Abstract—In this paper we analyze the performance of multiple-input multiple-output (MIMO) multiplexing for BPSK and square $M$-QAM under Rayleigh fading. We derive exact closed-form expressions of the bit error rate (BER) assuming channel prediction error. Our analysis indicates that prediction error degrades MIMO multiplexing BER much more than MIMO beamforming BER. This is not surprising since prediction error in MIMO multiplexing leads to eigenchannel interference, which has a much greater impact on BER than the imperfect beamforming coefficients that result from such errors. Our numerical results also reveal that non-uniform rate allocation between ordered eigenchannels dramatically improves the BER.

I. INTRODUCTION

In MIMO systems, when perfect channel state information (CSI) is available at both the transmitter and the receiver, the channel can be decoupled into independent single-input single-output (SISO) channels without co-channel interference among the different data streams [1]. The CSI is estimated by the receiver and sent back to the transmitter. At the receiver, accurate CSI may be obtained through a combination of channel prediction and interpolation, and the accuracy increases as the estimation time grows. However, in transmission real-time CSI estimation is required, and this stringent delay constraint can lead to poor estimates. Imperfect CSI at the transmitter causes interference among the different SISO channels due to imperfect channel decomposition, which degrades system performance.

There has been much effort to derive expressions for bit error rate (BER) of MIMO systems due to imperfect CSI [2]–[7], mainly for adaptive modulation. However, all of these BER calculations rely on approximations, usually starting from exponential type bounds.

In this paper, exact closed-form BER expressions of MIMO multiplexing under imperfect channel prediction errors are derived. A fixed modulation scheme and transmit power is considered. The expressions obtained are valid for BPSK and square $M$-QAM assuming Rayleigh fading channels.

The remainder of this paper is organized as follows. Section II describes the system model. In Section III the BER expressions are derived. Section IV presents numerical results based on our analysis as well as simulation results which closely match the analytical ones. Finally, conclusions are provided in Section V.

II. SYSTEM MODEL

The system model for MIMO multiplexing is depicted in Figure 1. The following channel model is assumed. We consider $N_T$ transmit antennas and $N_R$ receive antennas, with channel gain represented by an $N_T \times N_T$ complex matrix $\mathbf{H}$, so that each entry $H_{i,j}$ is the channel gain between the $j$th transmit and the $i$th receive antenna. These channel gains exhibit frequency-flat slow fading. The entries $H_{i,j}$ are assumed to be independent identically-distributed (i.i.d.) complex circular symmetric normal random variables (RVs), with zero-mean and unity-variance, i.e. $H_{i,j} \sim \mathcal{CN}(0,1)$, where the symbol $\sim$ means statistically distributed as. Noise is modelled by an additive $N_R$-dimensional vector $\mathbf{z}$, whose entries $z_k$ are i.i.d. complex circularly symmetric normal RVs $\sim \mathcal{CN}(0,N_0)$. The received signal can be expressed as

$$\mathbf{y} = \mathbf{Hx} + \mathbf{z},$$

where $\mathbf{y}$ is the received $N_R$ dimensional complex vector and $\mathbf{x}$ is the transmitted $N_T$ dimensional complex vector. As in [8], our analysis adopts a slowly time-varying channel model in which the channel response remains invariant along the frame interval and the channel response correlation associated with different frame intervals is given by the well-known Jakes’ model [9]. At the receiver, two different channel estimation processes, which will be described later, are employed: prediction and interpolation. Channel prediction is used to obtain a real-time prediction of the channel matrix $\mathbf{\hat{H}}$. The singular value decomposition (SVD) of matrix $\mathbf{\hat{H}}$ yields two unitary matrices, $\mathbf{V}$ and $\mathbf{U}$. The first one is fed back to the transmitter to perform precoding, while the second one is used in the receiver for shaping. Channel interpolation is used to obtain an interpolated channel matrix $\mathbf{\tilde{H}}$ which is used to carry out the gain control.

The two channel estimation processes rely on a MIMO extension of classical pilot symbol assisted modulation (PSAM), reusing the same pilot sequences for both interpolation and prediction. A detailed description of MIMO PSAM can be found in [8]. In short, the data stream is parsed into frames...
of duration $PT_S$ seconds, where $T_S$ is the symbol interval and $P$ is the frame length. A known pilot symbol $s_p$ is inserted into each frame. The pilot symbol is spread out over $N_T$ symbol intervals, since orthogonal signatures are used to decouple the MIMO channel estimation problem into $N_T$ single transmit-antenna problems at each receiver branch. Note that the data rate penalty factor for MIMO channel prediction is $(P - N_T)/P$. Once the pilot symbols are extracted and decoupled, an initial channel estimate $\hat{H}$ is obtained by dividing the pilots by $s_p$. The channel interpolation and channel prediction are obtained by filtering the initial channel estimate $\hat{H}$.

The main difference between prediction and interpolation is that prediction uses causal filtering on a frame-by-frame basis, whereas interpolation results in a more accurate estimate due to its non-causal filtering on a symbol-by-symbol basis. Specifically, for channel interpolation the received signal is due to its non-causal filtering on a symbol-by-symbol basis, whereas interpolation results in a more accurate estimate. Prediction uses causal filtering on a frame-by-frame basis, i.e., $\hat{H}$ prediction are obtained by filtering the initial channel estimate $\hat{H}$.

From the initial channel estimate $\hat{H}$, the receiver performs optimal Wiener FIR filtering to predict the channel response $\hat{H}$. $\tau$ seconds ahead, where $\tau$ is the feedback delay which is a multiple of the frame duration $PT_S$. The channel prediction $\hat{H}$ can be expressed as follows

$$\hat{H} = \hat{H} + \hat{\Xi}, \quad \text{with} \quad \begin{cases} \hat{H}_{i,j} \sim \mathcal{CN}(0, 1 - \chi) \quad \text{i.i.d. RVs} \\ \hat{\Xi}_{i,j} \sim \mathcal{CN}(0, \chi) \quad \text{i.i.d. RVs} \end{cases},$$ (2)

where $\hat{\Xi}$ is the prediction error matrix and $\chi$ the minimum mean square error (MMSE) which includes the global effect of the PSAM prediction subsystem. The MMSE orthogonality principle guarantees that matrix entries $\hat{H}_{i,j}$ and $\hat{\Xi}_{i,j}$ are uncorrelated. The optimal Wiener FIR prediction yields

$$\chi = \chi(\hat{\gamma}_P; \mathcal{P}) = 1 - w(\mathcal{P})^T(W(\mathcal{P}) + \frac{1}{\hat{\gamma}_P})^{-1}w(\mathcal{P}),$$ (3)

where the elements of the matrix $W$ ($F \times F$-dimensional) are $W_{i,j} = J_0(2\pi(T_D|i - j|))$ ($F \times F$-dimensional) and the elements of the vector $w$ ($F$-dimensional) are $w_{i,j} = J_0(2\pi(T_D|i - j| + \tau_D))$ ($F$-dimensional). Note that $\gamma$ depends on the pilot symbols SNR $\hat{\gamma}_P$ and a certain set $\mathcal{P}$ of other prediction parameters. Constant power is usually employed for pilot symbols, thus, their SNR $\hat{\gamma}_P$ is strongly linked to the average channel SNR. The remaining prediction parameters are $\mathcal{P} = \{F, T_D, \tau_D\}$, where $F$ is the number of filter taps, $T_D = f_DT_S$ $P$ is the normalized signaling interval ($f_D$ is the Doppler spread) and $\tau_D = f_D\tau$ is the normalized adaptation delay.

Transmit and receiver MIMO processing based on SVD are briefly described below [1]. The predicted channel matrix is decomposed as $\hat{H} = U\Sigma V^H$, where $\bar{V}$ is the $N_T \times N_T$ unitary matrix used for transmit precoding, $U$ is the $N_R \times N_R$ unitary matrix required for receiver shaping and $\Sigma$ is an $N_R \times N_T$ diagonal matrix of singular values $\{\sqrt{\lambda_i}\}$. The MIMO multiplexing system is characterized by the $m$-dimensional vector $\hat{\lambda} = (\lambda_i)$ of eigenvalues of $\hat{H}^H \hat{H}$ with $m = \min\{N_T, N_R\}$ representing the equivalent number of parallel SISO channels. The eigenvalues $\lambda_i$ are the $m$ predicted eigenchannel power gains. The uncorrelated binary data from the input is multiplexed and mapped onto an $N_T$-dimensional vector $z$.
at the transmitter, then the \( N_T \)-dimensional vector \( \mathbf{x} = \mathbf{V} \mathbf{z} \) is sent across \( N_T \) antennas. At the receiver, the vector \( \mathbf{r} = \mathbf{U}^H \mathbf{y} \) is detected and demultiplexed to deliver the output data stream, i.e.,

\[
\mathbf{r} = \mathbf{U}^H [\hat{\mathbf{H}} + \hat{\mathbf{Z}}] \mathbf{V} \mathbf{z} + \mathbf{U}^H \mathbf{z} = \hat{\mathbf{z}} + \Psi \mathbf{z} + \mathbf{c}'
\]

(4)

where \( \Psi = \mathbf{U}^H \hat{\mathbf{Z}} \mathbf{V} \) is a complex normal matrix whose elements \( \Psi_{ij} \) are RVs \( \sim CN(0, \chi) \), since \( \mathbf{U} \) and \( \mathbf{V} \) are unitary matrices; and \( \mathbf{c}' \) is a complex normal vector whose elements \( \mathbf{c}'_k \) are RVs \( \sim CN(0, N_t) \). The value \( r_k \) detected at each SISO channel is then given by

\[
r_k = (\sqrt{\lambda_k} + \Psi_{kk}) z_k + \sum_{i\neq k} \Psi_{ik} z_i + \mathbf{c}'_k.
\]

(5)

It can be seen in 5 that the received symbol is perturbed by a noise term \( \mathbf{c}'_k \) and an interference term. The latter is due to the error in the predicted channel matrix \( \hat{\mathbf{H}} \), which causes the rest of the symbols to interfere in channel \( k \). Each SISO channel can use a different signal constellation or may not be used at all. The total energy \( E_S \) is uniformly distributed along the eigenchannels in use, each one with \( E_{S|k} \) energy.

The gain control (GC) needs to estimate the gain of each channel, \( g_k = \sqrt{\lambda_k} + \Psi_{kk} \), which is given by the diagonal elements of \( \mathbf{U}^H \mathbf{H} \mathbf{V} \). Since perfect channel interpolation is assumed, the GC performs perfect gain estimation. So, the input at the detector will be

\[
r'_k = \frac{r_k g_k^*}{|g_k|^2} = z_k + \mathbf{c}'_k
\]

(6)

where

\[
\mathbf{c}'_k = \frac{g_k^*}{|g_k|^2} \left( \sum_{i\neq k} \Psi_{ik} z_i + \mathbf{c}'_k \right)
\]

is the resultant noise. It is easy to show that the pdf of the resultant noise has two important properties: the real and imaginary parts have the same pdf, which is an even function.

We consider BPSK and square \( M \)-QAM modulations with different bits maps to the in-phase (I) and quadrature (Q) components. We assume the set of complex symbols of the signal constellation are defined as \( \{s_{u_k}, v_k\} = (2u_k - L_k - 1)d + j(2v_k - L_k - 1)d \}_{u_k, v_k = 1, ..., L_k} \), where \( 2d \) is the minimum distance between symbols and \( L_k \) represents the number of symbols for the I and Q components. Thus, the set of decision boundaries for the I and Q components are \( \{B_I(n) = (2n - L_k)d\}_{n=1, ..., L_k-1} \) and \( \{B_Q(n) = (2n - L_k)d\}_{n=1, ..., L_k-1} \), respectively.

III. BER ANALYSIS

The BER analysis presented in this section considers BPSK and square \( M \)-QAM with independent bit-mapping for the in-phase and quadrature components, e.g., Gray mapping. First, in Section III-A we present a general formulation of the BER. In Section III-B we derive, for our system model, the BER conditioned on a predicted eigenchannel power gain \( \hat{\lambda}_k \) for each SISO channel \( k \). Finally, in Section III-C, using the results of Section III-B, we obtain the final exact closed-form average BER expression by averaging over all predicted eigenchannel powers and all the eigenchannels.

A. Exact BER Formulation

The BER analysis can be significantly simplified in the same way as in [11] and [12]. This is possible due to the previously-mentioned fact that the real and imaginary parts of the noise expressions in (7) have the same distribution which is an even function. Using this fact, the BER for each SISO channel \( k \) can be expressed as

\[
BER_k = \sum_{n=1}^{L_k-1} \omega_k(n) \mathcal{I}^+_k(n)
\]

(8)

where \( \mathcal{I}^+_k(n) = \Pr\{|\mathbf{c}'_{k}^*| > D_n\} \) is the component of error probability (CEP), the coefficients \( \omega_k \) are calculated as in [11] or [13] and \( D_n = (2n - 1)d \) is the distance between the symbols \( s_{u_k}, v_k \) and the decision boundary \( B_I(n + n - 1) \). On the other hand, the distance \( D_n \) can be expressed as

\[
D_n = \sqrt{\kappa_n E_S}
\]

(10)

where

\[
\kappa_n = \begin{cases} 
(2n - 1)^2, & \text{for BPSK} \\
\frac{3(2n - 1)^2}{2L_k^2 - 1}, & \text{for M-QAM}.
\end{cases}
\]

(11)

B. Conditional BER Analysis

In this section we derive, for our system model, the expression of the BER conditioned on a predicted eigenchannel power gain \( \hat{\lambda}_k \) for each SISO channel \( k \). This conditional BER (CBER) is obtained using (8) as

\[
CBER_k(\hat{\lambda}_k) = \sum_{n=1}^{L_k-1} \omega_k(n) \mathcal{I}^+_k(n; \hat{\lambda}_k, s_k)
\]

(12)

where \( \mathcal{I}^+_k(n; \hat{\lambda}_k, s_k) \) is the CEP conditioned on a predicted eigenchannel power gain \( \hat{\lambda}_k \) and a set of interference symbols \( s_k = \{s_{u_k, v_k}\}_{i\neq k} \) and \( \mathcal{S}_k \) is the set of all possible interference symbols vectors \( s_k \). From equation (9), the conditional CEP (CCEP) is given by

\[
\mathcal{I}^+_k(n; \hat{\lambda}_k, s_k) = \Pr\{|\mathbf{c}'_{k}^*| > D_n\mid z = [s_{u_k, v_k}, s_k], \hat{\lambda}_k\}
\]

(13)

In order to evaluate the CCEP we use Proakis’ analysis of complex Gaussian quadratic forms [14]. Specifically, we adopt the compact expressions presented in [15]. According to our system model, when the symbol \( z_k = s_{u_k, v_k} \) is transmitted, the received symbol \( r_k \) and the interpolated channel gain \( g_k \) (which is assumed perfect) are given by

\[
r_k = (\sqrt{\lambda_k} + \Psi_{kk}) s_{u_k, v_k} + \sum_{i\neq k} \Psi_{ik} s_{u_i, v_i} + \mathbf{c}'_k
\]

(14)

\[
g_k = \sqrt{\lambda_k} + \Psi_{kk}
\]

(15)
where both \( r_k \) and \( g_k \) are complex normal variables conditioned on the predicted eigenchannel power gain \( \lambda_k \) and the set of interference symbols \( s_k \). Note that since \( \sqrt{\lambda_k} \) is predicted using a Wiener filter, it is orthogonal to the channel gain error \( \Psi_k \), and, therefore, the probability distribution of \( \Psi_k \) is independent of the \( \sqrt{\lambda_k} \) value. Defining the quadratic form \( D = x^H Q x \), for

\[
\mathbf{x} = \begin{bmatrix} r_k \\ g_k \end{bmatrix}, \quad \mathbf{Q} = \begin{bmatrix} 0 & -1/2 \\ -1/2 & B_f (u + n - 1) \end{bmatrix}
\]

(16) in a similar way to the quadratic form defined in [16], we have

\[
\begin{align*}
\text{Pr} \{ D < 0 \} &= \text{Pr} \{ \mathbf{R} \mathbf{r}_k \} > B_f (u + n - 1) | \mathbf{z} = [s_{u_k,v_k}, \mathbf{s}_k], \lambda_k \\
&= \text{Pr} \{ \mathbf{R} \mathbf{r}_k \} > \Delta_n | \mathbf{z} = [s_{u_k,v_k}, \mathbf{s}_k], \lambda_k \\
&= \mathcal{I}_k^+(n; \lambda_k, \mathbf{s}_k).
\end{align*}
\]

(17)

Now we can calculate the CCEP using Proakis' analysis of complex Gaussian quadratic forms [14]. This yields

\[
\mathcal{I}_k^+(n; \lambda_k, \mathbf{s}_k) = \text{Pr} \{ D < 0 \} = Q_1(a, b) - \frac{\eta}{1 + \eta} I_0(ab) \exp \left\{ -\frac{1}{2} (a^2 + b^2) \right\},
\]

(18)

where \( Q_1 \) is the first order Marcum-Q function, and \( I_0 \) is the zero order modified Bessel function of the first kind. The parameters \( a, b, \eta \) and \( C_p \) can be obtained using the expressions that appear in [14, pag. 944-947] or the following alternative expressions proposed in [15]:

\[
\eta = \left| \frac{\delta_1}{\delta_2} \right|
\]

(19)

\[
a = \sqrt{2\delta_2} \left( \frac{\mathbf{m}^H (\mathbf{Q} - \delta_1 \mathbf{R}^{-1} \mathbf{m})}{(\delta_1 - \delta_2)^2} \right),
\]

(20)

\[
b = \sqrt{2\delta_1} \left( \frac{\mathbf{m}^H (\mathbf{Q} - \delta_2 \mathbf{R}^{-1} \mathbf{m})}{(\delta_1 - \delta_2)^2} \right),
\]

(21)

\[
\mathbf{m} = E(\mathbf{x}) = \begin{bmatrix} \sqrt{\lambda_k} s_{u_k,v_k} \\ \sqrt{\lambda_k} s_{u_k,v_k} \end{bmatrix}
\]

(22)

and

\[
\mathbf{R} = E(\{ (\mathbf{x} - \mathbf{m})(\mathbf{x} - \mathbf{m})^H \})
\]

\[
= \begin{bmatrix}
\chi |s_{u_k,v_k}|^2 + ||s_k||^2 + N_0 & \chi s_{u_k,v_k} \\
\chi s_{u_k,v_k}^* & \chi
\end{bmatrix}
\]

(23)

are the mean and the covariance matrix of \( \mathbf{x} \), respectively; \( \delta_1 \) and \( \delta_2 \) are the eigenvalues of the matrix \( \mathbf{R} \mathbf{Q} \), and \( \delta_1 \geq \delta_2 \) by definition. After some algebra, we can calculate parameters \( a, b \) and \( \eta \) as

\[
a = \hat{a}(s_k, n) \sqrt{\lambda_k}
\]

(24)

\[
b = \hat{b}(s_k, n) \sqrt{\lambda_k}
\]

(24)

\[
\eta = \eta(s_k, n) = 1 + 2\gamma s_k \kappa_n + 2\sqrt{\gamma s_k \kappa_n (1 + \gamma s_k \kappa_n)}
\]

\[
\hat{a}(s_k, n) = \sqrt{\frac{1 + 2\gamma s_k \kappa_n - 2\sqrt{\gamma s_k \kappa_n (1 + \gamma s_k \kappa_n)}}{2(1 + \gamma s_k \kappa_n)\chi}}
\]

(25)

\[
\hat{b}(s_k, n) = \sqrt{\frac{1 + 2\gamma s_k \kappa_n + 2\sqrt{\gamma s_k \kappa_n (1 + \gamma s_k \kappa_n)}}{2(1 + \gamma s_k \kappa_n)\chi}}.
\]

In (24)-(25) \( \gamma s_k \) is the average signal to noise and interference ratio of the eigenchannel \( k \), defined as

\[
\gamma s_k = \frac{E_{S_k}}{N_0 + \chi ||s_k||^2} = \frac{1}{\gamma_k^{-1} + \chi ||s_k||^2}
\]

(26)

where \( \gamma_k = \frac{E_{S_k}}{N_0} \) is the average signal to noise ratio of any eigenchannel \( k \). Substituting parameters \( a, b \) and \( \eta \) into expression (18), the CCEP can be obtained; then, by substitution in (12), the CBER is expressed as

\[
CBER_k(\lambda_k) = \sum_{n=1}^{L_k-1} \sum_{s_k \in S_k} \omega_k(n)
\]

\[
\times \left[ Q_1(\hat{a}(s_k, n)\sqrt{\lambda_k}, \hat{b}(s_k, n)\sqrt{\lambda_k}) - \frac{\eta(s_k, n)}{1 + \eta(s_k, n)} \right]
\]

\[
\times I_0(\hat{a}(s_k, n)\hat{b}(s_k, n)\lambda_k)
\]

\[
\times \exp \left\{ -\frac{\lambda_k}{2} (\hat{a}(s_k, n)^2 + \hat{b}(s_k, n)^2) \right\}
\]

(27)

In the absence of prediction error, i.e. \( \Psi_k = 0 \), there is no interference among eigenchannels, and the resultant noise \( \zeta_k' \), given in (7), conditioned on \( \lambda_k \), is a Gaussian variable whose variance is \( N_0/\lambda_k \). Thus, the calculation of CCEP is equivalent to the calculation for a standard Gaussian noise channel, i.e. \( \mathcal{I}_k^+(n; \lambda_k) = Q \sqrt{2\lambda_k \kappa_n \gamma_k} \), where \( Q \) is the Gaussian Q-function. This yields the CBER, conditioned on \( \lambda_k \), as

\[
CBER(\lambda_k) = \sum_{n=1}^{L_k-1} \omega_k(n) Q \left( \sqrt{2\lambda_k \kappa_n \gamma_k} \right)
\]

(28)

C. Exact closed-form BER expressions

In this section we derive the BER expression assuming each eigenchannel can use a different signal constellation and corresponding bit rate \( R_k \). An eigenchannel may also not be used at all. The BER for each channel can be computed by averaging the CBER over the predicted eigenchannel power gain \( \lambda_k \), yielding

\[
BER = \frac{1}{R_T} \sum_{k=1}^{m} R_k \int_0^\infty CBER_k(\lambda_k) p(\lambda_k) d\lambda_k,
\]

(29)

where \( R_T = \sum_{k=1}^{m} R_k \). Assuming ordered eigenchannel gains, i.e., \( \lambda_1 > \lambda_2 \ldots > \lambda_m \), the probability density function
(pdf) of any \( \tilde{\lambda}_k \) is given in [17, eq. (13)]. For convenience, we express this pdf in a more tractable form as follows,

\[
p_u(\tilde{\lambda}_k) = \sum_{t=t_1}^{t_2} \sum_{q=q_1}^{q_2} B_k(t, q) \frac{\tilde{\lambda}_k^q}{(1-\gamma)^{q+1}} \exp\left(-\frac{t \tilde{\lambda}_k}{1-\gamma}\right),
\]

(30)

where the integer values \( t_1, t_2, q_1, q_2 \) and the fractional coefficients \( B_k \) are obtained by identifying the terms of the expression given in [17, eq. (13)]. In the case of the first eigenchannel gain \( \tilde{\lambda}_1 \), the integer values are \( t_1 = 1, t_2 = m, q_1 = N_d \) and \( q_2 = (N_d + 2(m-t))t \) with \( N_d \triangleq |N_T - N_R| \) [18]. When all eigenchannels use the same constellation, the expression (29) can be simplified as

\[
\text{BER} = \int_0^{\infty} CBER(\tilde{\lambda}_1)p_u(\tilde{\lambda}_1)d\tilde{\lambda}_1
\]

(31)

where \( p_u(\tilde{\lambda}_1) \) is the pdf of any unordered eigenchannel gain. This pdf has the same form as (30), but with different fractional coefficients \( B_k \) and, according to [5] and [19], with integer values \( t_1 = t_2 = 1, q_1 = N_d \) and \( q_2 = (N_d + 2(m-t))t \).

Using (27) and (30) in (29) we get

\[
\text{BER} = \sum_{k=1}^{m} \sum_{n=1}^{L_k-1} \sum_{t=t_1}^{t_2} \sum_{q=q_1}^{q_2} \frac{R_k \omega_k(n) B_k(t, q)}{R_T} \times \int_0^{\infty} Q_1(a\sqrt{\lambda_k}, b\sqrt{\lambda_k}) \left(\frac{\lambda_k}{1-\gamma}\right)^q \exp\left(-\frac{t \lambda_k}{1-\gamma}\right) d\lambda_k
\]

(32)

In order to simplify the notation, the dependence on \( s_k \) and \( n \) of parameters \( a, b, \eta \) has been omitted. Both integrals in (32) can be exactly calculated by using the results of [11, Appendix III], yielding the exact closed-form BER expression

\[
\text{BER} = \sum_{k=1}^{m} \sum_{n=1}^{L_k-1} \sum_{t=t_1}^{t_2} \sum_{q=q_1}^{q_2} \frac{R_k \omega_k(n) B_k(t, q)q!}{R_T (t+1) q!} \times \left\{1 + \frac{1}{2} \left[1 - \frac{\eta}{1 - \frac{\eta}{\rho}}\right] \sum_{l=0}^{q} \frac{2^l - \eta}{2^l - 1} \left[\frac{\rho}{\rho^2 - 1}\right]^l \left[\frac{1 + l + 1}{2l + 1}\right] - \left[\frac{P_{l+1}}{\rho^2 - 1}\right] - \left[\frac{P_l}{\rho^2 - 1}\right] \right\}
\]

(33)

where \( P_l(\cdot) \) are Legendre polynomials,

\[
\rho = \rho(s_k, n, t) = 1 + 2\kappa_n \tilde{\gamma}_s \chi + 2t \frac{\chi}{1-\chi} (1 + \kappa_n \tilde{\gamma}_s \chi) + \eta
\]

(34)

and \( \eta \) was defined in (24). Note that since \( q \) and \( l \) are non-negative integers, the associated Legendre polynomial \( P_l(\cdot) \) can be expressed as a finite sum of elementary functions [11, Appendix III] and thus the expression (33) is an exact closed-form.

Under perfect CSI, we substitute (28) into (29) to obtain

\[
\text{BER} = \sum_{k=1}^{m} \sum_{l=1}^{L_k-1} \sum_{t=t_1}^{t_2} \sum_{q=q_1}^{q_2} \frac{R_k \omega_k(n) B_k(t, q)}{R_T} \times \int_0^{\infty} Q_1(\sqrt{2\lambda_k \kappa_n \tilde{\gamma}_s}) \lambda_k^q \exp\left(-t \lambda_k\right) d\lambda_k
\]

(35)

\[
= \sum_{k=1}^{m} \sum_{l=1}^{L_k-1} \sum_{t=t_1}^{t_2} \sum_{q=q_1}^{q_2} \frac{R_k \omega_k(n) B_k(t, q)}{R_T (2t+1) q!} \times \left[1 - \sqrt{\frac{\kappa_n \tilde{\gamma}_s}{t + \kappa_n \tilde{\gamma}_s}} \sum_{l=0}^{q} \frac{t}{l} \left(\frac{1}{4 + t + \kappa_n \tilde{\gamma}_s}\right)^l\right]
\]

IV. NUMERICAL RESULTS

Figure 2 shows the BER in terms of the average SNR \( \bar{\gamma} \) for different parameterizations of eigenchannel constellations, channel prediction and \( 2 \times 2 \) MIMO channels. The predicted MMSE \( \chi \) is calculated with \( \tilde{\gamma}_p = \gamma, f_D T_S = 1/1000 \) and pilot insertion interval \( P = 64 \), i.e. \( T_D = 64 \cdot 10^{-3} \). For example, this set of parameters corresponds to a 3 GHz wireless system with \( 1/T_S = 100 \) kHz and terminal speed \( v = 36 \) km/h. The predicted adaptation delay has been set to \( \tau_D = T_D \). We consider perfect CSI, using equation (35), and imperfect CSI with a 16-tap prediction filter, computed using the equation (33). Moreover, simulation results are also presented for the case of imperfect CSI with the same number of prediction filter taps. The bit rate is 8 bits per symbol interval, but distributed over ordered eigenchannels in different ways. Furthermore, the results obtained for the same parameters when beamforming is used instead of multiplexing is also shown using the BER expressions derived in [11] and [12]. The figure indicates that better results are achieved when the first eigenchannel, the one with the greatest power gain, uses a larger constellation than the second eigenchannel. We also see that beamforming achieves a significantly lower BER than multiplexing due to its eigenchannel interference from the imperfect CSI. These results are consistent with previous ones obtained based on approximate expressions [5]. The figure also indicates how multiplexing does not always obtain better results than beamforming in the perfect CSI case, due to the low gains achieved in the second eigenchannel.

Figure 3 presents BER curves for \( 4 \times 4 \) MIMO channels with the same parameters of the previous figure. We see from this figure that the distribution of the bit rates along the different eigenchannels significantly changes the BER results. The worst performance results when all 4 eigenchannels use a...
show how the impact of imperfect CSI on the BER is greater in MIMO multiplexing than in MIMO beamforming due to eigenchannel interference. Our BER results are consistent with previous studies based on approximate expressions. Furthermore, the BER is very dependent on the rate allocation between ordered eigenchannels. In particular, it is better to use higher rates in the eigenchannels with higher power gains.

ACKNOWLEDGEMENT

This work is partially supported by the Spanish Government under project TEC2007-67289/TCM and by AT4 wireless. The work of A. Goldsmith is supported by LG Electronics.

REFERENCES