Estimation of the continuously varying Doppler effect

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ABSTRACT
There are many applications for which it is important to resolve the location and motion of a target position. For the static situation in which a target transmitter and several receivers are not in motion, the target may be completely resolved by triangulation using relative time delays estimated by several receivers at known locations. These delays are normally estimated from the location of peaks in the magnitude of the cross-correlation function. For active radars, a transmitted signal is reflected by the target, and range and radial velocity are estimated from the delay and Doppler effects on the received signal. In this process, Doppler effects are conventionally modeled as a shift in frequency, and delay and Doppler are estimated from a cross-ambiguity function (CAF) in which delay and Doppler frequency shift are assumed to be independent and approximately constant. Delay and Doppler are jointly estimated as the location of the peak magnitude of the CAF plane. We present methods for accurately estimating delay for the static case and delay and the time-varying Doppler effects for non-static models, such as the radar model.

Keywords: correlation, cross ambiguity function, CAF, delay estimation, scale estimation, Doppler, super resolution, sub-sample delay

1. INTRODUCTION
There are many applications for which it is important to resolve the location and motion of a target position. For the static situation in which a target transmitter and several receivers are not in motion, the target may be completely resolved by triangulation using relative time delays estimated by several receivers at known locations. These delays are normally estimated from the location of peaks in the magnitude of the cross-correlation function [1]-[12]. For active radars, a transmitted signal is reflected by the target, and range and radial velocity are estimated from the delay and Doppler effects on the received signal. In this process, Doppler effects are conventionally modeled as a shift in frequency, and delay and Doppler are estimated from a cross-ambiguity function (CAF) in which delay and Doppler frequency shift are assumed to be independent and approximately constant [13]-[15]. Delay and Doppler are jointly estimated as the location of the peak magnitude of the CAF plane. We present methods for accurately estimating delay for the static case and delay and the time-varying Doppler effects for non-static models, such as the radar model.

We assume a simplified radar model in which a signal is transmitted and is reflected off a target. The returned signal is then received by a receiver that is co-located with the transmitter. The transmitted signal is then compared with the received signal to estimate time delay and Doppler effects. For a transmitted signal, \( s(t) \), the received signal differs from the transmitted signal by delay / Doppler effects and environmental effects, such as noise, interference and transmission loss. Typically the delay / Doppler effects are modeled by a narrowband approximation in which time delay is assumed approximately constant. The Doppler process is modeled as an approximately constant frequency shift [13]-[15]. A better approximation of the Doppler process is the wideband model in which the Doppler process is approximated as a stretch of the signal spectrum or equivalently the time-domain signal [16]-[22].

Both the narrowband frequency shift and the wideband frequency stretch models are only first order approximations of the true delay / Doppler process. Ignoring environmental effects, in the exact process, transmitted and received signals differ only by a time-varying delay. For the radar model, the instantaneous delay function represents the propagation time to the target and back and changes as the range of the target changes with time. The narrowband frequency shift and wideband frequency stretch are artifacts of the target radial velocity, which is the derivative, with respect to time, of the radial target distance.

The exact model of the Doppler process is universally valid. The wideband model provides a reasonable approximation of both the wideband and narrowband processes, while the narrowband model applies only to the
narrowband case where the ratio of the signal bandwidth to the signal RF is very small. The wideband model is almost never adopted due to the computational complexity of performing a scale correlation on a sufficiently fine scale lattice. We partially address the exact Doppler process, but we assume that the ratio of the signal bandwidth to the signal RF is small. With this assumption, we obtain a good estimate of the instantaneous narrowband Doppler shift (equivalently the instantaneous radial target velocity.) To obtain our estimates, we introduce a narrowband Doppler frequency estimation process that does not require joint estimation of delay and Doppler frequency from the two-dimensional CAF plane. This process is more accurate than conventional CAF-based estimates and is significantly more computationally efficient. In addition, we may efficiently estimate Doppler shift and delay within nearly the full respective Nyquist range and the full time duration of the signal. The process presented is an extension of our previous work [24]-[28]. In this present work, we present a brief statement of the methods and test results evaluating them. A full description/derivation of the processes is in preparation for a more complete subsequent work.

2. SIGNAL MODEL AND NOTATION

We assume a model in which a transmitted signal is reflected by a moving target and the returned signal is received by a receiver that is co-located with the radar transmitter. We assume further that the transmitted signal is band limited and is fairly narrowband, in the sense that the ratio of the signal bandwidth to the radio frequency (RF) is small. To simplify the discussion, we assume analytic representations of all signals. For a transmitted signal, \( s(t) \), the received signal has the form

\[
s_1(t) = e^{i\phi_1} a_1(t) s(t - \delta_1(t)) + \eta_1(t)
\]

(1)

\[
\approx e^{i\phi_1 + \omega_1(t_0)t} a_1(t) s(t - \delta_1(t_0)) + \eta_1(t)
\]

(2)

where \( s(t) \) is the transmitted signal, assumed to be band limited, and \( s_1, \phi_1, a_1, \delta_1 \) and \( \eta_1 \) are respectively the received signal, the relative difference between receiver transmitter phases, signal gain, propagation delay, and combined noise and interference at the receiver. The signal model for the standard CAF process is represented by Eq. (2), where \( t_0 \) is the assigned instant in time at which the delay, \( \delta_1(t_0) \), and Doppler frequency shift, \( \omega_1(t_0) \), are are assumed to be estimated.

For the stationary case in which there is no motion, the model reduces to

\[
s_1(t) = e^{i\phi_1} a_1(t) s(t - \delta_1) + \eta_1(t).
\]

(3)

We adopt the notation that time or time-delay signal representations are represented by lowercase letters, and frequency or time-frequency representations are represented by the corresponding uppercase letters. Bold face font is reserved for surface representations of the signal. With this notation, the Fourier transform and short time Fourier transform of the signal, \( s(t) \), have representations [23]

\[
S(\omega) = \int_{-\infty}^{\infty} s(t) e^{-i\omega t} dt
\]

(4)

\[
S_h(\omega,T) = \int s(t + T)h(t)e^{-i\omega t} dt,
\]

(5)

where \( h(t) \) is a time windowing function determining the interval of integration.

3. DELAY ESTIMATION

Before considering the case where the target is in motion, we consider the static case where there is no motion in the system. In this case, the transmitted and received signals are related by Eq. (3). The cross-spectrum, short time correlation function (STCF) and frequency dependent correlation function (FDCF) surfaces have the
representations

\[ R_h(s, s_1, \omega, T) = S_h(\omega, T)S_{ih}^*(\omega, T) \]  
\[ r_h(s, s_1, \delta, T) = \int s(t + T)s_1^*(t + T + \delta)h(t)dt \]  
\[ = \frac{1}{2\pi} \int R_h(s, s_1, \omega, T)e^{i\omega\delta}d\omega \]  
\[ r_{hH}(s, s_1, \delta, T, \Omega) \equiv \frac{1}{2\pi} \int R_h(s, s_1, \omega + \Omega, T)H(\omega)e^{i\omega\delta}d\omega. \]

Figure 1. Left: Mean delay error for phase-based, weighted linear least squares phase-based and parabolic interpolation methods. Results represent Gaussian signals with 1000 Monte Carlo trials at each SNR. Monte Carlo trials at each signal length. Right: Delay error standard deviation for the three methods.

The FTCF, is essentially a band-limited correlation function, with frequency response \( H(\omega - \Omega) \). As with the normal correlation magnitude, the index of the local maxima provide an estimate of the signal delay, \( \delta_1 \). The derivative with respect to \( \Omega \) of the argument of the FTCF at the local maxima provides a refined estimate of the signal delay, in a process fully described in [28]. Fig. 1 represents the results of Monte Carlo tests measuring the comparative performance of this estimator and conventional parabolic interpolation. In these tests, 1000 trials were performed at each SNR on band-limited Gaussian signals with randomly generated delays. In the figure, the phase derivative was evaluated both as the argument of the FTCF peak and as a weighted linear least squares (WLLS) fit of the phase function evaluated at the delay coordinate indicated by the phase of the FTCF peak. In the WLLS method, the abscissa was the argument of the FTCF function, with the magnitude of the FTCF used as the weighting function. The mean delay is a measure of the bias of the estimators, and the standard deviation is an estimate of the precision of the method. The WLLS method performed significantly better than either of the other methods.

To test performance of delay estimation as a function of signal length, Monte Carlo tests were performed on Gaussian signals with random delays. In these tests, represented in Fig. 2, there were 1000 trials at each SNR and each signal length. Three methods were tested, all based on the FTCF. Signal lengths in this series of tests were 50000, 10000, 1000 and 100. Performance in each delay estimation method demonstrated approximately 6dB improvement per decade of signal length. In these tests, amplitude interpolation resulted in better precision at the expense of being extremely biased.

4. DOPPLER ESTIMATION

We now consider the case in which there is target motion. In this case, the standard method for resolving the target is the cross-ambiguity function (CAF), which has a short time representation (STCAF)
Figure 2. Delay error mean and standard deviation for three estimation methods. Display represents Monte Carlo trials for signal lengths 50000, 10000, 1000 and 100 for 1000 Monte Carlo trials at each signal length.

\[ C_h(s, s_1, \omega, \delta, T) = \int s(t + T)s_1^*(t + T + \delta)h(t)e^{-i\omega t}dt. \]  (10)

The CAF is based on an assumption that the signal bandwidth is very small compared to the RF of the signal. This is true for most communication signals and is less true for very wideband signals, such as some radars. For the purposes of this paper, we will make the narrowband assumption. In applying the CAF, TDOA, \( \delta_1 \), and FDOA, \( \omega_1 \), (equivalently range and velocity) are normally estimated as

\[ (\omega_1, \delta_1) \approx \arg_{\omega, \delta} \max |C_h(s, s_1, \omega, \delta, T)|, \]  (11)

with an amplitude interpolation method, such as parabolic interpolation applied to both TDOA and FDOA axes to improve accuracy of the estimate. The magnitude of a portion of the CAF surface and the projections onto the TDOA and FDOA axes is represented in Fig. 3. In this representation, the signal environment consisted of three Gaussian signals of equal strength with random delays and random Doppler frequency offsets.

In the conventional CAF-based process, TDOA and FDOA are estimated jointly from the two dimensional CAF surface. We demonstrate a process that reduces this process to two one dimensional searches in which FDOA is first estimated and is applied to the received signal to mitigate the Doppler effects. TDOA is then estimated by the process outlined in the previous section. We present the basic method and demonstrate that we may accurately estimate the nominal TDOA and FDOA by this method and that the instantaneous phase of the STCF may also be estimated. A full discussion of the method, including the importance of the instantaneous STCF phase will appear in a future paper.

We first define the CIF surface c.f. [25]

\[ \text{CIF}_h(s, \omega, T) = |S_h(\omega, T)|^2e^{i\Phi(\omega, T)} \]  (12)

\[ \Phi'(\omega, T) = \frac{\partial}{\partial T} \arg \{S_h(\omega, T)\} \]  (13)

In practice, we may approximate the CIF surface by

\[ \text{CIF}_h(s, \omega, T) \approx S_h(\omega, T + \frac{\epsilon}{2})S_h^*(\omega, T - \frac{\epsilon}{2}), \]  (14)
where $\epsilon$ is a small time delay, typically one sample. We now define the ICIF by applying the inverse Fourier transform to the CIF

$$\text{ICIF}_h(s, \delta, T) = \int \text{CIF}_h(s, \omega, T) e^{i\omega\delta} d\omega$$

(15)

We define the spectral correlation function (SCF) as

$$\mathfrak{R}_h(s, s_1, \Omega, T) \equiv \int \text{ICIF}_h(s, \delta, T) \text{ICIF}_h^*(s_1, \delta, T) e^{i\Omega\delta} d\delta$$

(16)

For fixed $T_0$, the surface reduces to the “spectrum” $\mathfrak{R}_h(s, s_1, \Omega, T_0)$, with the property that the FDOA’s of signals in a multi-signal environment appear as peaks in this spectrum, and the expected values of the arguments of the spectral peaks are the angular FDOA of the received signals with respect to the transmitted signals. Fig. 4 represents the magnitude of $\mathfrak{R}_h(s, s_1, \Omega, T_0)$ for the superposition of three Gaussian signals represented in Fig. 3.

This representation has several advantages over the conventional CAF. It does not require the computation of the two dimensional CAF surface. Moreover, it provides FDOA estimates for the full range of frequencies within the Nyquist limits, unlike the CAF surface that must be delay and frequency limited for computational efficiency and because of memory constraints.

4.1. Doppler Residual
Finally, we demonstrate estimation of the instantaneous correlation phase residual. The importance and application of this function will be more fully discussed in a future paper. To compute the correlation phase residual, we first estimate the nominal FDOA and TDOA from either the CAF surface or the SCF, $\mathcal{R}_h(s, s, \Omega, T_0)$ and correlation function. The function,

$$C_h(s, s_1, \tilde{\omega}_1, \tilde{\delta}_1, T) \approx \max_{\omega, \delta} \{ C_h(s, s_1, \omega, \delta, T) \}, \quad (17)$$

where max is the complex value of the element of the surface having the largest local magnitude. The argument of $C_h(s, s_1, \tilde{\omega}_1, \tilde{\delta}_1, T)$ provides an estimate of the time-varying phase residual of the instantaneous correlation of the transmitted and received signals, the derivative of which is the instantaneous FDOA residual. Fig. 5 represents the instantaneous phase residual and the unwrapped phase residual. Fig. 6 represents the instantaneous phase difference between a synthesized Gaussian signal, $s(t)$, of length 100,000 and the received signal with a random initial delay and a time-varying phase function driven by a $5^{th}$ degree polynomial. If we denote by $\tilde{s}_1(t)$ the received signal after all of the described corrections, Fig. 6 is a plot of the estimated residual phase error of the transmitted signal and the corrected received signal.
5. CONCLUSIONS

We have presented methods for blind estimation of Doppler frequency (FDOA) and signal delay (TDOA). Unlike the conventional CAF process, where these parameters are estimated jointly, the methods presented make it possible to these parameters by independent sequential processes. In addition, we have presented a method to recover the instantaneous correlation phase (equivalently the instantaneous FDOA function.) Moreover, the methods presented provide sub-sample super resolution of the estimated parameters.

REFERENCES