Risk Analysis of Combustion System Using Vague Ranking Method

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Abstract

A new approach for vague risk analysis based on the ranking of trapezoidal vague sets is proposed. Firstly, a new method for ranking of vague sets is presented. Then, the proposed method is applied to develop a new method for dealing with vague risk analysis problems. This analysis helps us to find out the probability of failure of each component of combustion system, which could be used for managerial decision making and future system maintenance strategy. The proposed method provides a useful way for handling vague risk analysis problems.

Keywords: Ranking function, Vague sets, Fuzzy sets, Vague risk analysis.

1 Introduction

In past research the fuzzy numbers are usually used for evaluating the values of risk of each sub-components of the system [4, 5, 11, 22]. Schmucker [22] proposed a method for fuzzy risk analysis based on fuzzy number arithmetic operations. Kangari and Riggs [11] proposed a method for constructing risk assessment using linguistic terms. Chen and Chen [4] proposed a method for fuzzy risk analysis based on similarity measures between generalized fuzzy numbers. Chen and Chen [5] presented a method for ranking generalized trapezoidal fuzzy numbers for handling fuzzy risk analysis problems. In recent years, the methods for ranking generalized fuzzy numbers have been extensively researched and used for dealing with fuzzy risk analysis problems. Ranking of fuzzy sets is first proposed by Jain [10]. After word many authors developed several approaches for the ranking fuzzy sets. Ramli and Mohamad [21] give the comprehensive survey of different ranking method.

In the real world there are situations where due to insufficiency in the information available, the evaluation of membership values is not possible up to our satisfaction. Due to the some reason evaluation of non-membership values is not also always possible and consequently there remains a part indeterministic on which hesitation survives therefore to handle these type of situations fuzzy sets theory [25] is not appropriate to deal. A possible solution is to use intuitionistic fuzzy set [2] and vague set [9]. The ranking of vague sets plays a main role in real life problems involving vague decision-making, vague clustering...
ample of a combustion system. The final section makes conclusions.

2 Preliminaries

In this section some basic definitions related to vague sets and arithmetic operations between vague sets are presented.

2.1 Vague sets

Definition 2.1 [9] A vague set \( \tilde{A} = \{(x, \mu_{\tilde{A}}(x)), 1 - \nu_{\tilde{A}}(x)) | x \in X \} \) on the universal set \( X \) is characterized by a truth membership function \( \mu_{\tilde{A}} : X \to [0, 1] \) and a false membership function \( \nu_{\tilde{A}} : X \to [0, 1] \). The values \( \mu_{\tilde{A}}(x) \) and \( \nu_{\tilde{A}}(x) \) represents the degree of membership and degree of non-membership of \( x \) in \( X \) and always satisfies the condition \( \mu_{\tilde{A}}(x) + \nu_{\tilde{A}}(x) \leq 1 \ \forall \ x \in X \). The value \( 1 - \mu_{\tilde{A}}(x) - \nu_{\tilde{A}}(x) \) represents the degree of hesitation of \( x \) in \( \tilde{A} \).

Definition 2.2 [9] A vague set \( \tilde{A} \), defined on the universal set of real numbers \( \mathbb{R} \), denoted as \( \tilde{A} = \{(a, b, c); \delta, \rho) \rangle \), where \( a \leq b \leq c \) and \( \delta \leq \rho \), is said to be a triangle vague set if degree of membership, \( \mu_{\tilde{A}}(x) \), and complement of the degree of non-membership, \( 1 - \nu_{\tilde{A}}(x) \), are given by

\[
\mu_{\tilde{A}}(x) = \begin{cases} \frac{\delta(x-a)}{(b-a)}, & a \leq x < b \\ \frac{\delta(x-c)}{(c-b)}, & b \leq x \leq c \\ 0, & otherwise \end{cases} \quad \text{and} \quad (1 - \nu_{\tilde{A}}(x)) = \begin{cases} \frac{\rho(x-a)}{(b-a)}, & a \leq x < b \\ \frac{\rho(x-c)}{(b-c)}, & b \leq x \leq c \\ 0, & otherwise \end{cases}
\]

where, \( \delta = \supremum\{\mu_{\tilde{A}}(x) : x \in R\} \) and \( \rho = \infimum\{1 - \nu_{\tilde{A}}(x) : x \in R\} \)

2.2 Arithmetic operations

Let \( \tilde{A} = \langle [(a_1, b_1, c_1, d_1); \delta_1, \rho_1) \rangle > \) and \( \tilde{B} = \langle [(a_2, b_2, c_2, d_2); \delta_2, \rho_2) \rangle > \) be two trapezoidal vague sets then

(i) \( \tilde{A} \odot \tilde{B} = \langle [(a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2); \min(\delta_1, \delta_2), \min(\rho_1, \rho_2)] \rangle > \)

(ii) \( \tilde{A} \odot \tilde{B} = \langle [(a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2); \min(\delta_1, \delta_2), \min(\rho_1, \rho_2)] \rangle > \)

(iii) \( \gamma \tilde{A} = \langle [(\gamma a_1, \gamma b_1, \gamma c_1, \gamma d_1); \delta_1, \rho_1) \rangle \rangle \)

(iv) \( \tilde{A} \oplus \tilde{B} = \langle [(a_1 + b_2, b_1 + c_2, c_1 + b_2, d_1 + a_2); \min(\delta_1, \delta_2), \min(\rho_1, \rho_2)] \rangle > \)

In vague sets division, in order to avoid 0, we must limit 0 \( \notin [a_2, d_2] \).

3 A new approach for ranking of vague sets

Let \( \tilde{A} = \langle [(a_1, b_1, c_1, d_1); \delta_1, \rho_1) \rangle > \) and \( \tilde{B} = \langle [(a_2, b_2, c_2, d_2); \delta_2, \rho_2) \rangle > \) be two triangular vague sets, where \( a_1 \leq b_1 \leq c_1 \leq d_1 \), \( \delta_1 \leq \rho_1 \) and \( a_2 \leq b_2 \leq c_2 \leq d_2 \), \( \delta_2 \leq \rho_2 \) then use the following steps to compare \( \tilde{A} \) and \( \tilde{B} \):

Step 1 Transform \( \tilde{A} \), \( \tilde{B} \) into \( \tilde{A}^*, \tilde{B}^* \) as follows:

\[
\tilde{A}^* = \langle [(a_1, b_1, c_1, d_1); \delta, \rho) \rangle >, \tilde{B}^* = \langle [(a_2, b_2, c_2, d_2); \delta, \rho) \rangle >
\]

where, \( \delta = \min(\delta_1, \delta_2) \) and \( \rho = \min(\rho_1, \rho_2) \)

Step 2 Calculate \( \mathcal{R}_{\nu}^\lambda(\tilde{A}^*_\delta) = \frac{1}{\delta} \int (a_1 + (b_1 - a_1) \lambda) dx + \int (1 - \lambda)(d_1 + (c_1 - d_1) \lambda) dx \)

\[
\Rightarrow \mathcal{R}_{\nu}^\lambda(\tilde{A}^*_\delta) = \lambda \delta (\frac{a_1 + b_1}{2}) + (1 - \lambda) \delta (\frac{c_1 + d_1}{2}) \quad (2)
\]

Now \( \mathcal{R}_{\nu}^\lambda(\tilde{A}^*_\rho) = \mathcal{R}_{\nu}^\lambda(\tilde{A}^*_\rho, \delta) \)

\[
= \lambda (\frac{a_1 + b_1}{2}) + (1 - \lambda) (\frac{(b_1 - a_1)}{2} \lambda (\frac{c_1 - b_1}{2}) + (1 - \lambda) (\frac{(c_1 + d_1)}{2} \lambda (\frac{d_1 - c_1}{2}))
\]

Now \( \mathcal{R}_{\nu}^\lambda(\tilde{A}^*_\rho) = \mathcal{R}_{\nu}^\lambda(\tilde{B}^*_\rho) + \mathcal{R}_{\nu}^\lambda(\tilde{B}^*_\rho, \delta) \)

\[
\mathcal{R}_{\nu}^\lambda(\tilde{A}^*) = \mathcal{R}_{\nu}^\lambda(\tilde{B}^*) + \mathcal{R}_{\nu}^\lambda(\tilde{B}^* \delta)
\]

Case (i) If \( \mathcal{R}(\tilde{A}) > \mathcal{R}(\tilde{B}) \) then \( \tilde{A} > \tilde{B} \)

Case (ii) If \( \mathcal{R}(\tilde{A}) < \mathcal{R}(\tilde{B}) \) then \( \tilde{A} < \tilde{B} \)

Case (iii) If \( \mathcal{R}(\tilde{A}) = \mathcal{R}(\tilde{B}) \) then \( \tilde{A} \sim \tilde{B} \)

3.1 Particular cases:

It can be easily shown that the proposed ranking formula for the generalization of existing ranking approaches [1, 16].

Case (i) If \( 0 \leq \delta = \rho \leq 1 \) then (5) reduces to

\[
\mathcal{R}(\tilde{A}) = \frac{1}{2} (\lambda (a_1 + d_1) + (b_1 - a_1 + c_1 - d_1) (\lambda)) = \lambda \frac{(a_1 + b_1 + c_1 + d_1)}{4}
\]

which is the ranking formula for generalized trapezoidal fuzzy numbers [16].
Case (ii) If $0 \leq \delta = \rho \leq 1$ and $b_1 = c_1$ then (5) reduces to
\[ \mathcal{R}(\bar{A}) = \frac{1}{2} \left( \lambda(a_1 + d_1) + \frac{(b_1 - a_1 + b_1 - d_1)}{2}(\lambda) \right) = \frac{\lambda(a_1 + 2b_1 + d_1)}{4} \]
which is the ranking formula for generalized triangular fuzzy numbers [16].

Case (iii) If $0 \leq \delta = \rho = 1$ then (5) reduces to
\[ \mathcal{R}(\bar{A}) = \frac{1}{2} \left( (a_1 + d_1) + \frac{(b_1 - a_1 + b_1 - d_1)}{2} \right) = \frac{(a_1 + 2b_1 + d_1)}{4} \]
which is the ranking formula for normal trapezoidal fuzzy numbers [1, 16].

Case (iv) If $0 \leq \delta = \rho = 1 \leq 1$ and $b_1 = c_1$ then (5) reduces to
\[ \mathcal{R}(\bar{A}) = \frac{1}{2} \left( (a_1 + d_1) + \frac{(b_1 - a_1 + b_1 - d_1)}{2} \right) = \frac{(a_1 + 2b_1 + d_1)}{4} \]
which is the ranking formula for normal triangular fuzzy numbers [1, 16].

4 Validation of proposed ranking function

Some examples are chosen to show that the proposed ranking function satisfies the reasonable properties [23].

Example 4.1 Let $\tilde{A} = (0.1, 0.3, 0.3, 0.5; 0.8)$ and $\tilde{B} = (0.1, 0.3, 0.3, 0.5; 1)$ be two generalized trapezoidal fuzzy numbers then $\bar{A} \odot \tilde{B} = (-0.4, 0, 0, 0.4; 0.8)$ and $\tilde{B} \odot \bar{B} = (-0.4, 0, 0, 0.4; 1)$

Step 1 $\min(0.8, 1) = 0.8$
Step 2 $\mathcal{R}(\bar{A} \odot \tilde{B}) = 0.4(0.4 - 0.8\alpha)$ and $\mathcal{R}(\tilde{B} \odot \bar{B}) = 0.4(0.4 - 0.8\alpha)$

Since $\mathcal{R}(\bar{A} \odot \tilde{B}) = \mathcal{R}(\tilde{B} \odot \bar{B}) \forall \alpha$, so $\bar{A} \odot \tilde{B} \sim \tilde{B} \odot \bar{B}$.

Example 4.2 Let $\tilde{A} = (0.1, 0.3, 0.3, 0.5; 1)$ and $\tilde{B} = (0.3, 0.5, 0.5, 0.7; 1)$ be two generalized trapezoidal fuzzy numbers then $\bar{A} \odot \tilde{B} = (-0.6, -0.2, -0.2, 0.2; 1)$ and $\tilde{B} \odot \bar{B} = (-0.4, 0, 0, 0.4; 1)$

Step 1 $\min(1, 1) = 1$
Step 2 $\mathcal{R}(\bar{A} \odot \tilde{B}) = \frac{1}{2}(-0.8\alpha)$ and $\mathcal{R}(\tilde{B} \odot \bar{B}) = \frac{1}{2}(0.4 - 0.8\alpha)$

For a pessimistic decision maker, with $\alpha = 0$, $\mathcal{R}(\bar{A} \odot \tilde{B}) = 0$ and $\mathcal{R}(\tilde{B} \odot \bar{B}) = 0.2$. Since $\mathcal{R}(\bar{A} \odot \tilde{B}) < \mathcal{R}(\tilde{B} \odot \bar{B})$, so $(\bar{A} \odot \tilde{B}) < (\tilde{B} \odot \bar{B})$.

For optimistic decision maker, with $\alpha = 1$, $\mathcal{R}(\bar{A} \odot \tilde{B}) = -0.4$ and $\mathcal{R}(\tilde{B} \odot \bar{B}) = -0.2$. Since $\mathcal{R}(\bar{A} \odot \tilde{B}) < \mathcal{R}(\tilde{B} \odot \bar{B})$, so $(\bar{A} \odot \tilde{B}) < (\tilde{B} \odot \bar{B})$.

For moderate decision maker, with $\alpha = 0.5$, $\mathcal{R}(\bar{A} \odot \tilde{B}) = -0.2$ and $\mathcal{R}(\tilde{B} \odot \bar{B}) = 0$. Since $\mathcal{R}(\bar{A} \odot \tilde{B}) < \mathcal{R}(\tilde{B} \odot \bar{B})$, so $(\bar{A} \odot \tilde{B}) < (\tilde{B} \odot \bar{B})$.

Example 4.3 Let $\tilde{A} = (-0.5, -0.3, -0.3, -0.1; 1)$ and $\tilde{B} = (0.1, 0.3, 0.3, 0.7; 1)$ be two generalized trapezoidal fuzzy numbers then $\bar{A} \odot \tilde{B} = (-0.6, -0.2, -0.2, 0.2; 1)$ and $\tilde{B} \odot \bar{B} = (-0.4, 0, 0, 0.4; 1)$

Step 1 $\min(1, 1) = 1$
Step 2 $\mathcal{R}(\bar{A} \odot \tilde{B}) = \frac{1}{2}(-0.8 + 2.4\alpha)$ and $\mathcal{R}(\tilde{B} \odot \bar{B}) = \frac{1}{2}(0.4 - 0.8\alpha)$

For a pessimistic decision maker, with $\alpha = 0$, $\mathcal{R}(\bar{A} \odot \tilde{B}) = -0.4$ and $\mathcal{R}(\tilde{B} \odot \bar{B}) = 0.2$. Since $\mathcal{R}(\bar{A} \odot \tilde{B}) < \mathcal{R}(\tilde{B} \odot \bar{B})$, so $(\bar{A} \odot \tilde{B}) < (\tilde{B} \odot \bar{B})$.

For optimistic decision maker, with $\alpha = 1$, $\mathcal{R}(\bar{A} \odot \tilde{B}) = 0.2667$ and $\mathcal{R}(\tilde{B} \odot \bar{B}) = -0.2$. Since $\mathcal{R}(\bar{A} \odot \tilde{B}) > \mathcal{R}(\tilde{B} \odot \bar{B})$, so $(\bar{A} \odot \tilde{B}) > (\tilde{B} \odot \bar{B})$.

For moderate decision maker, with $\alpha = 0.5$, $\mathcal{R}(\bar{A} \odot \tilde{B}) = 0.2$ and $\mathcal{R}(\tilde{B} \odot \bar{B}) = 0$. Since $\mathcal{R}(\bar{A} \odot \tilde{B}) > \mathcal{R}(\tilde{B} \odot \bar{B})$, so $(\bar{A} \odot \tilde{B}) > (\tilde{B} \odot \bar{B})$.

Example 4.4 Let $\tilde{A} = (1, 1, 1, 1; w_1)$ and $\tilde{B} = (1, 1, 1, 1; w_2)$ be two generalized trapezoidal fuzzy numbers.

Step 1 $\min(w_1, w_2) = w$ (say)
Step 2 $\mathcal{R}(\bar{A}) = \frac{w}{2}((\alpha(2) + (1 - \alpha)(2)) = w$ and $\mathcal{R}(\tilde{B}) = \frac{w}{2}((\alpha(2) + (1 - \alpha)(2)) = w$.

Since $\mathcal{R}(\bar{A}) = \mathcal{R}(\tilde{B}) \forall \alpha$, so $\bar{A} \sim \tilde{B}$.

Example 4.5 Let $\tilde{A} = (-4, -2, -1, 7; w_1)$ and $\tilde{B} = (-4, -2, -1, 7; w_2)$ two generalized trapezoidal fuzzy numbers.

Step 1 $\min(w_1, w_2) = w$ (say)
Step 2 $\mathcal{R}(\bar{A}) = \frac{w}{2}(\alpha(-0.6) + (1 - \alpha)(0.6)) = 0.3w$ and $\mathcal{R}(\tilde{B}) = \frac{w}{2}(\alpha(-0.6) + (1 - \alpha)(0.6)) = 0.3w$.

Since $\mathcal{R}(\bar{A}) = \mathcal{R}(\tilde{B}) \forall \alpha$, so $\bar{A} \sim \tilde{B}$.

Example 4.6 Let $\tilde{A} =< (1, 3, 5; 0.3, 0.8) >$ and $\tilde{B} =< (4, 8, 9; 0.4, 0.9) >$ be two triangular vague sets, then $\bar{A} \odot \tilde{B} =< (-8, -5, 1; 0.3, 0.8) >$ and $\tilde{B} \odot \bar{B} =< (-5, 0, 5; 0.4, 0.9) >$

Step 1 $\min(0.3, 0.4) = 0.3$ and $\min(0.8, 0.9) =$
0.8

**Step 2** \( \mathcal{R}(\tilde{A} \odot \tilde{B}) = (-1.037 - 3.881\alpha) \) and \( \mathcal{R}(B \odot B) = (2.47 - 0.94\alpha) \)

For a pessimistic decision maker, with \( \alpha = 0 \),
\( \mathcal{R}(\tilde{A} \odot \tilde{B}) = -1.0375 \) and \( \mathcal{R}(\tilde{B} \odot \tilde{B}) = 2.47 \). Since
\( \mathcal{R}(\tilde{A} \odot \tilde{B}) < \mathcal{R}(\tilde{B} \odot \tilde{B}) \) so \( \tilde{A} \odot \tilde{B} < (B \odot B) \).

For an optimistic decision maker, with \( \alpha = 1 \),
\( \mathcal{R}(\tilde{A} \odot \tilde{B}) = -4.919 \) and \( \mathcal{R}(\tilde{B} \odot \tilde{B}) = 1.53 \). Since
\( \mathcal{R}(\tilde{A} \odot \tilde{B}) < \mathcal{R}(\tilde{B} \odot \tilde{B}) \), so \( \tilde{A} \odot \tilde{B} < (B \odot B) \).

For moderate decision maker, with \( \alpha = 0.5 \),
\( \mathcal{R}(\tilde{A} \odot \tilde{B}) = -2.978 \) and \( \mathcal{R}(\tilde{B} \odot \tilde{B}) = 2 \). Since
\( \mathcal{R}(\tilde{A} \odot \tilde{B}) < \mathcal{R}(\tilde{B} \odot \tilde{B}) \) so \( \tilde{A} \odot \tilde{B} < (B \odot B) \).

**Example 4.7** Let \( \tilde{A} = \left< (-2, 0.2; \lambda, \rho_1) \right> \) and \( B = \left< (-2, 0.2; \lambda, \rho_2) \right> \) be two triangular vague fuzzy sets, then \( A \odot B = \left< (\lambda, \rho_1) \right> \), where \( \lambda = \min(\lambda, \lambda_2), \rho_1 = \min(\rho_1, \rho_2) \) and \( B \odot B = < (-2, 0.2; \lambda, \rho_2) > \)

**Step 1** min \((\lambda_3, \lambda_2) = \lambda \) and min \((\rho_3, \rho_2) = \rho \)

**Step 2** \( \mathcal{R}(\tilde{A} \odot \tilde{B}) = 2\lambda + (\rho - \lambda)(2 + \frac{2\lambda}{\rho}) + \alpha(-4\lambda + (\rho - \lambda)(-4 + 2\lambda)) \) and \( \mathcal{R}(\tilde{B} \odot \tilde{B}) = 2\lambda + (\rho - \lambda)(2 + \frac{2\lambda}{\rho}) + \alpha(-4\lambda + (\rho - \lambda)(-4 + 2\lambda)) \)

Since \( \mathcal{R}(\tilde{A} \odot \tilde{B}) = \mathcal{R}(\tilde{B} \odot \tilde{B}) \) \( \forall \alpha \), so \( \tilde{A} \odot \tilde{B} \sim B \odot B \).

**Example 4.8** Let \( \tilde{A} = < (-8, 1, 4; \lambda_1, \rho_1) > \) and \( B = < (-8, 1, 4; \lambda_2, \rho_2) > \) be two triangular vague fuzzy sets, then \( A \odot B = < (-12, 0, 12; \lambda_3, \rho_3) > \), where \( \lambda_3 = \min(\lambda_1, \lambda_2), \rho_3 = \min(\rho_1, \rho_2) \) and \( B \odot B = < (-12, 0, 12; \lambda_2, \rho_2) > \)

**Step 1** min \((\lambda_3, \lambda_2) = \lambda \) and min \((\rho_3, \rho_2) = \rho \)

**Step 2** \( \mathcal{R}(\tilde{A} \odot \tilde{B}) = 6\lambda + (\rho - \lambda)(6 + \frac{6\lambda}{\rho}) + \alpha(-12\lambda + (\rho - \lambda)(-12 + 6\lambda)) \) and \( \mathcal{R}(\tilde{B} \odot \tilde{B}) = 6\lambda + (\rho - \lambda)(6 + \frac{6\lambda}{\rho}) + \alpha(-12\lambda + (\rho - \lambda)(-12 + 6\lambda)) \)

Since \( \mathcal{R}(\tilde{A} \odot \tilde{B}) = \mathcal{R}(\tilde{B} \odot \tilde{B}) \) \( \forall \alpha \), so \( \tilde{A} \odot \tilde{B} \sim B \odot B \).

**4.1 Comparative study**

In this section, the ranking of different fuzzy sets and vague sets are obtained using the proposed approach and the results are compared with exiting ranking approaches [6, 8, 12, 15, 16, 19, 20, 24].

**Set 1**

Let \( \tilde{A} = (0.1, 0.3, 0.3, 0.5; 0.8) \) and \( B = (0.1, 0.3, 0.3, 0.5; 1) \) be two trapezoidal fuzzy sets then \( \tilde{A} \) and \( B \) can be compared by using the following steps:

**Step 1** Using equation (1), \( \tilde{A}^* = (0.1, 0.3, 0.3, 0.5; 0.8) \) and \( B^* = (0.1, 0.3, 0.3, 0.5; 0.8) \)

**Step 2** Using equation (4), \( \mathcal{R}^{0.5}(\tilde{A}^*) = 0.24 \) and \( \mathcal{R}^{0.5}(\tilde{B}^*) = 0.24 \).

Since \( \mathcal{R}^{0.5}(\tilde{A}^*) = \mathcal{R}^{0.5}(\tilde{B}^*) \), so \( \tilde{A} \sim B \)

**Set 2**

Let \( \tilde{A} = (0.1, 0.3, 0.3, 0.5; 1) \) and \( B = (0.3, 0.5, 0.5, 0.7; 1) \) be two trapezoidal fuzzy sets

**Step 1** Using equation (1), \( \tilde{A}^* = (0.1, 0.3, 0.3, 0.5; 1) \) and \( B^* = (0.3, 0.5, 0.5, 0.7; 1) \)

**Step 2** Using equation (4), \( \mathcal{R}^{0.5}(\tilde{A}^*) = 0.3 \) and \( \mathcal{R}^{0.5}(\tilde{B}^*) = 0.5 \).

Since \( \mathcal{R}^{0.5}(\tilde{A}^*) < \mathcal{R}^{0.5}(\tilde{B}^*) \), so \( \tilde{A} \sim B \)

**Set 3**

Let \( \tilde{A} = (0.1, 0.3, 0.3, 0.5; 1) \) and \( B = (-0.5, -0.3, -0.3, -0.1; 1) \) be two trapezoidal fuzzy sets

**Step 1** Using equation (1), \( \tilde{A}^* = (0.1, 0.3, 0.3, 0.5; 1) \) and \( B^* = (-0.5, -0.3, -0.3, -0.1; 1) \)

**Step 2** Using equation (4), \( \mathcal{R}^{0.5}(\tilde{A}^*) = 0.3 \) and \( \mathcal{R}^{0.5}(\tilde{B}^*) = -0.3 \).

Since \( \mathcal{R}^{0.5}(\tilde{A}^*) \) \( > \mathcal{R}^{0.5}(\tilde{B}^*) \), so \( \tilde{A} > B \)

**Set 4**

Let \( \tilde{A} = < [(1, 2, 3, 4); 0.5, 0.6] > \) and \( B = \left< [(2, 4, 5, 6); 0.7, 0.9] \right> \) be two trapezoidal vague sets

**Step 1** Using equation (1), \( \tilde{A}^* = \left< [(1, 2, 3, 4); 0.5, 0.6] \right> \) and \( B^* = \left< [(2, 4, 5, 6); 0.7, 0.9] \right> \)
Figure 1: Five different fuzzy sets and vague sets
Step 2 Using equation (4), $\Re_{0.5}^{0.5}(\tilde{A}^*) = 15.42$ and $\Re_{0.5}^{0.5}(\tilde{B}^*) = 2.612$.

Since $\Re_{0.5}^{0.5}(\tilde{A}^*) > \Re_{0.5}^{0.5}(\tilde{B}^*)$, so $\tilde{A} \succ \tilde{B}$

Set 5

Let $\tilde{A} = \left\langle (1, 4, 5, 8); 0.6, 0.7 \right\rangle$ and $\tilde{B} = \left\langle (1, 3, 6, 8); 0.7, 0.8 \right\rangle$ be two trapezoidal vague sets.

Table 1
Comparison of proposed ranking approach with existing ranking approach

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<td>$\tilde{A} \sim \tilde{B}$</td>
<td>$\tilde{A} \sim \tilde{B}$</td>
<td>$\tilde{A} \succ \tilde{B}$</td>
<td>N.A</td>
<td>N.A</td>
</tr>
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<td>Liou and Wang [16]</td>
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<td>$\tilde{A} &lt; \tilde{B}$</td>
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<tr>
<td>Li [15]</td>
<td>$\tilde{A} &lt; \tilde{B}$</td>
<td>$\tilde{A} &lt; \tilde{B}$</td>
<td>$\tilde{A} \succ \tilde{B}$</td>
<td>N.A</td>
<td>N.A</td>
</tr>
<tr>
<td>Nehi [20]</td>
<td>N.A</td>
<td>$\tilde{A} &lt; \tilde{B}$</td>
<td>$\tilde{A} \succ \tilde{B}$</td>
<td>N.A</td>
<td>N.A</td>
</tr>
<tr>
<td>Kumar et al. [12, 13]</td>
<td>$\tilde{A} \sim \tilde{B}$</td>
<td>$\tilde{A} &lt; \tilde{B}$</td>
<td>$\tilde{A} \succ \tilde{B}$</td>
<td>N.A</td>
<td>N.A</td>
</tr>
<tr>
<td>Proposed approach</td>
<td>$\tilde{A} \sim \tilde{B}$</td>
<td>$\tilde{A} &lt; \tilde{B}$</td>
<td>$\tilde{A} \succ \tilde{B}$</td>
<td>$\tilde{A} \succ \tilde{B}$</td>
<td>$\tilde{A} \sim \tilde{B}$</td>
</tr>
</tbody>
</table>

where N.A denotes the not applicable.

5 Fuzzy risk analysis based on the proposed approach

In this analysis, application of proposed ranking method for calculating the rank of trapezoidal vague set is apply to the risk analysis problems. To illustrate the proposed approach given in this paper, a practical example of combustion system is consider. The first step is to construction of the fault tree of a combustion system that allows the definition of the functional/logical links between the equipment subsystems. The different components of a combustion are represented by different symbols that are shown in Table 2. The fault tree of a combustion system is shown in Figure 2, was divided into four main subsystems. Those subsystems are further divided into ten basic components as listed in Table 3, each one performing a specific function in connection with the subsystem main function. A failure in a component at the bottom of the fault tree affects all subsystems above it, causing a possible degradation in the system operation. The fault tree was developed according to the operation manual furnished by the manufacturer.

Table 2
Components of a combustion system
Figure 2: Fault tree of a compressor system

<table>
<thead>
<tr>
<th>Code</th>
<th>Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>Failure of total system</td>
</tr>
<tr>
<td>A</td>
<td>Failure of shell</td>
</tr>
<tr>
<td>B</td>
<td>Failure of igniter system</td>
</tr>
<tr>
<td>C</td>
<td>Failure of cooling system</td>
</tr>
<tr>
<td>D</td>
<td>Failure of basket of the shell</td>
</tr>
</tbody>
</table>

Table 3
Components of a combustion system

<table>
<thead>
<tr>
<th>Code</th>
<th>Component</th>
<th>Code</th>
<th>Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>Failure of flame tube of the shell</td>
<td>C1</td>
<td>Vessels of the cooling system blocked</td>
</tr>
<tr>
<td>A2</td>
<td>Failure of cylinder of the shell</td>
<td>C2</td>
<td>Bypass valve of the cooling system failed to open</td>
</tr>
<tr>
<td>B1</td>
<td>Spring of the igniter system got broken</td>
<td>C3</td>
<td>Control system of the cooling system failed</td>
</tr>
<tr>
<td>B2</td>
<td>Piston of the igniter system got broken</td>
<td>D1</td>
<td>Transition piece of the basket got damaged</td>
</tr>
<tr>
<td>B3</td>
<td>Failure of igniter of the igniter system</td>
<td>D2</td>
<td>Burner of the basket got damaged</td>
</tr>
</tbody>
</table>

Table 4
Linguistic terms and their corresponding vague set
The second step is to apply the vague set theory for evaluating the risk analysis of the combustion system. In this analysis, two evaluating terms \((R)\) and \((W)\) are used, where \((R)\) denotes the probability of failure and \((W)\) denotes the severity of loss. The probability of failure \((R)\) of a system is evaluated as a function of mean time between failure MTBF and severity of loss \((W)\) is assessed by the possible outcome of failure effects on the system i.e. MTTR. Depends upon the values of MTBF and MTTR values of the sub-components, the \((\bar{R}_i)\) and \((\bar{W}_i)\) may be regarded as linguistic terms (i.e. absolute-low, very-low etc) where, \(i = A, B, C,...\) so on and \(j = 1, 2, 3,...\) so on. These linguistic terms are further used to evaluate the vague probability of failure \((\bar{R}_i)\) of each component with respect to its sub-components as proposed in Chen and Chen [4] is shown below:

\[
\bar{R}_i = \frac{\sum_{j=1}^{n} \bar{R}_{ij} \otimes \bar{W}_{ij}}{\sum_{j=1}^{n} \bar{W}_{ij}} \tag{6}
\]

The seven-member linguistic terms set, shown in Table 4, is used for representing the linguistic terms and their corresponding vague sets. The linguistic values of evaluating items \((\bar{R}_{ij})\) and \((\bar{W}_{ij})\) of the sub-components are in shown in Table 5, where \((w_{\bar{R}_{ij}})\) denotes the degree of confidence of the experts opinion with respect to each sub-components of the system. The algorithm, proposed in Chen and Chen [4], is used to evaluate the vague probability of failure \((\bar{R}_i)\) of each components of the combustion system is presented as follow:

**Step 1** Aggregate the \(\bar{R}_{ij}\) and \(\bar{W}_{ij}\) of each sub-component of the fault tree based on vague set arithmetic operations given in sub-section 2.2 and equation (6) into the vague probability of failure \(\bar{R}_i\) of each component, shown as follows:

\[
\bar{R}_A = [\bar{R}_{A1} \otimes \bar{W}_{A1} \oplus \bar{R}_{A2} \otimes \bar{W}_{A2} \oplus \bar{R}_{D1} \otimes \bar{W}_{D1} \oplus \\
\bar{R}_{D2} \otimes \bar{W}_{D2}] \otimes [\bar{W}_{A1} \oplus \bar{W}_{A2} \oplus \bar{W}_{D1} \oplus \bar{W}_{D2}]
\]

**Table 5**

<table>
<thead>
<tr>
<th>Events</th>
<th>Sub components</th>
<th>Linguistic values of the severity of loss</th>
<th>Linguistic values of the probability of failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(A_1)</td>
<td>(W_{A1} = Very) - Low</td>
<td>(R_{A1} = Medium(w_{\bar{R}_{A1}} = 0.4; 0.8))</td>
</tr>
<tr>
<td></td>
<td>(A_2)</td>
<td>(W_{A2} = Medium)</td>
<td>(R_{A2} = High(w_{\bar{R}_{A2}} = 0.35; 0.7))</td>
</tr>
<tr>
<td>D</td>
<td>(D_1)</td>
<td>(W_{D1} = Medium)</td>
<td>(R_{D1} = Very - Low(w_{\bar{R}_{D1}} = 0.4; 0.8))</td>
</tr>
<tr>
<td></td>
<td>(D_2)</td>
<td>(W_{D2} = High)</td>
<td>(R_{D2} = Very - Low(w_{\bar{R}_{D2}} = 0.425; 0.85))</td>
</tr>
<tr>
<td>B</td>
<td>(B_1)</td>
<td>(W_{B1} = Low)</td>
<td>(R_{B1} = Very - Low(w_{\bar{R}_{B1}} = 0.45; 0.9))</td>
</tr>
<tr>
<td></td>
<td>(B_2)</td>
<td>(W_{B2} = High)</td>
<td>(R_{B2} = Very - Low(w_{\bar{R}_{B2}} = 0.425; 0.85))</td>
</tr>
<tr>
<td></td>
<td>(B_3)</td>
<td>(W_{B3} = Medium)</td>
<td>(R_{B3} = High(w_{\bar{R}_{B3}} = 0.45; 0.9))</td>
</tr>
<tr>
<td>C</td>
<td>(C_1)</td>
<td>(W_{C1} = Low)</td>
<td>(R_{C1} = Low(w_{\bar{R}_{C1}} = 0.5; 0.1))</td>
</tr>
<tr>
<td></td>
<td>(C_2)</td>
<td>(W_{C2} = Very) - Low</td>
<td>(R_{C2} = Medium(w_{\bar{R}_{C2}} = 0.4; 0.8))</td>
</tr>
<tr>
<td></td>
<td>(C_3)</td>
<td>(W_{C3} = Very) - High</td>
<td>(R_{C3} = Very - Low(w_{\bar{R}_{C3}} = 0.375; 0.75))</td>
</tr>
<tr>
<td>D</td>
<td>(D_1)</td>
<td>(W_{D1} = Medium)</td>
<td>(R_{D1} = Very - Low(w_{\bar{R}_{D1}} = 0.4; 0.8))</td>
</tr>
<tr>
<td></td>
<td>(D_2)</td>
<td>(W_{D2} = High)</td>
<td>(R_{D2} = Very - Low(w_{\bar{R}_{D2}} = 0.425; 0.85))</td>
</tr>
</tbody>
</table>
Step 2: Using Table 4 and Table 5 the values of probability of failure $\tilde{R}_i$ of each components are:

$$
\tilde{R}_A = [0.10, 0.26, 0.26, 0.58] > 0.35, 0.7 > \\
\tilde{R}_B = [0.11, 0.27, 0.27, 0.58] > 0.45, 0.8 > \\
\tilde{R}_C = [0.049, 0.16, 0.16, 0.37] > 0.375, 0.65 > \\
\tilde{R}_D = [0.02, 0.11, 0.11, 0.23] > 0.4, 0.8 > 
$$

Step 3 Based on Equation (5), calculate the rank of $\Re(\tilde{R}_i)$ of each component of the turbine system. The larger the value of $\Re(\tilde{R}_i)$, the higher the probability of failure of component of a combustion system, shown as follows:

$$
\Re(\tilde{R}_A) = 0.3388; \Re(\tilde{R}_B) = 0.3455; \Re(\tilde{R}_C) = 0.2092; \Re(\tilde{R}_D) = 0.1345;
$$

Because of $\Re(\tilde{R}_B) > \Re(\tilde{R}_A) > \Re(\tilde{R}_C) > \Re(\tilde{R}_D)$, the ranking order of each vague set is $\Re_B > \Re_A > \Re_C > \Re_D$. This concludes that the component (B) has the highest probability of failure, then followed by (A,C,D).

6 Conclusion

In this paper, a new approach is proposed for the ranking of trapezoidal vague sets. The validation of proposed ranking function is also proved. Moreover, we also apply the proposed method to deal with vague risk analysis problems. According to the rank order the Igniter system’s probability of failure is highest which means this system needs more maintenance attention followed by Shell, Cooling system, basket of the shell. The proposed method provides a useful way for handling vague risk analysis problems.

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