A Replicator Dynamics Weighted Control Technique for a DC-DC Converter

Eduardo Mojica-Nava
Electronics and Telecommunications Engineering Program
Universidad Católica de Colombia
Diagonal 47 No 15-50, Bogotá, Colombia
E-mail: eamojica@ucatolica.edu.co

Nicanor Quijano
Department of Electrical and Electronics Engineering
Universidad de Los Andes
Carrera 1 Este No 19A - 40, Bogotá, Colombia
E-mail: nquijano@uniandes.edu.co

Abstract—A power electronics benchmark problem involving several practical scenarios for a fixed-frequency synchronous step-down DC-DC converter is investigated. A new weighted control technique that uses the replicator dynamics concepts to weight the operation of different controllers tuned to operate in different modes, and with different control objectives is presented. Simulations for four critical process conditions are shown to illustrate the performance of this novel technique.

Index Terms—Replicator Dynamics, Linear Quadratic Regulator, DC-DC Converter

I. INTRODUCTION

Evolution can be seen as learning by cultural interactions between different entities (agents) [6]. One area that mathematically describes how the agents evolve over time is evolutionary game theory (EGT) [14], [7]. In EGT, several approaches have been developed over the years, where the replicator dynamics is the most widely used concept in different fields, from biology to engineering, because its implementation is simple. One of the reasons for that is because the replicator dynamics uses zero-level type of agents, which implies that the actions learnt by the agent use very few data from the environment [13]. The replicator dynamics were introduced originally by Taylor and Jonker in [12], and it is a simple model inspired by the biological evolution of homogenous agents. In general, they describe how the proportion of agents in different habitats is affected by the differential of fitness produced by the strategies selected by each of the individuals. These dynamics are based on three main assumptions: i) each individual is genetically programmed to use the same pure strategy throughout its lifetime; ii) when a given individual has an offspring, it transfers the same strategy that it has used; and iii) the fitness of each individual changes proportionally to the net reproduction rate of the population [6]. In this paper, we propose another technique based on dynamically weight different controllers that have been tuned to perfectly operate in specific regions, so that we can give more emphasis to the optimal controller that gives a better performance. In this work, we use the ideas in [3], [9], and some specific analogies, in order to dynamically change the weight that each controller should have depending on the region where the converter is.

This work is organized as follows. In Section II we present a DC-DC buck converter model used in this work. Section III is dedicated to the replicator dynamics concepts, which are then used to develop a weighted control strategy. In Section IV the weighted control strategy is applied to the DC-DC converter model, and simulations results are presented to validate our approach. Finally in Section V some conclusions are given.

II. DC-DC BUCK CONVERTER MODEL

The control strategy proposed in this work is represented over an input affine non linear system. We consider the topology of the fixed-frequency step-down converter (buck converter) as it is shown in Fig. 1. We use the basic ideas of state-space averaging and the so-called small signal modeling for control design purposes [11]. The state-space average model of the converter is described by substituting the duty ratio feedback function $u$ in place of the actual switch control function $s$ in Fig. 1, and may be expressed as

$$
\dot{x}(t) = Fx(t) + Bu(t) \\
y = Gx(t)
$$

(1)
where $F$ and $B$ are given by

$$F = \begin{bmatrix} -\frac{1}{x_i} (r_1 + \frac{r_{1c}}{r_o + r_{1c}}) & -\frac{1}{x_o} (r_{oc} + \frac{1}{r_o + r_{oc}}) \end{bmatrix}, \quad B = \begin{bmatrix} \frac{1}{x_i} \\
\end{bmatrix}$$

and $x(t) = [x_1 \quad x_2]^T = [i_l(t) \quad v_c(t)]^T$ are the states, where $i_l(t)$ is the inductor current and $v_c(t)$ the capacitor voltage, and $u \in [0, 1]$ is the control input. The output $y$ is the output voltage $v_o(t)$ across the output load $r_o$, which is expressed as a function of the states through $G$ as

$$G = \begin{bmatrix} \frac{r_{1c} r_o}{r_o + r_{1c}} \\
\frac{r_{oc} r_o}{r_o + r_{oc}} \\
\end{bmatrix}$$

We are considering in the model the parasitic elements, i.e., the internal resistance of the inductor, $r_1$, and the equivalent series resistance of the capacitor, $r_c$. The parameters $x_1$ and $x_c$ represent the inductance and the capacitance of the low-pass filtering stage, respectively, and $v_s$ is the input voltage. The parameters used in the simulations are chosen to describe a 50V to 25V step-down DC-DC converter using the relation from [1] expressed in the per unit input voltage. They are given by $r_1 = 1\Omega, r_c = 0.15\Omega, x_1 = 2mH, x_c = 100\mu F, v_s = 50V, v_{ref} = 25V$, and if not otherwise stated $r_o = 50\Omega$.

In the last few years, it has been shown that a control strategy for the DC-DC step-down converter needs to fulfill some prerequisites to have an impact on the power electronics researches and practitioners (see for instance [1], [8], among others). It must respect operation characteristics, feature favorable properties such as enable the designer to consider the constraints on the inductor current, have a global performance instead of concentrated on a specific operating point, and be easy to tune. Besides, the regulation must be accompanied by the minimum output voltage ripple, and be maintained despite changes in the load or the input voltage. In this work we consider that all states and parameters are measured.

### III. Weighted Control Strategy using Replicator Dynamics

#### A. Replicator Dynamics Model

Taylor and Jonker introduced in [12] the replicator dynamics concept, which is a model inspired by the biological evolution of how selection, via differential fitness, affects the proportion of agents using different strategies in a population. In other words, this model shows how the selection in an evolutionary process favors one species over another depending on their own fitness. For that, an evolutionary symmetric game between a homogenous population is played, where the individuals can only choose pure strategies that are transmitted without error from parent to child. One of the original assumptions states that the fitness of each individual changes proportionally to the total reproduction rate of the population, i.e., the population state changes depending on the fitness [6].

We assume that each agent has $q$ pure strategies. The total number of agents that are using the $i$th strategy is denoted by $\tau_i$, for all $i = 1, \ldots, q$. Let us assume that the number of agents is constant, i.e., $\sum_{j=1}^q \tau_j = T$ for some $T > 0$, and for all $t \geq 0$. Let

$$p_i = \frac{\tau_i}{\sum_{j=1}^q \tau_j}$$

represent the fraction of agents in a population playing the pure strategy $i$, $i = 1, \ldots, q$. Clearly $p_i \geq 0$ and $\sum_{j=1}^q p_j = 1$, for all $t \geq 0$.

The original assumptions in [12] can be combined in an exponential growth or decay expression, i.e.,

$$\frac{dp_i}{dt} = \tau_i f_i$$

where $f_i$ is the fitness, or from a biological point of view, the payoff at which each strategy reproduces. Hence, the replicator dynamics equation is given by

$$\frac{dp_i}{dt} = p_i \left(f_i(p) - \bar{f}(p)\right)$$

where the choice of the average fitness of the population

$$\bar{f} = \sum_{j=1}^q p_j f(j)$$

allows the invariance of the constraint set for the replicator dynamics. The simplex $\Delta = \{ p \in \mathbb{R}_+^q : \sum_{i=1}^q p_i = 1 \}$ (and hence the simplex $\Delta_+ = \{ p \in \mathbb{R}_+^q : \sum_{i=1}^q \tau_i = T \}$) is invariant under Equation (4) [7].

**Proposition 1**: If $p(0) \in \Delta - \delta \Delta$ (which implies that $p(t) \in \Delta - \delta \Delta$ for all $t > 0$) and if $f_i(p_i)$ is a monotonically decreasing function such that $f_i(p_i)$ is Lipschitz continuous, then the replicator equation (4) possesses a unique solution that lies strictly inside the simplex $\Delta_+$ (i.e., in $\Delta - \delta \Delta$). Notice that Proposition 1 (based on ideas presented in [7] and [14]) implies that $p(t) \neq 0$ for all $t \geq 0$. In other words, the system cannot be truncated, and the selection of the fitness will reflect this fact. For instance, in [10], the authors have shown that the equilibrium point reached is asymptotically stable for a special class of fitness function, similar to the one that we introduce next, which will lead to some robustness in the not truncated cases strategy.
B. Weighted Control Technique (WCT)

In Section III-A, we have described some basic ideas that we combine here. We use the replicator dynamics over a set of controllers to dynamically weight them. We develop a particular case where each of these controllers is designed using linear quadratic regulator (LQR) techniques for different operating points. The replicator dynamics will assign a weight to each of them, depending on a fitness function that we describe next. This fitness function depends on the performance of each LQR controller. The feedback vector is weighted by a constant \( p_i > 0 \), which represents a proportion such that \( \sum_{j=1}^{q} p_j = 1 \). In other words, the analogy that we are using here with respect to the replicator dynamics idea in Section III-A is as follows: each controller corresponds to a given strategy, i.e., when we say that we are “playing” strategy \( i \), we are saying that we are using the \( i^{th} \) controller. Furthermore, the proportion of the \( i^{th} \) population corresponds to the weight that we assign to controller \( i \) based on the difference of fitness in Equation (9).

The end purpose is to obtain a unique control signal from the set of controllers according to a decision block. In our case, the decision maker is constructed by the replicator dynamics defined previously. The selection criteria are based on the assignment of a fitness function for each controller, where the term fitness can be understood as a performance measurement for each one of them. In other words, the idea is to have a strategy that gives a higher payoff to those controllers whose fitness functions represent the controllers that are more suitable to well-perform in a given region. However, as it was pointed out before, all the controllers will be on at all time, even though some of them will have a small weight.

We define the finite set of controllers as \( \mathcal{U} = \{u_1, u_2, \ldots, u_q\} \), where each controller is designed for a different operation region. We design each controller for different operation points, each one of them based on optimal linear controllers using LQR techniques. We use an LQR controller due to its simplicity of design, robustness, and optimality. The optimality criteria for each controller is given by,

\[
J_i = \int_{0}^{\infty} x^T(t)Q_ix(t) + u_i^T(t)R_iu_i(t) \, dt
\]

where \( Q_i \) and \( R_i \) are symmetric positive definite matrices, and with \( x(t) \in \mathbb{R}^n \) and \( u_i(t) \in \mathcal{U} \).

We obtain a linear system for each operation point of the form

\[
\begin{align*}
\dot{x} &= A_ix + B_iu_i \\
y &= C_ix
\end{align*}
\]

According to the theory, we can find the feedback matrices \( K_i \) such that the input control variables \( u_i(t) = N_ir(t) - K_ix(t) \) minimizes (6), where \( r(t) \) is the reference signal for the state variables and \( N_i \) the matrix associated with this reference. The matrices \( K_i \) are obtained by \( K_i = R_i^{-1}B_i^TP_i \), where \( P_i \) is the matrix solving Ricatti’s algebraic equation \( A_i^TP_i + P_iA_i - P_iB_iR_i^{-1}B_i^TP_i + Q_i = 0 \), as it is shown in classical control theory [4].

We design each controller choosing \( Q_i \) and \( R_i \) according to the design conditions and model restrictions described below.

We use Bryson’s rule [5] to estimate the initial parameter and then, we use a trial and error procedure to finally adjust them. We assume that there is a finite set of controllers \( \mathcal{U} \) designed for \( q \) different operating points. Hence, we obtain \( q \) feedback vectors \( K_i \), with \( i = \{1, \ldots, q\} \). The design criteria to find the feedback vector are the same for all of them. First, we choose \( Q_i \) as a diagonal matrix with the first element ten times greater than the others, giving priority to the first state. Then, we choose \( R_i = 1 \) so that the control input signal is not going to be saturated.

We describe the replicator dynamics as a strategy to dynamically weight each feedback control related to each operation point. Consider the controllers as it is stated so far

\[
u_i(t) = N_ir(t) - K_ix(t)
\]

In order to implement the replicator dynamics approach, first we define a fitness function based on the particular problem under consideration. The success of any pure strategy is then evaluated by the difference between the controller-specific fitness function and the average fitness. Considering this analogy, we suggest the following definition.

**Definition 2:** The fitness function is a payoff function associated with the population of the agents, as well as with respect to the dynamics of the process to be controlled. For this case we assume that the following restrictions are satisfied:

\[
f_i(p_i, x_i) > 0,
\]

and

\[
\partial f_i(p_i, x_i)/\partial p_i < 0, \forall p_i \in (0, 1)
\]

We then propose a weighted control signal of the form

\[
u = \sum_{i=1}^{N} p_iu_i, \tag{8}\]

where \( u_i \) is the control signal from the \( i \)-controller, which is designed to deal with specific regions of the system that have similar dynamics. The weights \( p_i \) of each control signal are given by the replicator equation

\[
p_i = \beta p_i(f_i - \bar{f}), \tag{9}\]

where we have introduced a factor \( \beta \), which is a speed parameter for the convergence of the replicator dynamics. The fitness function, \( f_i \), is selected based on the criteria stated in Definition 2, and \( \bar{f} \) is the average fitness defined in Equation (5). The replicator dynamics assigns weights to the controllers depending on the control objectives, i.e., it weights the control signals. These weights are constrained to the set \( \Delta - \partial \Delta \), which implies that for all \( t \geq 0 \), the \( p_i \)'s will be different from zero, and they will dynamically evolve strictly inside the simplex \( \Delta \). This is summarized in the following proposition.

**Proposition 3:** Given a single input affine nonlinear system represented by Equation (1), there are fitness functions \( f_i(p_i, x_i) \) such that the evolutionary dynamics expressed by
the replicator dynamics in Equation (9), generate suitable coefficients \( p_i \in (0, 1) \) that satisfy the control law \( u \),

\[
u = \sum_{i=1}^{q} p_i u_i = p_1 u_1 + p_2 u_2 + \ldots + p_q u_q,
\]

where \( u_1, u_2, \ldots, u_q \) are the control signals for different controllers.

The controllers are designed for acting in specific regions where similar behaviors could be assumed. The set of regions is chosen in such a way that the control is achieved for the nonlinear system described above. In Fig. 2 a schematic of the weighted controller, where the weights are settled by the replicator dynamics technique, is shown.

The key point in this approach is the selection of the fitness function. A fitness function is chosen for each controller such that different performances are attained by each one of them. In this way, the weights for those controllers that are showing a better response at the output of the process will increase, while for the other controllers this weight will decrease (however, the weight cannot be equal to zero as it was stated before). The initial conditions need to be different from zero. This process is done dynamically, i.e., if there are some variations in the process, then the weights will change their values. In this case, it is even possible to see a drastically change of weight between controllers, because one of them would be more suitable to perform better in a given operating region.

The proportion that we assign to the \( i^{th} \) controller is related to the error defined as follows,

\[
e(t) = \| y_{ref} - y_i \|
\]

where \( y_{ref} \) is the desired value for the most influential variable to be controlled. These values are chosen in such a way that they are equal for all the system, and \( y_i \) is the output variable when the control \( u_i \) is applied to the process. Therefore, we use a fitness function of a power-law form for each controller defined as

\[
f_i(p_i) = \frac{U_B - \| y_{ref} - y_i \|}{p_i}
\]

Here, \( U_B > 0 \) is an upper bound such that \( U_B - \| y_{ref} - y_i \| > 0 \) for all time, which guarantees that \( f_i(p_i) > 0 \), for all \( i \).

Considering (5), (4), and (11), the dynamic weight of each controller is given by,

\[
\dot{p}_i = \beta \left( U_B - \| y_{ref} - y_i \| - p_i \sum_{j=1}^{q} (U_B - \| y_{ref} - y_j \|) \right)
\]

In the next section the weighted control technique applied to a DC-DC step-down converter is presented.

IV. Weighted Control Applied to a Step-down Converter

A. The WCT Technique for the Step-down Converter

The design criteria are based on the requirements presented in Section II, and are of very different nature. The resulting closed loop system should ensure that the control and state constraints are satisfied. Four case studies are used for evaluating the performance of the proposed control approach. These cases represent different scenarios of practical interest and they have been used to see the performance of the control strategies for this benchmark. The case studies are (i) the start-up condition from zero initial conditions; (ii) the response to input voltage variations; (iii) the response to step changes in the output load; and (iv) a critical operation of the controller, the protection of the system against excessive load currents (in this case the load drops from its nominal value to a very small one, \( r_o = 2.5\Omega \), almost creating a short circuit at the output). In the last case, the controller must respect the current limit and lead the output voltage to drop to a level needed in order to keep the current bounded. We then have to select and design two main components for this approach. First, the regulators for different control objectives, considering the case studies we have designed two LQR controller, one related to maintain the output voltage as close to the reference as possible. It can be interpreted as dealing with case studies (i), (ii), and (iii). Another LQR is designed for the special case (iv), when it turns out to be more important to keep the system with the load current bounded despite of the output voltage reference.

1) LQR Designs for Different Operation Regions: Two LQRs are obtained from classical methods. For the first regulator, i.e., an LQR at normal operation, we obtain a linear model of the system with the parameters of normal operation. For this case we use \( r_o = 50\Omega \), \( v_{ref} = 25V \), \( v_o = 50V \),

\[
R_1 = 1, \quad Q_1 = \begin{bmatrix} 1 & 0 \\ 0 & 10 \end{bmatrix}
\]

Thus, the control input would be

\[
u_1 = N_1 v_{ref} - K_1 x = 3.162 v_{ref} - 1.874 x_1 - 3.1 x_2
\]

where, as we stated before \( x_1 = i_l \) and \( x_2 = v_c \).

For the second LQR (i.e., an LQR at critical condition), we obtain a linear model with \( r_o = 2.5\Omega \), \( i_{Lref} = 4A \), \( v_o = 50V \),

\[
R_2 = 1, \quad Q_2 = \begin{bmatrix} 100 & 0 \\ 0 & 1 \end{bmatrix}
\]

This results in a control law of the form,

\[
u = N_2 i_{Lref} - K_2 x = 3.6 i_{Lref} - 9.98 x_1 - 0.1 x_2
\]
Notice that this second controller differs from the first one since we force the system to follow a reference for the inductor current. That is why the inductor current $i_L$ is the most important state variable. As we have mentioned, these two controllers are designed to perform adequately in a certain region. However, the control methodology we are proposing weights each controller to give more importance to the controller more suitable in a certain region. For that, as we said before, a fitness function should be selected in order to improve the global performance of the converter.

2) Fitness Selection: Given the dynamics expressed in Equations (1), (2), and (3), we need a controller that, depending on the system response, can accomplish the different variations smoothly. Then, when the system response is near to the normal operation point the control $u_1$ should have more weight, i.e., $p_1$ should be bigger than $p_2$. Otherwise, when the system is near to the critical operation point, i.e., when $r_o$ is a small value, the control $u_2$ has more weight, i.e., $p_2$ is now bigger than $p_1$. We have used Definition 2 and Proposition 3 to construct a fitness function that accomplishes the two different control objectives, which is based on the error between the reference and the output (see Equation (11)). For the first control objective case we have

$$f_1(p_1, v_{ref}) = \frac{28 - \|v_{ref} - v_o\|}{p_1}$$

where $U_{B}$ is chosen such a way that the fitness is always positive. For the second control objective case we have the fitness function defined by

$$f_2(p_2, i_{L,ref}) = \frac{28 - \|i_{L,ref} - i_L\|}{p_2}$$

where $p_2 + p_1 = 1$. We can now state the weight control law as

$$u = \begin{cases} 
  p_1 u_1 + p_2 u_2 \\
  p_1 (3.162 r_{ref} - 1.874 x_1 - 3.1 x_2) + \\
  p_2 (3.6 i_{L,ref} - 9.98 x_1 - 0.1 x_2) \\
  \beta p_1 (f_1 - (p_1 f_1 + p_2 f_2)) \\
  \beta p_2 (f_2 - (p_1 f_1 + p_2 f_2)) 
\end{cases}$$

(13)

The parameter $\beta$, in the replicator dynamics equation, corresponds to a scaling factor that affects the convergence velocity of the weights assigned to each controller. For the simulations shown next, this value is chosen to be $\beta = 10$.

B. Simulation Results

As we have mentioned above, four different cases are considered. In general we have observed that a good value for the initial conditions of the weights is 0.5 for both, which can be interpreted as follows. If we choose a initial conditions near to $p_1$ the controller $u_1$ has more weight from the beginning and the response is closer to the response when controller $u_1$ is applied alone, which implies more overshoot and more settling time. On the other hand, when the weights are started with the same initial conditions, the second controller improve the transient response of the global system.

- Start-up: This case corresponds to starting-up the converter from zero initial conditions. During this start-up the inductor current must respect the constraint. The initial state is given by $x(0) = [0 \ 0]$. The initial condition for the weights is $p(0) = [0.5 \ 0.5]$. The simulation results for this case are shown in Fig. 3 and 4. Fig. 3 shows the weights for the two controllers, starting from its initial condition. $p_1$ converges to a value near to 1, while $p_2$ goes to a small value near to zero. It means that for normal operation the first controller has better performance than the second one, obtained by means of the replicator dynamics. Fig.4 shows the output voltage of the system stabilizing to the reference value without any overshoot and the inductor current constraint is largely respected, and has very small oscillations. However, the traditional LQR controller, tuned around the normal operation set-up, shows an overshoot around 50%, decreasing the quality of the response.

- Input Voltage Variation: At steady state a step change in the input voltage from $v_s = 50V$ to $v_s = 90V$ is applied with $v_{ref} = 25V$. The simulations results show that there is no significant variations from the steady state of the output voltage with respect to the reference. As it can be seen, the duty cycle takes the appropriate value to keep the output voltage in the reference, which can be seen as a sort of robustness. In terms of the the weights, $p_1$ is closer to 1, while $p_2$ is near to zero as it can be seen in Fig. 5.

- Output Load Variation: At steady state the load drops from its nominal value to $r_o = 25\Omega$ around $t = 0.02s$. As it is shown in Fig. 9, there is a small variation in the response of the system to a load resistance drop. We can observe that the weight of the second controller $p_2$ increases its value, while $p_1$ decreases in order to regulate the output voltage (see Fig. 7).

- Almost Short Circuit: Starting from steady state the load drops from its nominal value to $r_o = 2.5\Omega$, almost creating a short circuit at the output. The controller must respect the current limit and drive the output voltage to drop to the level that is needed in order to keep the current bounded. The weights for the worst-case scenario...
are shown in Fig. 8. In this case, the LQR designed for the critical condition acts with more proportion than the other, i.e., $p_2$ is now greater than $p_1$. However, the output voltage reduces its value to keep the current bounded, see Fig. 9. The response for the traditional LQR presents an overshoot in the current, and then it is settled at a stable value greater than the WTC technique.
V. CONCLUSIONS

A simple weighted controller technique based on replicator dynamics and classical linear quadratic regulator has been introduced. Simulation results, through four case studies demonstrate the effectiveness of our weighted control technique in a DC-DC step-down converter, which is widely known to be a challenging control problem. In this work, we have assumed that all states and parameters are available. However, this assumption is not entirely realistic, and one of the future directions of this work would be the estimation and relaxation of this issue.

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