Numerical solution of the coupled viscous Burgers equations by Chebyshev–Legendre Pseudo-Spectral method

Abdur Rashid a,*, Muhammad Abbas b,*, Ahmad Izani Md. Ismail c, Ahmad Abd Majid c

a Department of Mathematics, Gomal University, 29100 Dera Ismail Khan, Pakistan
b Department of Mathematics, University of Sargodha, 40100 Sargodha, Pakistan
c School of Mathematical Sciences, Universiti Sains Malaysia, 11800 USM Penang, Malaysia

Abstract

In this paper, we consider Chebyshev–Legendre Pseudo-Spectral (CLPS) method for solving coupled viscous Burgers (VB) equations. A leapfrog scheme is used in time direction, while CLPS method is used for space direction. Chebyshev–Gauss–Lobatto (CGL) nodes are used for practical computation. The error estimates of semi-discrete and fully-discrete of CLPS method for coupled VB equations are obtained by energy estimation method. The numerical results of the present method are compared with the exact solution for two test problems.

Keywords:
Coupled viscous Burgers equations
Chebyshev–Legendre Pseudo-Spectral method
Semi-discrete scheme and fully-discrete scheme

1. Introduction

The coupled VB equations was derived by Esipov [1] to study the model of sedimentation. The partial differential equations (PDEs) of coupled VB equations are as follow:

\[
\frac{\partial \Psi}{\partial t} + \frac{\partial^2 \Psi}{\partial x^2} + \gamma \frac{\partial \Psi}{\partial x} \frac{\partial \Phi}{\partial x} + \alpha \frac{\partial^2 \Phi}{\partial x^2} = 0, \quad x \in \Omega, \quad t \in [0, T],
\]

\[
\frac{\partial \Phi}{\partial t} + \frac{\partial^2 \Phi}{\partial x^2} + \delta \frac{\partial \Phi}{\partial x} \frac{\partial \Phi}{\partial x} + \beta \frac{\partial^2 \Phi}{\partial x^2} = 0, \quad x \in \Omega, \quad t \in [0, T],
\]

with initial conditions

\[
\Psi(x, 0) = f(x), \quad \Phi(x, 0) = g(x), \quad x \in \Omega,
\]

and the boundary conditions

\[
\Psi(-L, t) = \Psi(L, t), \quad -\Phi(-L, t) = \Phi(L, t), \quad t \in [0, T],
\]

where \( \Omega = [-L, L] \), \( \gamma \) and \( \delta \) are real constants, \( \alpha \) and \( \beta \) are arbitrary constants.

There has been continued interest in the solution of the VB equations. La Bryer et al. [2] developed a new method optimal spatiotemporal reduced order modelling to improve the order of accuracy for VB equations. Huilin and Chang Feng [3] solved the coupled VB equations by using new lattice Boltzmann model in 2014. They investigated the accuracy and stability of the new model in detail. Hossein used Laplace transform and homotopy perturbation methods to find the analytical approximation for the coupled VB equations [4].