Almost all modern imperative programming languages include operations for dynamically manipulating the heap, for example by allocating and deallocating objects, and by updating reference fields. In the presence of recursive procedures and local variables the interactions of a program with the heap can become rather complex, as an unbounded number of objects can be allocated either on the call stack using local variables, or, anonymously, on the heap using reference fields. As such a static analysis is, in general, undecidable.

In this paper we study the verification of recursive programs with unbounded allocation of objects, in a simple imperative language for heap manipulation. We present an improved semantics for this language, using an abstraction that is precise. For any program with a bounded visible heap, meaning that the number of objects reachable from variables at any point of execution is bounded, this abstraction is a finitary representation of its behaviour, even though an unbounded number of objects can appear in the state. As a consequence, for such programs model checking is decidable. Finally we introduce a specification language for temporal properties of the heap, and discuss model checking these properties against heap-manipulating programs.

1 Introduction

One of the major problems in model checking recursive programs which manipulate dynamic linked structures is that the state space is infinite, since programs may allocate an unbounded number of objects during execution by updating reference fields (pointers). Indeed model checking and reachability for such programs are undecidable, in general. Consequently to allow a restricted form of model checking we need to impose either some syntactic restrictions on the program [7] or some suitable bounds on its model. A natural bound for model checking programs without necessarily restricting their capability of allocating an unbounded number of objects is to impose constraints on the size of the visible heap [4]. The visible heap consists of those objects which are reachable from the variables in the scope of the currently executed procedure. Such a bound still allows for storage of an unbounded number of objects onto the call-stack, using local variables.

In this paper we introduce a method for model checking sequential imperative programs with pointers and recursive procedure calls. In order to allow implementation of model checking of unbounded object allocation in the context of a bounded visible heap, we introduce a new mechanism for the generation of fresh object identities which allows for the reuse of object identities and which includes a renaming
scheme to resolve possible resulting name clashes. We introduce a formal operational semantics based on this mechanism for an abstract programming language, called Shylock. Subsequently we introduce a logic for reasoning about properties of the heap, where we use atomic propositions defined as regular expressions in what is basically a Kleene algebra with tests [13]. Namely, the global and local variables of a program are used as nominals, whereas the pointers (reference fields) constitute the set of basic actions.

Our renaming mechanism allows a different kind of reuse of object identities than usual garbage collection techniques. A garbage collector typically reuses object identities from the heap and considers objects on the call stack as still in use. In contrast our technique is more tailored towards model checking, and as such, we need to reuse as much object identities as possible to guarantee a representation of program behaviour in terms of a finite pushdown system with a finite stack alphabet. In fact, our mechanism allows to reuse objects allocated in the call stack, that may become active when procedures return.

Structure of the paper  In the next paragraph we briefly discuss related work. We introduce Shylock and its formal semantics in Section 2. In Section 3 the abstraction of this semantics is introduced, together with a proof of its correctness. Then in Section 4 we define a logic for temporal properties of heaps, and finally in Section 5 we conclude.

Related work  We introduce a novel technique for resolving name clashes in the context of reuse of object identities. It is based on the concept of cut points as introduced in [19] to support static analysis via abstract interpretation techniques. Cut points are objects in the heap that are referred to from both local and global variables, and as such are subject to modifications during a procedure call. Recording cut points in extra logical variables allows for a precise abstract execution of the program, which in case of a bound on the visible heap can be represented by a finitary structure, namely that of a finite pushdown system.

In [4] a language is studied with the same features as our Shylock programs extended with a bounded form of concurrency. Because concurrency is an orthogonal dimension to the vertical growing of the number of objects due to recursion and the horizontal growing due to the anonymous field update, we have decided to not incorporate it in our Shylock language. In fact, bounded concurrency could easily be handled with a technique similar to the one used in [4]. The novelty of our work is not in the decidability result, that is indeed similar to the one obtained in [4], but in the technique we used to obtain it. While [4] uses finite graphs and graph isomorphisms to represent heaps and avoid name clashes, respectively, our approach is purely symbolic, and, therefore, directly usable for model checking temporal properties of heaps. We discuss the relationship with [4] in more details in the final paragraph of Section 4.

Currently there are several model checkers for object oriented languages. Java Path Finder [12] is basically a Java Virtual Machine that executes a Java program not just once but in all possible ways, using backtracking and restoring the state during the state-space exploration. Even if Java Path Finder is capable of checking every Java program, the number of states stored during the exploration is a limit on what can be effectively checked. As with JCAT [8], Java source code can be translated into Promela, the input language of SPIN. Since Promela does not support dynamic data structures, fixed-size heaps and stacks have to be allocated.

Bandera [6] is an integrated collection of tools for model checking concurrent Java software using state-of-the art abstraction, partial order reductions and slicing techniques to reduce the state space. It compiles Java source code into a reduced program model expressed in the input language of other existing verification tools. For example, it can be combined with the SAL (Symbolic Analysis Laboratory) model
checker [17] that uses unbounded arrays whose sizes vary dynamically to store objects. In order to explore all reachable states model checking is restricted to Java programs with a bounded (but not fixed a priori) number of created objects.

TOPICS [14, 11] is a tool which aims to find certain types of bugs in non-recursive C programs which manipulate a restricted type of heaps containing only single-linked lists. The faults detected by TOPICS are of several types, namely: memory leak, segmentation faults, array out of bounds errors, and usage of undefined objects in tests. The method used for bug-detection is reachability, and the models on which reachability is performed are the counter machines. The transformation of a C program into a counter machine goes through the intermediate representation of pointer machines which abstracts away the contents of the cells of the linked lists.

The problem of reusing object identities has been already faced when defining semantic models for the pi-calculus. Most notably, in history-dependent automata [16], a model based on the theory of named sets capable of finite-state verification of processes that can allocate fresh resources [5]. Model checking of a possibly unbounded number of objects with pointers but for a language with a restricted form of recursion (tail recursion) and no block structure has been studied using high level allocation Büchi automata [9] that allow for a finite state symbolic semantics very similar to ours. Full recursion, but with a fixed-size number of objects is instead considered in jMoped [10], using a pushdown structure to generate an infinite state system.

The techniques described in this paper aim at verifying programs by model checking. This is fundamentally different from other tools and techniques for verifying programs manipulating the heap by deductive verification methods, such as separation logic [18]. Automated methods for proving annotated programs is a very active area of research (see e.g. [15, 2, 3]). For a more detailed discussion we refer to [3].

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2 Shylock: A language to manipulate the heap

In this section we introduce Shylock, a simple imperative programming language that allows us to focus on dynamic pointer structures in the context of recursive procedures with local variables. Programs consist of a set of recursive procedures that can create new objects, and store them into their local or global variables. Besides being dynamically allocated, objects can be referenced by other objects via object fields, and exist as long as they are reachable in the heap from some other object or from a variable. To simplify the presentation objects are the only data structure of Shylock.

We assume an infinite set $V$ of variables ranged over by $x, y$, including a finite collection $G$ of global program variables $\{g_1, g_2, \ldots, g_n\}$, and a disjoint finite set $L$ of local program variables $\{l_1, l_2, \ldots, l_m\}$. We denote by $C$ the infinite set $V \setminus (G \cup L)$ of cut point variables ranged over by $c_1, c_2, \ldots$. Further, we assume a distinguished element $\text{nil} \in G$, used as a constant to refer to the undefined object. A Shylock program acts only on global and local variables, cut point variables will be used later in the abstract semantics as a kind of “freeze” variables for storing relevant points of the heap during procedure call. For simplicity we assume that all objects have the same set of fields $F = \{f_1, \ldots, f_k\}$. We denote by $\bar{g}, \bar{l}, \bar{c}, \bar{f}$ the sequences of all global, local, cut point variables and fields, respectively.
For \( P \) a finite set of \textit{procedure names} \( \{p_0, \ldots, p_l\} \), a program is a set of \textit{procedure declarations} of the form \( p_i : \cdot B_i \), where \( B_i \), denoting the \textit{body} of the procedure \( p_i \), is a statement defined by the following grammar:

\[
B ::= x.f := y \mid x := y.f \mid x := \text{new} \mid [x = y]B \mid [x \neq y]B \mid B + B \mid B ; B \mid p
\]

Here \( x \) and \( y \) are (local or global) program variables ranging over \( G \cup L \), \( f \) is a field in \( F \) and \( p \) is a procedure name in \( P \). We assume a distinguished \( p_0 \in P \), called the \textit{initial} procedure of a program.

The \textit{assignment} statements \( x.f := y \) or \( x := y.f \) assign the identity of the object referenced at right hand side of the assignment to the field or variable, respectively, at the left hand side. The statement \( x := \text{new} \) \textit{creates} a new object that will be referenced by the program variable \( x \). All fields of \( x \) will reference the undefined object \textit{nil}. We restrict to programs in which the variable \textit{nil} does not appear in the left-hand side of an assignment or object creation, i.e., \textit{nil} is a constant. Conditional statements \([x = y]B \) and \([x \neq y]B \), nondeterministic choice \( B_1 + B_2 \), and sequential composition \( B_1 ; B_2 \), have the standard interpretation. A \textit{procedure call} \( p \) means that the body \( B \) associated with \( p \) is executed next on the same global state but on a new fresh local state. After the procedure body terminates, its local state is destroyed forever and the previous local state (from which the procedure has been called) is restored. Changes to the global state, however, remain.

Notice that variable assignment \( x := y.f \) and field update \( x.f := y \) suffice, as more general expressions and updates can be encoded. For example, a statement \( x := y.f_1 \ldots f_k \) is encoded as \( x := y.f_1; \ldots; x := x.f_k \). A basic variable assignment \( x := y \) can be encoded as \( z.f := y; x := z.f \).

More general boolean expressions in conditional statements can be obtained by using sequential composition and nondeterministic choice. In fact \( (b_1 \land b_2)B \) can be written as \( (b_1)B_2B \), whereas \( (b_1 \lor b_2)B \) as \( (b_1)B + (b_2)B \). Negation of a boolean expression \( b \) can be obtained by transforming \( b \) into an equivalent boolean expression in conjunctive disjunctive normal form, for which negation of the simple expression \([x = y]\) and \([x \neq y]\) is defined as expected. Ordinary while, skip, and if-then-else statements can be expressed easily in the language, using recursive procedures, conditional statements and nondeterministic choice. For the sake of simplicity, we allow creation and assignment of a single object identity only; generalizations to simultaneous assignments and object creation can be added in a straightforward manner.

The language does not directly support parameter passing. However, it is worthwhile to note that we can model procedures with call-by-value parameters by means of global variables. Let \( p(v_1, \ldots, v_n) \) be a procedure with formal parameters \( v_1, \ldots, v_n \). We see the formal parameters as local variables and introduce for each parameter \( v_i \) a corresponding global variable \( g_i \) (which does not appear in the given program). Every procedure call \( p(v_1, \ldots, v_n) \) can then be encoded by the statement \( g_1 := v_1; \ldots; g_n := v_n; p \) whereas the body \( B \) of \( p(v_1, \ldots, v_n) \) can be encoded by \( v_1 := g_1; \ldots; v_n := g_n; B \). A similar approach can be taken to model procedures with return values. Finally, method calls \( x.m(x_1, \ldots, x_n) \) can be modeled by introducing the called object \( x \) as an additional parameter of the procedure \( m \).

\textbf{Example 1.} We consider a simple example program which opens a file, passes it to some procedure which returns again a file, and finally tries to close this returned file.\(^\text{1}\) It consists of the procedures \textit{main}, \textit{q}, \textit{open} and \textit{close}, and the sets of global and local variables are \( G = \{\text{nil}\} \) and \( L = \{x, y\} \) respectively. The procedures \textit{main}, \textit{open} and \textit{close} are defined as follows:

\[
\text{main} :: \text{open}(x); \ y := q(x); \ \text{close}(y)
\]

\[
\text{open} :: x := \text{new}
\]

\[
\text{close}(z) :: \ [z \neq \text{nil}] z := \text{nil}
\]

\( \text{1} \)This idea was suggested to us by Dilian Gurov.
The definition of $q$ is left open. Recall from the above discussion that while parameter passing and return values are not directly in the syntax of the language, they can easily be encoded. The intuition of this program is as follows. We start by executing main. Then first the program opens a file, modeled by an object creation, and then it passes the reference $x$ to this file on to a procedure $q$. This procedure $q$ then performs some calculations and passes back a file reference $y$. Finally we try to close the file referenced by $y$. Closing a file is modeled by first checking if the given reference is not nil, and then simply setting the reference to nil. If we pass close a reference to nil, then the program crashes. 

In order to describe the formal semantics of Shylock programs, we first formalize some relevant notions related to the heap. To represent object identities we use the set $\mathbb{N}_\bot = \mathbb{N} \cup \{\bot\}$ of natural numbers extended with an element $\bot$ and ranged over by $n,m$. Let $s : V \to \mathbb{N}_\bot$ be a variable assignment and $h : F \to (\mathbb{N}_\bot \to \mathbb{N}_\bot)$ be a field assignment such that for all $f$, $h(f)(\bot) = \bot$ and the set of objects for which $h(f)(n) \neq \bot$ is finite. A heap $H$ is a pair $(s,h)$ of a variable- and a field assignment. We write $H(x)$ for $s(x)$, and $H(f)$ for $h(f)$. For a subset of variables $Var \subseteq V$ we denote with $\mathcal{R}_H(Var)$ the set of objects reachable from objects labeled by these variables in $H$ via any of the (functional) transition relations $H(f)$, for $f \in F$. Formally it is defined as the least fixpoint of the equation

$$\mathcal{R}_H(Var) = \{H(x)|x \in Var\} \cup \{H(f)(n)|f \in F, n \in \mathcal{R}_H(Var)\}$$

If $Var = V$ we write the set of reachable objects of $H$ by $\mathcal{R}_H$. Further we define the “purely local” part of a heap $H$ as $\mathcal{R}_H = \mathcal{R}_H(L \cup C) \setminus \mathcal{R}_H(G)$. Intuitively $\mathcal{R}_H$ contains all objects which are reachable from a local (or cut point) variable, but not from a global variable.

We denote variable update by $H[x := n]$, global field update by $H[f := \varphi]$, where $\varphi : \mathbb{N}_\bot \to \mathbb{N}_\bot$ such that $\varphi(\bot) = \bot$, and local field update by $H[f := \varphi[n := m]]$. We use the standard notation and definition of simultaneous assignments and updates. A renaming $\rho$ of a subset $N \subseteq \mathbb{N}_\bot$ is an injective function in $\mathbb{N}_\bot \to \mathbb{N}_\bot$ such that $\rho(n) = n$ for all $n \notin N$, and otherwise $\rho(n) \in N$. Clearly it has an inverse, denoted by $\rho^{-1}$. Given a renaming $\rho$ we define its application on a heap $H$ as $\rho(H)(x) = \rho(H(x))$ and $\rho(H)(f)(n) = \rho(H(f)(\rho^{-1}(n)))$.

A configuration is a tuple $(H,\Gamma)$ where $H$ is the current heap and $\Gamma$ is a stack of statements and heaps. The head of a stack is separated from the tail by means of the right-associative operator $\bullet$, while the empty stack is represented by $\epsilon$. The current statement to be executed is on the top of the stack. When there are no statements but an heap on the top of the stack, then a procedure returns, and the state on the stack has to be restored as current state. A computation is a (possibly infinite) sequence $C_0 \to C_1 \to \ldots$ of transitions, where $\to$ is a relation between configurations which we will now define by cases on the top of the stack. To this end let $\Gamma$ be a stack of statements and heaps. Assignments to a variable or to a field update the current heap structure as expected:

$$\langle H, x := y.f \bullet \Gamma \rangle \to \langle H[x := H(f)(H(y))], \Gamma \rangle$$

$$\langle H, x.f := y \bullet \Gamma \rangle \to \langle H[f := H(f)[H(x) := H(y)]], \Gamma \rangle$$

To model object creation we assume a distinguished global “system” variable $oc$ which is used as a counter, and does not appear in a program. We implicitly assume that $H(oc) \neq \bot$, for every heap $H$. The semantics of the operator “new” is:

$$\langle H, x := \text{new} \bullet \Gamma \rangle \to \langle H[x, oc := H(oc), H(oc) + 1][f := \varphi], \Gamma \rangle$$

where $\varphi$ is the sequence such that $\varphi = H(f_1)[H(x) := \bot]$. Conditional statements are executing depending on the evaluation of the condition:

$$H(x) = H(y) \quad \langle H, [x = y]B \bullet \Gamma \rangle \to \langle H, B \bullet \Gamma \rangle$$

$$H(x) \neq H(y) \quad \langle H, [x \neq y]B \bullet \Gamma \rangle \to \langle H, B \bullet \Gamma \rangle$$
Sequential composition adds on top of the stack the next statements to be executed, while nondeterministic choice selects just one of the two statements.

\[ \langle H, B_1; B_2 \cdot \Gamma \rangle \rightarrow \langle H, B_1 \cdot B_2 \cdot \Gamma \rangle \quad \langle H, B_1 + B_2 \cdot \Gamma \rangle \rightarrow \langle H, B_1 \cdot \Gamma \rangle \quad i \in \{0, 1\} \]

Finally, procedure call and return are modeled as follows:

\[ \langle H, p \cdot \Gamma \rangle \rightarrow \langle H[\bar{l} := \bot], B \cdot H \cdot \Gamma \rangle \quad \langle H, H' \cdot \Gamma \rangle \rightarrow \langle H[\bar{l} := H'(\bar{l})], \Gamma \rangle \]

where \( H'(\bar{l}) \) denotes the pointwise application of \( H' \) to the local variables \( \bar{l} \), and \( B \) is the body of the procedure \( p \). Recall that for technical convenience there is a single sequence of local variables \( \bar{l} \) shared by the procedures.

This section is concluded with the notion of properness, which is a formalization of some operational properties of configurations appearing during the execution of programs. Suppose \( H \) and \( H' \) are heaps which appear in the stack, meaning they are pending heaps from a procedure call, and \( H \) appears higher up in the stack than \( H' \) (so \( H' \) was put on the stack before \( H \)). Then (1) the structure of the purely local part of \( H' \) is preserved in \( H \), since it could not have been accessed in between. Moreover (2) if some object is reachable in both \( H \) and \( H' \), then it must be reachable from a global variable in \( H' \). Both conditions intuitively capture the property that reachable objects remain reachable in a recursive call only if they were already reachable from global variables.

**Definition 1.** The set of proper stacks is defined inductively as follows:

- the empty stack is proper
- if \( \Gamma \) is proper then \( B \cdot \Gamma \) is proper for statements \( B \)
- if \( \Gamma \) is proper and \( H \) is a heap such that for every \( H' \) occurring in \( \Gamma \) the following holds:
  - for all \( f \in F, n \in \mathbb{R}^H_f \): \( H(f)(n) = H'(f)(n) \)
  - \( \mathbb{R}^H \cap \mathbb{R}^H \subseteq \mathbb{R}^H'(G) \)
  then \( H \cdot \Gamma \) is proper.

A configuration \( \langle H, \Gamma \rangle \) is proper if \( H \cdot \Gamma \) is a proper stack.

For example every configuration \( \langle H, p_0 \rangle \) is proper. Further, all transition steps preserve proper configurations. Thus every configuration in a computation starting from a proper one is proper.

3 Improving the semantics

The semantics introduced may generate, for a given program, a transition system with infinitely many configurations. This is not only because of the unbounded stack size, but, more problematically, also because each time a new object is allocated a new natural number is used. Thus, the number of heaps needed is also unbounded. Consider for example the Shylock program consisting of a single procedure \( p \) with as body the statement

\[ x := \text{new}; \quad p \]

where \( x \) is a local variable. Each time the statement \( x := \text{new} \) is executed, a new natural number is assigned to \( x \). This has the unfortunate consequence that infinitely many heaps are needed, and thus the usual model checking techniques for recursive systems \([10, 21]\) cannot be guaranteed to terminate. In this section we introduce an abstract semantics for Shylock, based on reuse of natural numbers for...
objects. Identities of objects that are not in use in the current heap will be reused instead of using a new identity each time a new object is allocated. More concretely, when creating a new object we will choose the minimal unused identity available, as formally expressed by the following rule:

\[ \langle H, x := \text{new} \cdot \Gamma \rangle \rightarrow \langle H[x := n][f := \phi], \Gamma \rangle \]

where \( n = \min(\mathbb{N} \setminus \mathcal{R}_H) \), and, for each \( i \), \( \phi_i = H(f_i)[n := \bot] \). The intuition here is that numbers are no longer concrete object identities, but instead they represent equivalence classes. However, this adapted rule may introduce name clashes with objects in the local state pending on the stack. We illustrate this problem and our solution with an example. Consider the following heap:

Here \( x : n \) represents the identity \( n \) to which the variable \( x \) refers (so technically the figure represents a heap \( H \) for which \( H(l) = 0, H(f)(0) = 1 \), etc.). Further \( l \) is a local variable, \( g \) is a global variable, and \( f \) is the single field. Let us consider first the execution of a call to a procedure \( p :: g := \text{new} \). Starting from the above heap, on the call a copy is placed onto the stack and the local variable \( l \) is initialized to \( \bot \), so the procedure \( p \) is executed on the following heap:

When executing \( g := \text{new} \) we take for \( g \) the minimal object identity unreachable from the current variables, which is 0. Then, on procedure return, we see that there is a name clash: both \( g \) and \( l \) point to the object with identity 0, while they should obviously not be identified. A solution is to rename the object \( n \) to which \( g \) points, i.e., to make \( g \) point to an identity \( m \) which is used neither by the current nor the caller’s stored heap, and updating the fields of this new object according to the fields of \( n \). Then we can just take the union of the global part of the new heap, and the local part of the stored heap. For example we could rename, in the current heap, the object 1 to 2, which is free, take the union with the (local part of the) stored heap of the caller, and combine the two heaps as follows:

However, consider now the execution of a procedure \( p' :: g := \text{new}; g := \text{new} \), starting from the same heap as before (the first figure above). After executing the first object creation statement in the procedure, \( g \) again points to 0. But then after the second time we execute \( g := \text{new} \), \( g \) is assigned the minimal index available, which is 1 at that point. Thus on procedure return, the heap is exactly the same as in the beginning of the procedure execution. So two object creations are in this case indistinguishable from no creation at all. On the procedure return, when combining the current heap with the stored heap it is thus not clear whether \( g \) should keep pointing to 1 (when no object creation statements were executed), or if it should be renamed to a new identity separate from the others (when two object creation statements were executed).
Our solution to this problem is a non-trivial extension of the semantics of procedure call and -return based on the identification of so-called cut points, which are the object identities at the “edge” of the global and the local part of the heap, representing exactly the point where the local part “enters” the global part. On a procedure call, these cut points are identified, and we assign their values to a set of distinguished cut point variables, in the heap of the callee. Then, on procedure return, the cut point variables “connect” the current heap with the stored one, giving us precisely the information about how to combine the two. Returning to our example, consider the following heap:

\[
g, c : 1 \quad f \quad l, nil : \bot
\]

This heap represents the initial heap of the callee, extended with the only cut point of the caller heap, in the form of the cut point variable \(c\). Now if we execute \(g := \text{new}; g := \text{new}\), the global variable \(g\) is not assigned the identity 1 since it is already in use. So now on procedure return we can distinguish between the case that \(g\) was newly created (in which case it will have a new identity), and the case that it was not (in which case it will have the same identity as before).

Formally, for a given heap \(H\), the set \(CP_H\) of cut points is defined as follows:

\[
\mathcal{R}_H(G) \cap (H(L \cup C) \cup F(\mathcal{R}_H))
\]

where \(F(N) = \{H(f)(n) | n \in N, f \in F\}\). Further, \(H(V)\) means applying \(H\) point-wise to \(V\), hence \(H(L \cup C) = \{H(v) | v \in L \cup C\}\). Note that the definition involves the cut point variables; recall from the above discussion that these variables represent the cut points of the previous heap. Further recall that \(\mathcal{R}_H(G)\) is the global part of the heap, while \(\mathcal{R}_H\) is the “purely local” part of the heap. Intuitively, \(F(\mathcal{R}_H)\) represents the objects which are pointed to by a field from an object which is purely local. Further \(H(L \cup C) \cap \mathcal{R}_H(G)\) is the set containing objects pointed to directly by the local variables, which are also reachable from global variables. On the other hand, \(F(\mathcal{R}_H) \cap \mathcal{R}_H(G)\) contains objects adjacent to the purely local (reachable) nodes (where the node adjacency is provided by the field transitions).

Now the procedure call of the improved semantics is modeled by the following rule.

\[
\langle H, p; \Gamma \rangle \rightarrow \langle H[\tilde{v} := \bar{1}][\tilde{c} := \bar{n}], B; \bullet H; \bullet \Gamma \rangle,
\]

where \(\tilde{v}\) is the sequence of local variables and cut point variables \(c\) for which \(H(c) \neq \bot\). Further \(\tilde{c}\) is a sequence of cut point variables of the same length as the sequence \(\bar{n}\) of cut points \(CP_H\). Note that, given a heap \(H_c\), if on top of the stack we have a heap \(H_l\), by the way we modeled the procedure call, the cut points in the stacked heap \(H_l\) correspond exactly to the cut point variables in the current heap \(H_c\).

We proceed to discuss the construction of the return heap, say \(H_r\). We first rename all objects of the purely local part of \(H_l\) which conflict with \(H_c\), meaning that they are also reachable from global variables in \(H_c\). When this is done, we can just copy all the local variables directly to \(H_c\) and update the fields of the purely local part of \(H_l\) in \(H_c\). In order to formalize this process we define the set of name clashes \(N\) of \(H_c\) and \(H_l\):

\[
N = \mathcal{R}_H(\mathcal{R}_H(G))
\]

Remember that \(\mathcal{R}_H\) only contains the objects which are reachable from a local variable (or from a previous cut point variable), and are not reachable from any global variable. Now the return rule is formalized as follows:

\[
\langle H_c, H_l; \bullet \Gamma \rangle \rightarrow \langle \theta \odot \rho(H_c), \Gamma \rangle
\]

where
\begin{itemize}
\item $\rho$ is a renaming, monotonic on $\mathbb{N}$, such that $\rho(n) \in \mathbb{N} \setminus \mathcal{R}_{H_i}$ if $n \in \mathbb{N}$, and $\rho(n) = n$ otherwise, and $\rho$ is minimal w.r.t pointwise comparison between renaming functions.
\item $\theta$ resets the purely local part:
\[
\theta(H) = H[\vec{t}, \vec{c} := H_l(\vec{t}), H_l(\vec{c})][f := \phi]
\]
where $\vec{c}$ is the sequence of cut point variables $c$ for which $H_l(c) \neq \bot$, and $\phi$ is the sequence defined, for all $n \in \mathbb{N}_\bot$, as follows:
\[
\phi_i(n) := \begin{cases}
H_l(f_i)(n) & \text{if } n \in \mathcal{R}_{H_i} \\
H_i(f_i)(n) & \text{otherwise}
\end{cases}
\]
\end{itemize}

**Correctness** We provide a proof of the equivalence between the concrete semantics of Shylock, and the abstract semantics defined in the previous section. To this end we adapt the concrete semantics to take into account the initialization and restoration of the cut point variables, similar to the abstract semantics. Note that this does not affect the behaviour of programs, as they are assumed not to contain cut point variables. First we need the basic notion of heap isomorphism:

**Definition 2.** Two heaps $H$ and $H'$ are isomorphic, denoted $H \sim H'$, if there exists a function $\alpha : \mathcal{R}_H \rightarrow \mathcal{R}_{H'}$ such that
\begin{itemize}
\item $\alpha$ is a bijection.
\item For each $x \in V$: $\alpha(H(x)) = H'(x)$.
\item For each $f \in F$ and $n \in \mathcal{R}_{H}$: $\alpha(H(f)(n)) = H'(f)(\alpha(n))$.
\end{itemize}

Note that since fields are deterministic, such a function $\alpha$, if it exists, is unique. In order to proceed, we introduce the important notion of cut point identification. Recall that on a procedure call, the new heap $H_c$ represents in its cut point variables the cut points of the caller heap $H_l$. Suppose now we have other heaps $H'_c$ and $H'_l$ such that $H_c \sim H'_c$ and $H_l \sim H'_l$. Note that cut points are preserved by isomorphisms. Cut point identification now formalizes the representation of cut points of $H_l$ and $H'_l$ in $H_c$ and $H'_c$ respectively, using for each cut point in $H_l$ and $H'_l$ the same variable to represent it in $H_c$ and $H'_c$.

**Definition 3** (Cut point identification). Let $H_c, H_l, H'_c, H'_l$ be heaps such that $H_c \sim_{\alpha_c} H'_c$, $H_l \sim_{\alpha_l} H'_l$. Let $\{n_1, \ldots, n_k\} = CP_{H_l}$ be the cut points of $H_l$. We define
\[
(H_c, H_l) \bowtie (H'_c, H'_l)
\]
iff there exists a sequence of cut point variables $c_1, \ldots, c_k$ such that for all $i \leq k$:
\[
H_c(c_i) = n_i \text{ and } H'_c(c_i) = \alpha_c(n_i)
\]

From this definition we immediately deduce that the two isomorphisms agree on cut points:

**Corollary 1.** If $(H_c, H_l) \bowtie (H'_c, H'_l)$ then for all $n \in CP_{H_l}$: $\alpha_l(n) = \alpha_c(n)$.

**Proof.** For all $n_i \in CP_{H_l}$ we have $\alpha_l(n_i) = H'_c(c_i) = \alpha_c(H_c(c_i)) = \alpha_c(n_i)$. \hfill \square

Now we are ready to introduce a strong notion of equivalence, based on heap isomorphism, which also takes along the main operational properties characterizing configurations appearing in computations:

**Definition 4.** Given stacks $\Gamma, \Gamma'$ we define $\Gamma \sim \Gamma'$ inductively as follows:
• if $\Gamma$ and $\Gamma'$ are both empty then $\Gamma \sim \Gamma'$
• if $\Gamma \sim \Gamma'$ then $B \cdot \Gamma \sim B \cdot \Gamma'$
• if 
  - $\Gamma \sim \Gamma', H \sim H'$
  - $(H, \nu(\Gamma)) \triangleright (H', \nu(\Gamma'))$
  - $H \cdot \Gamma$ is proper
then $H \cdot \Gamma \sim H' \cdot \Gamma'$

where $\nu(\Gamma)$ extracts from the stack the top heap. Now for configurations we define $\langle H, \Gamma \rangle \sim \langle H', \Gamma' \rangle$ iff $H \cdot \Gamma \sim H' \cdot \Gamma'$.

The following lemma states how the current heap and the stacked heap are combined on a procedure return in the concrete semantics. More precisely it expresses that any identity which becomes reachable right after a procedure returns, is in the purely local part of the heap of the caller procedure.

**Lemma 1.** Suppose $\langle H_c, H_l \cdot \Gamma \rangle \sim \langle H'_c, H'_l \cdot \Gamma' \rangle$. Let $H_r = H_c[\bar{I} := H_l(\bar{I})]$. Then for all $n \in \mathcal{R}_{H_r}$: if $n \notin \mathcal{R}_{H_c} \cup \mathcal{R}_{H_l}$ then $n \in \mathcal{R}_{H_r}$.

**Proof.** Let $H_c, H_l, H_r$ be as above. Let $n \in \mathcal{R}_{H_r}$ and assume $n \notin \mathcal{R}_{H_c} \cup \mathcal{R}_{H_l}$, so $n$ is reachable in $H_r$ from a local variable. We prove by induction that any path in $H_r$ reaching such an $n$ is reflected in the same path in $H_l$ which lies entirely in its purely local part $\mathcal{R}_{H_l}$.

Suppose first that $n = H_r(l)$ for some local variable $l$. Then $n = H_r(l) = H_c[\bar{I} := H_l(\bar{I})](l) = H_l(l)$. Now by assumption (that is in the relation $\sim$), $H_c$ has cut point variables precisely on the cut points of $H_l$. We may then conclude that $n$ is not reachable from a global variable in $H_l$; otherwise it would, by definition (of cut points) be on a cut point of $H_l$, and consequently on a cut point variable in $H_c$ which contradicts our assumption on the reachability of $n$. Thus $n \in \mathcal{R}_{H_l}$.

Now let $n = H_r(f_1) \circ \ldots \circ H_r(f_i)(H_l(l))$ such that $n \notin \mathcal{R}_{H_c} \cup \mathcal{R}_{H_l}$, $n = H_l(f_1) \circ \ldots \circ H_l(f_i)(H_l(l))$ and $n \in \mathcal{R}_{H_r}$. Suppose $H_r(f)(n) \notin \mathcal{R}_{H_r} \cup \mathcal{R}_{H_l}$ for some field $f$. Since $n \in \mathcal{R}_{H_r}$ and $H_c \cdot H_l$ is proper we have $H_l(f)(n) = H_r(f)(n)$. Now $H_r(f)(n) \notin H_c(c)$ for all cut point variables $c$; otherwise $n$ would be reachable from such an $H_c(c)$ which would be a contradiction with our assumption. But then by the cut point identification of $H_l$ and $H_r$, $H_r(f)(n)$ is not on a cut point of $H_l$, which implies that the global state has not been entered yet, i.e., $H_l(f)(n) \in \mathcal{R}_{H_l}$ as desired. 

We are now ready for the main theorem of this section, stating that the concrete and the abstract semantics are equivalent.

**Theorem 1** (Bisimulation). Let $C_1$ and $C_2$ be configurations such that $C_1 \sim C_2$. Denote with $\rightarrow_c$ and $\rightarrow_a$ the transition relations corresponding to the concrete and the abstract semantics, respectively. If $C_1 \rightarrow_c C'_1$ then there exists a configuration $C'_2$ such that $C_2 \rightarrow_a C'_2$ and $C'_1 \sim C'_2$, and vice versa.

**Proof.** We only discuss the isomorphism of the resulting heaps on procedure return. Suppose $\langle H_c, H_l \cdot \Gamma \rangle \sim \langle H'_c, H'_l \cdot \Gamma' \rangle$.

By definition of the concrete and the abstract semantics from these respective configurations the enabled transitions are $\langle H_c, H_l \cdot \Gamma \rangle \rightarrow_c \langle H_r, \Gamma \rangle$ and $\langle H'_c, H'_l \cdot \Gamma' \rangle \rightarrow_a \langle H'_r, \Gamma' \rangle$. 

where $H_i' = \theta \circ \rho(H_i')$. By definition of $\sim$ there are isomorphisms $H_c \sim_{\alpha_c} H_i'$ and $H_i \sim_{\alpha_i} H_i'$. We explicitly define an isomorphism $\alpha : \mathcal{R}_{H_c} \rightarrow \mathcal{R}_{H_i'}$ as follows:

$$\alpha(n) = \begin{cases} 
\rho \circ \alpha_c(n) & \text{if } n \in \mathcal{R}_{H_c}(G \cup C) \\
\alpha_i(n) & \text{otherwise}
\end{cases}$$

Note that by Lemma [1] if $n \notin \mathcal{R}_{H_c}(G \cup C)$ then $n \in \mathcal{R}_{H_c}^\infty$ so $\alpha$ is well-defined. To see that $\alpha$ is an isomorphism, intuitively, note that $\alpha_c$ is an isomorphism on $\mathcal{R}_{H_c}(G \cup C)$ and $\alpha_i$ is an isomorphism on $\mathcal{R}_{H_i}^\infty$, and by cut point identification we known from Corollary [1] that $\alpha_c(n) = \alpha_i(n)$ for all $n \in CP_{H_c}$.

## 4 Model checking Shylock programs

In this section we present a framework for model checking Shylock programs. We first turn our abstract semantics into a pushdown system, then we introduce a linear time temporal logic for heap structures, and finally we shortly recall the actual model checking procedure.

**Programs as pushdown systems** A pushdown system can be considered as a pushdown automaton without an input alphabet. Formally a pushdown system $\mathcal{P}$ is a triple $(\Delta, \Sigma, \rightarrow)$ where $\Delta$ is a set of control locations, $\Sigma$ is a stack alphabet, and $\rightarrow$ is a subset of $\langle \Delta \times \Sigma \rangle \times (\Delta \times \Sigma^*)$ representing the set of rules. A pushdown system is said to be finite when the above three sets are all finite.

The behaviour of any Shylock program $P = \{p_0 :: B_0, \ldots, p_l :: B_l\}$ can be represented by a pushdown system $\mathcal{P}_P = (\Delta, \Sigma, \rightarrow)$, where $\Delta$ is the set of all heaps, and $\Sigma = \Delta \cup cl(P) \cup \{Z\}$, where $Z$ is an element which does not occur in $\Delta$ and $cl(P)$. Here $cl(P)$ is the set of all possibly reachable statements in $P$, and it is defined as the union of all $cl(B_i)$, for all $0 \leq i \leq l$, with $cl(B)$ given inductively by:

- $cl(x.f := y) = \{x.f := y\}$
- $cl(x := new) = \{x := new\}$
- $cl([x = y]B) = \{x = y\} \cup cl(B)$
- $cl([x \neq y]B) = \{x \neq y\} \cup cl(B)$
- $cl(B_1 + B_2) = cl(B_1) \cup cl(B_2) \cup \{B_1 + B_2\}$
- $cl(B_1; B_2) = cl(B_1) \cup cl(B_2)$

The rules of the pushdown system are specified using the abstract semantics as follows:

$$\langle H, \gamma \rangle \rightarrow \langle H', w \rangle \iff \langle H, \gamma \bullet \Gamma \rangle \rightarrow \langle H', w \bullet \Gamma \rangle$$

where $H$ ranges over heaps. Further we add rules $\langle H, \gamma \rangle \rightarrow \langle H, Z \rangle$ for any configuration $\langle H, \gamma \rangle$ which does not have outgoing transitions to complete with stuttering steps terminating computations starting from $\langle H_0, p_0 \bullet Z \rangle$. Because there are infinitely many heaps, the pushdown system constructed above will in general be infinite. Consequently existing model checking techniques can not be applied. In order to allow model checking we consider a subclass of programs. First, we need the following definition:

**Definition 5.** A heap $H$ is $k$-bounded if $|\mathcal{R}_H(G \cup L)| \leq k$. A computation $\langle H_0, \Gamma_0 \rangle \rightarrow \langle H_1, \Gamma_1 \rangle \rightarrow \ldots$ (where the transition steps are according to the abstract semantics) is $k$-bounded if $|\mathcal{R}_H|$ is $k$-bounded for all $i$. A program $P$ with main procedure $p_0$ is $k$-bounded if every computation $\langle H, p_0 \rangle \rightarrow \ldots$ is $k$-bounded.

**Example 2.** As an example of a program which is bounded in this sense, recall the program with a single procedure defined as $p :: x := new; p$, where $x$ is a local variable. Indeed only one object identity is
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needed to represent the object to which \( x \) refers, so this program is 1-bounded. Nevertheless, since \( x \) is local, during the execution of the program an unbounded number of objects are stored on the stack.

For another example, recall the program from Example[1] which opens and closes a file. This program is \( k \)-bounded iff the visible heap during execution of the procedure \( q(x) \) (with \( x \) a fresh object) is \( k \)-bounded.

Now if we restrict to \( k \)-bounded programs, the abstract semantics, because of its reuse of object identities, allows us to represent the behaviour of a program as a pushdown system as above, but using as control states only \( k \)-bounded heaps. By Theorem[1] we then have precise abstractions of \( k \)-bounded programs as finite pushdown systems. More precisely, given a \( k \)-bounded program \( P \) we define the pushdown system \( k\cdot P = (\Delta_k, \Sigma_k, \rightarrow_k) \) obtained as a restriction from \( P \) as follows. First, \( \Delta_k = \{ H \mid |H_{H}(G \cup L)| \leq k \} \cup \{ \top \} \) is the subset of all \( k \)-bounded heaps. The stack alphabet \( \Sigma_k \) is given by \( \Delta_k \cup cl(P) \cup \{ Z \} \), and the relation \( \rightarrow_k \) is the restriction of \( \rightarrow \) to \( \Delta_k \) together with the two out-of-bound rules below

\[
\begin{align*}
\langle H, \gamma \rangle &\rightarrow \langle H', w \rangle \mid |\mathcal{R}_{H'}| > k \\
\langle H, \gamma \rangle &\rightarrow_k \langle \top, \gamma \rangle \\
\langle \top, \gamma \rangle &\rightarrow_k \langle \top, \gamma \rangle
\end{align*}
\]

Note that in fact for any program \( P, k\cdot P \) is a finite pushdown system. However it is a precise abstraction of \( P \) only if \( P \) is \( k \)-bounded.

Specifications in \( \mathcal{L}^r\mathcal{T}\mathcal{L}^r \) In order to do a precise pointer analysis of Shylock programs, we introduce a linear time temporal logic (\( \mathcal{L}^r\mathcal{T}\mathcal{L}^r \)) for describing the evolution of the heap structure. We first introduce the language of the properties satisfied by a given heap, which will form the atomic properties of the linear temporal logic. We do so by the introduction of expressions of the Kleene algebra with tests[13] over fields and variables. More precisely, let \( \text{Rite} \) be the smallest set defined by the following grammar:

\[
r ::= \mathcal{E} \mid x \mid \neg x \mid f \mid r.r \mid r + r \mid r^*
\]

where \( x \) ranges over variable names (to be used as tests) and \( f \) over field names (to be used as actions). The regular expressions introduced by \( \text{Rite} \) are similar to the heap patterns used in matching logic [20] and separation logic [18]. We define a transition relation \( n \rightarrow_H m \) between objects of a heap \( H \) as the least relation such that

\[
\begin{align*}
n &\xrightarrow{\mathcal{E}} H n \\
n &\xrightarrow{x} H n \quad \text{if } H(x) = n \\
n &\xrightarrow{\neg x} H n \quad \text{if } H(x) \neq n \\
n &\xrightarrow{f} H m \quad \text{if } H(f)(n) = m \\
n &\xrightarrow{n_1 + n_2} H m \quad \text{if } n \xrightarrow{n_1} m \text{ or } n \xrightarrow{n_2} m \\
n &\xrightarrow{n_1.n_2} H m \quad \text{if exists an object } n' \text{ such that } n \xrightarrow{n_1} H n' \text{ and } n' \xrightarrow{n_2} H m \\
n &\xrightarrow{n'} H m \quad \text{if either } n = m \text{ or there exists an object } n' \text{ such that } n \xrightarrow{n'} H n' \text{ and } n' \xrightarrow{n'} H m 
\end{align*}
\]

Further we introduce the following modal interpretation of regular expressions:

\[
H \models r \text{ if and only if for each reachable object } n \in \mathcal{R}_H \text{ there exists } m \text{ such that } n \xrightarrow{r}_H m.
\]

(Note that this coincides with the truth definition of \( H \models (r)\text{true} \) in dynamic logic.) For instance, the regular expression \( \text{first}.n^*.\text{last} + \neg \text{first} \) is satisfied by a heap \( H \) if and only if the object referred to by
the variable *first* is linked via a chain of fields *next* with the object referred to by the variable *last*, or *first* is not allocated.

Our $L'TIL$ formulas are built according to the following grammar:

$$\phi ::= true \mid r \mid \neg \phi \mid \phi_1 \land \phi_2 \mid X\phi \mid \phi_1 U\phi_2$$

where $r$ ranges over $Rite$. Other propositional connectives $\lor$, $\rightarrow$ are defined in terms of $\land$ and $\neg$. Further we define $F\phi = true \lor \phi$ and $G\phi = \neg F(\neg \phi)$. We define $At(\phi)$ to be the set of atomic propositions $r \in Rite$ which appear in $\phi$. Clearly $At(\phi)$ is always a finite set.

For a set $X$ we denote its powerset by $2^X$. With $(2^{At(\phi)})^\omega = \{w_0w_1w_2\ldots | w_i \in 2^{At(\phi)} \text{ for all } i \geq 0\}$ we denote the set of infinite words over sets of expressions in $At(\Phi)$. Given such an infinite word $w = w_0w_1w_2\ldots$ we denote with $w_i$ the $i$-th element, and with $w[i\ldots]$ the subsequence $w_iw_{i+1}\ldots$ starting from the $i$-th element of $w$. For a $L'TIL$ formula $\phi$ and an infinite word $w \in (2^{At(\phi)})^\omega$ we denote that $w$ satisfies $\phi$ by $w \models \phi$. This satisfaction relation is defined inductively on the structure of $\phi$ according to the standard semantics of LTL [1]:

- $w \models true$ iff $r \in w$
- $w \models r$ iff $w \models \phi_1 \land \phi_2$ iff $w \models \phi_1$ and $w \models \phi_2$
- $w \models \neg \phi$ iff $w \not\models \phi$
- $w \models X\phi$ iff $w[1\ldots \models \phi$
- $w \models \phi_1 U\phi_2$ iff $\exists j \geq 0. w[j\ldots \models \phi_2$ and $w[i\ldots \models \phi_1$ for all $0 \leq i < j$

Let $\pi$ be an infinite sequence of $k$-bounded heaps for some $k$, i.e., $\pi \in \{H_0H_1H_2\ldots | H_i \in \Delta_k \text{ for all } i \geq 0\}$. Intuitively, $\pi$ represents a trace of heaps which we encounter during a particular computation of a $k$-bounded program. We say that $\pi \models \phi$ if and only if there exists a sequence $w \in (2^{At(\phi)})^\omega$ such that $w \models \phi$ and for all $i \geq 0$:

$$\pi_i \models r \text{ for all } r \in w_i$$

Finally the above relation $\models$ is pointwise extended to sets of infinite sequences of $k$-bounded heaps.

**Model checking** Recall that a Büchi automaton $\mathcal{B} = (\nabla,A,\rightsquigarrow,Q_0,F)$ consists of a finite set of states $\nabla$, an input alphabet $A$, a transition relation $\rightsquigarrow \subseteq \nabla \times A \rightarrow \nabla$, a set of initial states $Q_0$ and a set of final states $F$. The language $L(\mathcal{B})$ accepted by $\mathcal{B}$ is the set of all infinite words $w$ over $A$ such that there is an infinite path via $\rightsquigarrow$ labeled by $w$, starting from a state $q_0 \in Q_0$, and visiting an accepting state in $F$ infinitely often. Given a $L'TIL$ formula $\phi$, one can effectively construct a Büchi automaton $\mathcal{B}_\phi$ which recognizes exactly the set of words $w$ over sets of expressions in $At(\phi)$ satisfying $\phi$ (see e.g. [1] for details).

Let $\phi$ be a formula of our temporal logic for heaps, and $P$ be a $k$-bounded Shylock program. To check if $P$ satisfies the formula $\phi$, we have to check if all the computations of the pushdown system $k-\mathcal{P}_P$ starting from the initial configuration $(H_0,p_0\bullet Z)$ satisfy $\phi$. This amounts to synchronizing the pushdown system $k-\mathcal{P}_P$ with the Büchi automaton $\mathcal{B}_{\neg \phi}$ and checking if the resulting Büchi pushdown system has an accepting run, i.e., a run starting from the initial states of the two systems which visits infinitely often configurations whose control locations projected into the states of the Büchi automaton are final [10,21].

The problem of finding an accepting run of a Büchi pushdown system can be reduced to that of finding a repeated head reachable from the initial configuration [10,21]. Computing the repeating heads
is typically developed in two phases. In the first phase, one constructs the head reachability graph \( G \) associated with a Büchi pushdown system, while in the second phase, \( G \) is analyzed to identify those nodes of the graph which are repeated heads. To avoid redundant computations, it suffices to construct the head reachability graph \( G \) restricted to those configurations reachable from an initial configuration of the Büchi pushdown system. This can be done using forward reachability analysis, by using the so called post* method. For more details, see e.g. Chapter 3 in [21].

**Shylock and [4]** As discussed in the introduction, the related work closest to our approach is presented in [4]. While [4] considers only reachability, this could be extended to a full model checking procedure along the lines discussed in this paper. A first view on the two works shows that [4] has a very high level solution to the problem while we give an explicit solution to it. The semantics of procedure call and specifically of the procedure return as given in [4] are stated in terms of abstract graph isomorphisms, and it is not clear how they should be implemented. In contrast, in this paper we give a purely symbolic characterization of procedure call and return.

Now we present a more detailed view on the difference between the semantics given in the two works. The procedure call in [4] employs the cut point mechanism as well, but it also cleans the heap of the currently unreachable objects. Hence, their procedure call passes to the callee only the strictly visible heap of the caller, actually an isomorphic instance of it. In contrast, Shylock’s procedure call relies on the cut points as well, but it doesn’t necessarily clean the heap (though it could). Instead, Shylock reuses object identities on demand, during the object creation statements of the procedure, while [4] doesn’t pay attention to the body of the procedure because of the initial cleaning. Though Shylock may seem lazy w.r.t. cleaning, it’s memory reuse mechanism acts in fact as a localized cleaning. This pays off during the execution of the procedure returns. Namely, in [4] the procedure return has to proceed by renaming the entire visible object space, such that it can synchronize the current heap with the caller’s heap. Meanwhile, Shylock renames only the name clashes, i.e., the objects at the intersection of the caller’s global heap and the callee’s current purely local heap. These differences induce a different reasoning during the verification phase. Namely, while Shylock can afford to use heap equality during the model checking phase, the reachability procedure in [4] has to be performed on normal forms of the heaps (i.e., on the representatives of the graph isomorphic equivalence class). We are not sure if Shylock maintains strictly one representative of each isomorphic class, but we plan to study this particular aspect in the near future.

**5 Conclusions**

In the presence of recursive procedures and local variables, an unbounded number of objects can be allocated either on the call stack using local variables, or, anonymously, on the heap using reference fields. In this paper we discussed Shylock, a language which supports these features, together with a formal abstract semantics which allows model checking in the context of bounded visible heaps. We introduced a temporal logic for specifying properties of the heap, and discussed a procedure for checking these properties against Shylock programs.

**Future work** Shylock’s improved semantics has been implemented in the \( \mathbb{K} \) framework. We are currently implementing a general model checking technique for recursive programs defined in \( \mathbb{K} \), from which we would obtain a Shylock model checker along the lines described in this paper. Further we are investigating the expressive power of programs with a bounded visible heap.
References


