Abstract—In this paper we present an improved shape from shading method using improved fast marching method. We commence by showing how to recover 3D shape from a single image using an improved fast marching method for solving SFS problem. Then we use the level set method constrained by energy minimization to evolve the 3D shape. Finally we show that the method can recover stable surface estimates from both synthetic and real world images of complex objects. The experimental results show that the resulting method is both robust and accurate.

Keywords—shape from shading; fast marching method; level set; 3D shape;

I. INTRODUCTION

Shape from shading is a classical problem and has attracted over three decades of research [1]. The most common problem is that the recovered field of surface height or normals is subject to concave-convex shape ambiguities, and this can lead to the implosion of shape features. This effect can lead to shape features becoming inverted. The problem not only frustrates accurate surface shape recovery, it also means that subsequent shape analysis can not rely on shape from shading as an input [2]. There have been several attempts at eliminating concave-convex ambiguities. For instance, Balanz et al. [3] construct a morphable model of appearance based on the principal components analysis of 3D head shape and texture collected using a laser range finder. Given a single input image they simultaneously estimate pose, lighting, camera properties, face shape and face texture, by optimizing the error between predicted and measured appearance. Zhao and Chelleppa [4] exploit the approximate bilateral symmetry of faces. Their geometric symmetry constraint simplifies the problem considerable, but fails to recover shape of high fidelity. More recently, Smith and Hancock [2] successfully solve the concave-convex ambiguities of the nose by using a robust fit of the principal geodesics as a statistical constraint for shape from shading. However, the recovered facial shapes are very similar because the average statistical model dominates the process of shape from shading.

The aim of this paper is to present an improved shape from shading algorithm that can overcome the concave-convex ambiguities problem and can be used to accurately recover 3D shape.

II. SHAPE FROM SHADING

We deal with Lambertian scenes and suppose that the albedo is constant and everywhere equal to 1. Assume that the surface we aim to recover is specified as a height-function $z(x, y) : R^2 \rightarrow R$ of the position co-ordinate $(x, y)$, and whose surface normal at the location $(x, y)$ is given by unit vector $n = (n_x, n_y, n_z)^T$. Let $I(x, y)$ be the normalized image brightness at the point with image co-ordinates $(x, y)$. The light source direction is denoted as unit vector $L = (L_x, L_y, L_z)^T$. From Lambert’s law, we have

$$I = n \cdot L. \quad (1)$$

Given a surface $z(x, y)$ with surface gradients $z_x = \frac{\partial z(x,y)}{\partial x}$ and $z_y = \frac{\partial z(x,y)}{\partial y}$, then equation (1) can be rewritten as

$$I = \frac{z_x L_x + z_y L_y + L_z}{\sqrt{z_x^2 + z_y^2 + 1}} \quad (2)$$

since $z_z = 1$. For the case when the light source is vertical, i.e. $L = (0, 0, 1)$, equation (2) can be simplified as

$$\| \nabla z(x,y) \| = \sqrt{\frac{1}{I(x,y)^2} - 1} \quad (3)$$

This equation is known as the Eikonal equation.

III. IMPROVED FAST MARCHING

From the above discussion, the shape from shading problem involves solving the Eikonal equation (3), which the solution can be found using the fast marching method (FMM) [5]. Although FMM provides a stable and consistent solution to the Eikonal equation, it still has some limitations. For instance, at each grid point $z$, the method employs only the 4-connected directly adjacent vertical and horizontal neighbors to update $z$. Thus it ignores the information provided by diagonal points in the 8-neighbourhood. As a consequence, FMM can suffer from a large numerical error along the diagonal directions [5]. In order to reduce the error, we consider the information provided by diagonal neighbours.
For this paper we use FMM for shape from shading from a single image, and as a result we consider only the 2D FMM. We rewrite equation (3) as follows:

\[
a_1 \| \nabla z(x, y) \|^2 + a_2 \| \nabla z(x, y) \|^2 = f(x, y) \tag{4}
\]

where \( a_1 + a_2 = 1 \), \( a_1 \geq 0 \), \( a_2 \geq 0 \) and \( f(x, y) = \sqrt{1 / l(x, y)^2} - 1 \). There are two terms on the left hand of the equation (4), the first term \( \| \nabla z(x, y) \|^2 \) represent the contribution of the 4-connected neighbors, and which gives the same results as the classical FMM algorithm. The second term \( \| \nabla z(x, y) \|^2 \) represents the contribution of the 8-connected neighbors, and provides additional information along the diagonal directions of the pixel lattice. The coefficients \( a_1 \) and \( a_2 \) represent the relative weight of the two terms respectively. Thus, we may rewrite the numerical approximation as follow:

\[
a_1 \sum_{n=1}^2 \max(\frac{Z - Z_n}{\Delta n}, 0)^2 + a_2 \sum_{n=1}^2 \max(\frac{Z - Z_n'}{\Delta n}, 0)^2 = F^2 \tag{5}
\]

where a) \( \Delta_1 = \Delta x \) and \( \Delta_2 = \Delta y \) are the spacings of the sample points used to estimate \( \nabla z(x, y) \) on the pixel lattice in the x and y directions. b) \( Z = z_{i,j} \) is the height sampled at the centre neighbourhood location and \( F = f_{i,j} = \sqrt{1 / l_{i,j}^2 - 1} \), \( I_{i,j} \) is the normalized image brightness of point \((i, j)\). c) \( Z_1 = \min(z_{i-1,j}, z_{i+1,j}) \) and \( Z_2 = \min(z_{i-1,j-1}, z_{i+1,j+1}) \) are height estimates for the horizontal and vertical neighbours, and d) \( Z'_1 = \min(z_{i-1,j-1}, z_{i+1,j+1}) \) and \( Z'_2 = \min(z_{i+1,j-1}, z_{i-1,j+1}) \) are the height estimates for the two diagonal neighbours.

The solution of equation (5) is also the solution of SFS. We refer to denote this method as IFFM, and although it makes a small improvement to FMM, it can reduce the error and this is proved later by experiment. The complexity of IFFM is the same as the complexity of FMM.

IV. CONSTRAINED ENERGY MINIMIZATION USING LEVEL SETS

Shape from shading via IFFM can not avoid concave-convex ambiguities. In order to recover accurate 3D shape, we require a postprocessing step to be applied subsequent to surface recovery. We define the initially recovered shape as a closed deformable surface in three dimensions, i.e. \( \Gamma(t) \subseteq \mathbb{R}^3 \). We aim to develop an update process that will evolve the initial surface \( \Gamma(t) \) to a new and more accurate representation under the effects of its geometrical features and singular points. In particular we aim to formulate this update process so as to resolve concave-convex ambiguities. This process can be regarded as one of surface energy minimization.

\[
\Gamma(t) = \arg \min_{\Gamma} E(\Gamma(t)) \tag{6}
\]

where \( E(\Gamma(t)) \) is the surface energy which can be decomposed into internal and external energy terms,

\[
E(\Gamma(t)) = E_{\text{int}} + E_{\text{ext}} \tag{7}
\]

where \( E_{\text{int}} \) represents the internal energy of the surface and \( E_{\text{ext}} \) represents the externa potential energy relative to the surface.

The internal energy \( E_{\text{int}} \) depends on the geometrical features of the surface. In order to simplify calculation, \( E_{\text{int}} \) can be approximated as follow [7]

\[
E_{\text{int}}(\Gamma) = \int_{\Gamma} (\Gamma_{xx}^2 + 2\Gamma_{xy}^2 + \Gamma_{yy}^2) dx \, dy \tag{8}
\]

where \( \Gamma_{xx}, \Gamma_{xy} \) and \( \Gamma_{yy} \) are the second-order derivatives of \( \Gamma \).

To calculate \( E_{\text{ext}} \), we note that changes of the potential energy due to surface deformation must satisfy the curve constraint. Suppose we have a curve \( l_0(s), s \in L \) that shrinks to the target curve \( l(s) \) on the surface. Our idea is to constrain the evolution of the surface using the distance between \( l_0(s) \) and \( l(s) \). The potential energy associated with the shrinkage of the curve is \( \frac{\|l(s) - l_0(s)\|^2}{c} \). As a result the the potential energy associated with the surface \( \Gamma \) is

\[
E_{\text{ext}}(\Gamma) = c \int_{L} (\|l(s) - l_0(s)\|^2) ds \tag{9}
\]

where \( c \) is a constant weight factor and \( L \) is integral interval. In order to solve the surface deformation process, we discretize \( \Gamma \) into a three dimension space and represent it as a signed distance function, denoted by \( \phi(x, t) \). The function \( \phi(x, t) \) represents the distance between the point \( x \) and the target curve \( l(s) \). Thus, \( \Gamma(t) \) can be denoted as a level set of \( \phi(x, t) = 0 \), i.e.

\[
\phi(\Gamma(t), t) \equiv 0. \tag{10}
\]

The time evolution of \( \phi \) is given by

\[
\phi_t + \frac{d\Gamma(t)}{dt} \| \nabla \phi \| = 0 \Leftrightarrow \phi_t + v_n \| \nabla \phi \| = 0. \tag{11}
\]

where \( v_n \) is the evolution velocity of \( \Gamma(t) \) along its normal direction. In order to ensure the surface evolves to the target, we control \( v_n \) by minimizing the total energy \( E(\Gamma(t)) \). Based on this idea, we firstly obtain the gradient of \( \phi \) via \( E_{\text{int}} \) and \( E_{\text{ext}} \), and then construct \( v_n \). From the definition of \( E_{\text{ext}} \) we obtain

\[
\frac{\partial E_{\text{ext}}}{\partial \phi} = 2c \sum_{i=1}^n \|l(i) - l_0(i)\| \tag{12}
\]

where the index \( i \) runs over the sample points on the surface curve. Since \( v_n \) is inversely proportional with the gradient of \( E_{\text{ext}} \), we have

\[
\frac{d\phi}{dt}_{\text{ext}} = -a \sum_{i=1}^n \|l(i) - l_0(i)\| = v_{\text{ext}}. \tag{13}
\]
where $\alpha > 0$. Hence, the rate of surface evolution is controlled by the amount of curve shrinkage required. For $E_{\text{int}}$, we follow [8] and allow a curvature dependant evolution process. As a result

$$\frac{d\phi}{dt}_{\text{int}} = k = v_{\text{int}}.$$  \hspace{1cm} (14)

where $k$ is the mean curvature. As a result, we have a diffusion process in which high curvature surface locations are associated with the greatest rate of change. This will have the effect of smoothing away local surface noise. Using (13) and (14), we can rewrite equation (11) as follows

$$\phi_t + v_{\text{ext}} \| \nabla \phi \| + v_{\text{int}} \| \nabla \phi \| = 0$$  \hspace{1cm} (15)

Solving equation (15) can evolve the initial surface under internal and external energy constraints.

V. EXPERIMENTS

We now demonstrate the results of applying our algorithm to both synthetic images and real world images. The algorithm runs on a laptop with Intel Centrino Duo T2400 1.83GHZ processor and 1GB RAM. For all the experiments the execution time is less than 5 seconds, which gives some idea of the efficiency of the numerical algorithm.

A. Recovering synthetic shape

In this experiment, we will prove that IFFM has higher accuracy than FFM. The input image is of size $100 \times 100$ pixels, and contains a hemisphere of radius 50 pixels. The experimental result is shown in Fig. 1. From Fig. 1, it is clear that the surface recovered by IFFM is better than the surface recovered by FFM. The radius recovered by IFFM is about 49.6 pixels, the radius recovered by FFM is about 41.3 pixels. Using IFFM is therefore more accurate.

B. Recovering complex object shape

In this experiment, we will demonstrate that our method can recover an accurate representation of complex shape from a single input image. The experimental result is shown in Fig. 2. From Fig. 2, it is clear that shape from shading using IFFM is prone to concave-convex ambiguities. However, with the addition of our shape evolution method, we can obtain accurate 3D shape.

C. Recovering shape from real world images

In this experiment, we will show that our method can recover accurate 3D shape from a wide variety of real world imagery. The input images are a hair dryer and a vase. The experimental results are shown in Fig. 3 and Fig. 4. Only the input image and the recovered shapes, which have been evolved by our shape evolution method, are shown. For Fig. 3, besides the air outlet, the hair dryer is not concave. For Fig. 4, besides the bottleneck, the vase is not concave. The results are relatively accurate.

D. Recovering from synthetic image

In this experiment, we will show that our method can recover complex 3D shape from a synthetic image. The input image is image of a bunny. The experimental result is shown in Fig. 5. Only the input image and the recovered shapes, which have been evolved by our shape evolution method, are shown. For Fig. 5, besides some white curve, the shape is not concave. The recovered 3D shape is relatively accurate.

VI. CONCLUSION

We have introduced a robust shape from shading algorithm which recovers stable 3D shape from a wide variety of real world and synthetic imagery. Experimental results have shown that our method for 3D shape recovery is both feasible and effective. We will improve the evolution process in the next work.

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Figure 2. Recovered facial shape using IFFM and shape evolution. The different rows are for different faces. The first column are the input images, the second and third column (from two different viewpoints) show the facial shape recovered using IFFM, the final two columns (again from two different view points) are the more accurate facial shapes after shape evolution.

Figure 5. Recovered shape from bunny image. The left is the input image, the middle and right (two view points) are the recovered 3D shape by our method.

REFERENCES


