If the linear term indicates a low frequency inter-area mode with strong interaction with others modes, the system behaviour indicates the onset the inter-area mode phenomenon [3].

IV. SECOND ORDER MODAL POWER FLOW

A. Second Order Representation

The above approach has been extended to compute the second order modal power distribution along selected network elements. To this end, the classical power system representation is modified to preserve network structure. Following Messina et al. [13], let the dynamic behaviour of system generators be represented by the second order model:

\[
x = W x + \frac{1}{2} x^T W_H x + N \Delta v
\]

(16)

and

\[
\Delta i = W x + \frac{1}{2} x^T W_H x + N \Delta v
\]

(17)

where \( \Delta i \) is the vector of incremental modal injections and \( \Delta v \) is the vector of incremental network bus voltages; submatrices \( W \) and \( N \) represent appropriate sensitivity relations. In addition, the network is represented by node current injection equations as:

\[
\Delta i = Y \Delta v
\]

(18)

where \( Y \) is the network admittance matrix. Loads are represented as constant impedances.

B. Second Order Sensitivities

Second order modal voltage and current sensitivities to changes on system states are obtained by substituting (18) into (17) as follows:

\[
\Delta v = [Y - N]^T \left[ W x + \frac{1}{2} x^T W_H x \right]
\]

(19)

At \( t=0 \), (19) allows to express modal voltages as a function of the linear combination of state variables of interest. Further, modal current sensitivities to changes in system states can be obtained from (19) and (17). Once modal voltage and current deviations are determined, power deviations can be determined from:

\[
P_k = V_D k I_D k + V_E k I_E k
\]

(20)

This approach is being used in this research to determine operating scenarios to stress the system as well as to identify critical contingencies that could lead to the inter-area phenomenon.

V. APPLICATION

The procedures developed in sections II through IV were tested on a practical 46-machine, 191-bus test system. A simplified diagram of major system elements is shown in fig. 1. The main characteristics of this system are described elsewhere [15].
The MIS model exhibits two critical inter-area modes at approximately 0.34 and 0.68 Hz associated with the interaction of geographically widespread machines [15]. Table 1 summarises the main characteristics of these modes showing their associated frequencies and dominant machines as obtained from the analysis of linear participation factors. These modes are described in this work as the North-south and East-west modes due to their physical characteristics.

Table 1. Critical inter-area modes for the MIS model

<table>
<thead>
<tr>
<th>Mode</th>
<th>Eigenvalue</th>
<th>Freq. (Hz)</th>
<th>Oscillation pattern</th>
<th>Dominant machines</th>
</tr>
</thead>
<tbody>
<tr>
<td>North-south</td>
<td>0.00±j2.180</td>
<td>0.3476</td>
<td>REC(1.00) vs. North systems MTY(0.755) vs. South systems HU(0.371)</td>
<td></td>
</tr>
<tr>
<td>East-west</td>
<td>0.00±j6.660</td>
<td>0.6856</td>
<td>MM(1.00) vs. Western and central system MP(0.590) vs. South-eastern system ANG(0.563)</td>
<td></td>
</tr>
</tbody>
</table>

Modal power flow analyses were undertaken to gain insight into the nature of modal power flow distribution. Table 2 gives the transmission lines showing the largest participation on these modes. Values are normalised with respect to the largest entry. Examination of the relative magnitudes of power oscillation flow in Table 2 shows a close agreement with the mode shape analysis. As expected from conventional eigen-analysis and time-domain studies, transmission lines linking the north and south systems (tie-lines Guemez-HUI and PRD-ALT) show the largest participation on the North-south mode while transmission lines linking the south-eastern network with the central system (Temascal-PBD) show a large participation on the East-west mode. These paths were selected for contingency analysis as well as to identify operating scenarios to stress the system.

Table 2. Top modal power flow associated with critical system modes

<table>
<thead>
<tr>
<th>Mode</th>
<th>Transmission line</th>
<th>Modal power flow*</th>
</tr>
</thead>
<tbody>
<tr>
<td>North-south</td>
<td>Guemez-ALT</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>Guemez-HUI</td>
<td>0.538</td>
</tr>
<tr>
<td></td>
<td>PRD-ALT</td>
<td>0.876</td>
</tr>
<tr>
<td></td>
<td>PRD-TUX</td>
<td>0.314</td>
</tr>
<tr>
<td></td>
<td>PRD-LGV</td>
<td>0.283</td>
</tr>
<tr>
<td>East-west</td>
<td>Temascal-PBD</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>Temascal-MID</td>
<td>0.977</td>
</tr>
<tr>
<td></td>
<td>TUL-TEX</td>
<td>0.798</td>
</tr>
<tr>
<td></td>
<td>MCD-MID</td>
<td>0.855</td>
</tr>
</tbody>
</table>

Based on this analysis, several contingencies were selected to excite the inter-area modes. Cases of special interest in this research included: 1) system perturbations on the 400 kV network, linking the south-eastern system with the central and north-eastern networks, and 2) contingencies close to the north-south interface (refer to Fig. 1). These perturbations were expected to excite the East-west mode and the North-south modes, respectively as indicated by the modal power flow analysis.

A solid three-phase fault at bus Guemez, cleared in 0.13 seconds by opening one circuit of the tie-line to bus ALT was simulated. Reclosing was not considered.

Normal form results for this fault were computed using first and second order approximations. The results were compared with detailed step-by-step simulations (SBSS) obtained from a transient stability program. Fig. 2 provides a comparison of the time domain solutions with the normal form technique.

Further, Table 3 shows the interaction indices for this fault indicating the main participating modes. As expected, the second order approximation singles out the north-south mode (mode 3 in Table 3) as the most relevant mode in terms of the non-linear effect of the second order terms. The analysis of participation factors in Table 4, reveals that the low frequency mode #3 represents an inter-area mode which exhibits a strong interaction with modes #48 and #75, both essentially of local nature.
Table 3. Interaction indices for case 1

<table>
<thead>
<tr>
<th>Mode</th>
<th>( \lambda_j )</th>
<th>Index1</th>
<th>( k )</th>
<th>( m )</th>
<th>( \lambda_k )</th>
<th>( \lambda_m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.218i</td>
<td>11.697</td>
<td>48</td>
<td>75</td>
<td>0-8.60i</td>
<td>0+10.71i</td>
</tr>
<tr>
<td>11</td>
<td>0.433i</td>
<td>3.5459</td>
<td>4</td>
<td>4</td>
<td>0-3.84i</td>
<td>0-3.84i</td>
</tr>
<tr>
<td>47</td>
<td>0.860i</td>
<td>1.3695</td>
<td>4</td>
<td>75</td>
<td>0-3.84i</td>
<td>0+10.71i</td>
</tr>
<tr>
<td>75</td>
<td>0+10.71i</td>
<td>0.4636</td>
<td>3</td>
<td>47</td>
<td>0-2.18i</td>
<td>0-8.60i</td>
</tr>
</tbody>
</table>

Table 4. Second order participation factors associated with modes #3, #48 and #75

<p>|</p>
<table>
<thead>
<tr>
<th>Machine</th>
<th>Mode#3</th>
<th>Mode#48</th>
<th>Mode#75</th>
</tr>
</thead>
<tbody>
<tr>
<td>REC</td>
<td>0.940 (1.000)</td>
<td>0.101 (0.078)</td>
<td>--</td>
</tr>
<tr>
<td>MTY</td>
<td>0.960 (0.755)</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>HUI</td>
<td>1.000 (0.371)</td>
<td>1.000 (1.000)</td>
<td></td>
</tr>
<tr>
<td>CRB</td>
<td>x (0.288)</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>MZD</td>
<td>x (0.196)</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>GPL</td>
<td>x (0.195)</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>LEK</td>
<td>0.114 (0.185)</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>RIB</td>
<td>x (0.180)</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>TUX</td>
<td>0.292 (x)</td>
<td>--</td>
<td>0.299 (0.258)</td>
</tr>
<tr>
<td>ALT</td>
<td>0.141 (x)</td>
<td>1.000 (1.000)</td>
<td></td>
</tr>
<tr>
<td>SYC</td>
<td>0.104 (x)</td>
<td>--</td>
<td>0.371 (0.311)</td>
</tr>
<tr>
<td>PRI</td>
<td>--</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>TUL</td>
<td>--</td>
<td>--</td>
<td>0.257 (0.258)</td>
</tr>
<tr>
<td>AMI</td>
<td>--</td>
<td>--</td>
<td></td>
</tr>
</tbody>
</table>

* bracketed numbers correspond to linear participations

To investigate the nature of the East-west mode, a three phase fault at substation JUI, cleared at 0.10 seconds by opening one of the circuits of the tie-line Temascal-JUI was simulated. Fig. 3 compares the normal form solution with the SBSS and linear approximations, whilst Table 5 shows interaction indices for this case.

Examination of the results in Table 5 shows that this case excites the East-west inter-area mode as expected from modal power flow studies.

Also of interest, Table 6 shows second order participation factors for this case. It is seen that in this case linear participations provide a fair estimate of dominant machines.

Table 5. Interaction indices for case 2

<table>
<thead>
<tr>
<th>Mode</th>
<th>( \lambda_j )</th>
<th>Index1</th>
<th>( k )</th>
<th>( m )</th>
<th>( \lambda_k )</th>
<th>( \lambda_m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0+4.16i</td>
<td>0.6632</td>
<td>5</td>
<td>6</td>
<td>0-4.16i</td>
<td>0-4.16i</td>
</tr>
<tr>
<td>1</td>
<td>0+2.45i</td>
<td>0.3907</td>
<td>2</td>
<td>5</td>
<td>0-2.45i</td>
<td>0+4.16i</td>
</tr>
</tbody>
</table>

Table 6. Second order participation factors associated with mode #5 given for Index1, in case 2.

<table>
<thead>
<tr>
<th>Machine</th>
<th>Mode#5</th>
</tr>
</thead>
<tbody>
<tr>
<td>MM1</td>
<td>1.000 (1.000)</td>
</tr>
<tr>
<td>MFD</td>
<td>0.600 (0.595)</td>
</tr>
<tr>
<td>ANG</td>
<td>0.515 (0.565)</td>
</tr>
<tr>
<td>PE A</td>
<td>0.274 (0.170)</td>
</tr>
<tr>
<td>MNZ</td>
<td>0.107 (0.090)</td>
</tr>
<tr>
<td>SLM</td>
<td>0.097 (0.095)</td>
</tr>
</tbody>
</table>

* bracketed numbers correspond to linear participations

Table 7 and 8 synthesise the main results obtained from these studies.

Table 7. The oscillation modes detected by Index1

<table>
<thead>
<tr>
<th>Case</th>
<th>Mode</th>
<th>Freq.</th>
<th>Oscillation Pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>5</td>
<td>0.3476</td>
<td>North system vs. South system.</td>
</tr>
<tr>
<td>1</td>
<td>48</td>
<td>1.3695</td>
<td>North-eastern</td>
</tr>
<tr>
<td>75</td>
<td>1.7167</td>
<td>South-eastern system</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>5,6,5</td>
<td>0.6631</td>
<td>Central and western systems vs. South-eastern system</td>
</tr>
</tbody>
</table>

Table 8. Comparison of Index2 in the Interactions Dominates

<table>
<thead>
<tr>
<th>Case</th>
<th>( j )</th>
<th>( k )</th>
<th>( m )</th>
<th>Index2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>48</td>
<td>75</td>
<td>3.5356</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>6</td>
<td>5</td>
<td>1.3508</td>
</tr>
</tbody>
</table>

The results in Tables 7 and 8 clearly show that in the first case a fault on line Guemez-ALT leads to strong modal interaction as indicated by Index2. This means that the terms of high order dominate on lineal term. On the other hand, case 2 associated with line TMD-JUI exhibits less non-linear interaction between the modes \((j, k, m)\) as compared to case 1. This can be verified in the solutions in the time shown in the figures 2 and 3.
VI. CONCLUSIONS

This paper presents several advances on the development of analytical techniques to analyse inter-area oscillations in complex power systems. Two different approaches to assess the nature of inter-area oscillations are investigated and implemented in a production-grade computer program. The first consists of studying the nature of the motion modes of the system using conventional linear analysis. The second approach focuses on the analysis of non-linear interaction under stressed operating conditions using the normal forms of vector fields.

Current advances in this research include the representation of second order effects in the system model and the analysis of machine groupings and nonlinear interaction between states and critical system oscillation modes. Emphasis is being placed on the analysis of weakly interconnected power systems characterised by several critical oscillation modes.

The application of these methodologies to the study of a complex power system has shown the appropriateness of the developed models. Study results show that critical machines associated with critical system modes can differ from those obtained from first order participation factors. While the present study results correlate well with the observed system behaviour, other system operating conditions and models need to be considered.

W. REFERENCES


VII. BIOGRAPHIES

E. Barocio received the M.Sc in Electrical Engineering-Power Systems from the University of Guadalajara, Mexico in 1997. He is currently working towards his Ph.D at the Centre for Research and Advanced Studies (CINVESTAV) of the National Polytechnic Institute (IPN) of Mexico.

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