 Quadratic-Programming Based Self-Motion Planning with No Target-Configuration Assigned for Planar Robot Arms

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Abstract—For better manipulability, a criterion in the form of a quadratic function is presented for the self-motion planning (SMP) of redundant manipulators with no target-configuration assigned. Such SMP scheme could automatically select the desirable configuration so that the manipulator could be more flexible and maneuverable. As physical limits generally exist in actual redundant manipulators, both joint limits and joint velocity limits are taken into consideration in the presented SMP scheme for practical purposes. Computer simulations based on two types of multi-link planar robot arms substantiate the efficiency and reliability of the presented scheme. Moreover, theoretical analysis of the presented quadratic performance index is conducted and proved via two different approaches, i.e., gradient-descent and Zhang et al.’s neural-dynamic methods.

I. INTRODUCTION

According to the number of degrees of freedom (DOF), manipulators could be divided into redundant manipulators and non-redundant manipulators. A manipulator is said to be (kinematically and/or functionally) redundant when more DOF are available than the minimum number of DOF required to execute a specific end-effector primary task [1]–[3]. Compared to non-redundant manipulators, redundant manipulators naturally have wider operational space and extra degrees to configure the manipulators to meet additional functional constraints without affecting the position (and/or orientation) of the end-effector [1]–[3].

One fundamental issue in operating redundant robot systems is the redundancy-resolution problem [1], [2]. It could generally be described as "given the desired Cartesian trajectory \( r(t) \in \mathbb{R}^m \) for the manipulator’s end-effector, we need to generate the corresponding joint trajectory \( \theta(t) \in \mathbb{R}^n \) in real time \( t \)". The conventional solution to such an inverse-kinematic problem is the pseudo-inverse-type formulation, i.e., one minimum-norm particular solution plus a homogeneous solution. Moreover, the research of recent fifteen years [2], [4] show that this problem could be solved in a more favorable manner via online optimization techniques [specifically, the quadratic programming (QP) technique].

Self-motion is one of significant characteristics of redundant manipulators [3], [5]–[8]. That is, keeping the end-effector at a certain position/orientation [4], with \( r(t) = 0 \in \mathbb{R}^m \), the manipulator could adjust its configuration in joint space from one state [e.g., an initial joint state, \( \theta(0) \in \mathbb{R}^n \)] to another state [e.g., a desired joint state, \( \theta_d \in \mathbb{R}^n \)]. By performing self-motion, the mechanism could obtain more maneuverability and achieve different secondary tasks, such as, avoiding joint limits, singularities and obstacles, and achieving cyclicity [3], [5]–[10]. Therefore, self-motion planning is evidently very essential in redundant robot control, and it has significant physical and biological meanings [3], [5]–[10].

During a long-time extensive study, much effort has been devoted to the development of self-motion planning [3], [5]–[8]. Various computational methods and algorithms have been proposed for such a topic in robotic applications, such as, a reduction-type solution [5], a discontinuous switching control scheme [6], and a bi-directional self-motion path-planning scheme [7]. However, among those techniques, the physical constraints such as joint limits and joint velocity limits are usually not taken into consideration, which may lead to the saturation and infeasibility of the redundancy solution. As a result, the tracking error may increase considerably, not to mention the physical damage possibly caused when a commanded joint or joint velocity hits its physical bound [4], [8]. In addition, the illustrative simulation of those schemes are almost all based on three-link planar robot arms. To higher DOF multi-link planar robot arms, the applicability of these algorithms still remains to be studied intensively.

In this paper, in order to overcome the deficiencies of the above schemes, a self-motion-planning (SMP) scheme is presented for redundant manipulators, which takes both joint limits and joint-velocity limits into consideration. Such a scheme is reformulated in the form of quadratic program (QP). In view of the advantages of neural networks, we apply the primal-dual neural network based on linear variational inequalities (LVI-PDNN) [8], [11] to solving online such a QP scheme. Computer simulations based on multi-link planar manipulators (i.e., five-link and seven-link robot arms) are carried out to substantiate the efficacy of the presented SMP scheme. In addition, two different methods are presented to analyze theoretically the effectiveness and validity of the SMP performance index.

II. PROBLEM FORMULATION

The forward kinematics, which is a relation from joint variable vector \( \theta(t) \in \mathbb{R}^n \) to end-effector position-and-orientation vector \( r(t) \in \mathbb{R}^m \), could be formulated as follows:

\[
r = f(\theta),
\]

(1)
where \( f(\cdot) \) is a continuous nonlinear mapping with a known structure and parameters for a given manipulator. Due to the nonlinearity of \( f(\cdot) \), the inverse-kinematic problem is difficult to solve directly via the inverse mapping of forward-kinematic equation (1). Such a redundancy-resolution problem is thus usually considered at the joint-velocity level by differentiating (1):

\[
J(\theta)\dot{\theta} = \dot{r},
\]

where \( J(\theta) \) is the Jacobian matrix which is defined as \( J(\theta) = \partial f(\theta)/\partial \theta \). For redundant manipulators (i.e., \( n > m \)), equations (1) and (2) are generally both under-determined and thus admit an infinite number of feasible solutions. This, however, could be used to avoid obstacles, joint physical limits, singularity points, as well as to perform self-motion.

In order to facilitate follow-up discussions, the concept of maneuverability (or termed, manipulability) should be introduced here. The maneuverability measure at state \( \theta \) with respect to manipulation vector \( r \) could be defined as [9]:

\[
M(\theta) = \sqrt{\det(J(\theta)J^T(\theta))}, \tag{3}
\]

where “det” denotes the determinant of a matrix, and the superscript \( T \) denotes the matrix or vector transpose. With \( M(\theta) \) defined as a performance criterion to be optimized, it would be helpful for detailed evaluation of the manipulation ability of robotic mechanisms. For instance, \( M(\theta) \) could give an overall measure of the directional uniformity of the ellipsoid and the upper bound of the velocity with which the end-effector could be moved in any directions [9]. Therefore, it could be used as an important reference factor on the configurations selection of redundant manipulators. However, it is worth pointing out that the derivative of \( M(\theta) \) (i.e., \( \partial M(\theta)/\partial \theta \)) is difficult to derive, and it is thus not directly exploited in this paper as a performance index to be explicitly optimized.

A. Quadratic-Programming Formulation

In this paper, for self-motion-planning purposes, the following minimization of quadratic performance index is presented to achieve a suitable target-state from any initial state \( \theta(0) \in \mathbb{R}^n \): \( (\dot{\theta} + c)^T(\dot{\theta} + c)/2 \) with \( c = \lambda(\theta - \theta_m) \), where \( \theta_m \) is the middle-configuration joint-vector (or termed as the desired joint configuration), and \( \lambda > 0 \) is a positive design-parameter used to scale the magnitude of the manipulator response to such joint-drifts. Because almost all manipulators are physically constrained by their joint limits and joint velocity limits, it is more realistic and useful to consider the avoidance of joint physical limits. Hence, we could have the following scheme-formulation for self-motion planning of redundant robot manipulators:

\[
\text{minimize} \quad (\dot{\theta} + c)^T(\dot{\theta} + c)/2 \quad \text{with} \quad c = \lambda(\theta - \theta_m), \tag{4}
\]

subject to

\[
J(\theta)\dot{\theta} = 0, \quad \tag{5}
\]

\[
\theta^- \leq \dot{\theta} \leq \theta^+, \quad \tag{6}
\]

\[
\theta^- \leq \theta \leq \theta^+. \quad \tag{7}
\]

In bound constraints (6) and (7), superscripts \( ^+ \) and \( ^- \) denote respectively the upper and lower limits of the joint variable vector (e.g., joint vector \( \theta \) and joint velocity vector \( \dot{\theta} \)).

It is worth mentioning that the above SMP scheme (4)-(7) for physically-constrained robot arms could finally be reformulated as the quadratic program below [4], [8], [11]:

\[
\text{minimize} \quad \frac{1}{2} \dot{\theta}^T W \dot{\theta} + c^T \dot{\theta}, \quad \tag{8}
\]

subject to

\[
J \dot{\theta} = b, \quad \tag{9}
\]

\[
\xi^- \leq \dot{\theta} \leq \xi^+, \quad \tag{10}
\]

where coefficients \( W := I, \quad b := 0, \quad c = \lambda(\theta - \theta_m), \) and the \( i \)th elements of new bounds \( \xi^- \) and \( \xi^+ \) can be defined respectively as \( \max\{\dot{\theta}^-i, \mu(\theta^-i - \theta_i)\} \) and \( \min\{\dot{\theta}^+i, \mu(\theta^+i - \theta_i)\} \), with intensity coefficient \( \mu > 0 \) (e.g., \( \mu = 20 \) in the ensuing computer simulation) used to adjust the feasible region of \( \dot{\theta} \). Note that the above quadratic program (8)-(10) could be solved readily by using MATLAB routine “QUADPROG”, but preferably by using neural networks because of their parallel and iteratively processing nature as well as the convenience of hardware realization. In this paper, a primal-dual neural network based on linear variational inequalities [8], [11] is exploited in the computer simulation.

B. Desired Configuration Selection

For self-motion planning, the problem of selecting the desired (or to say, target) configuration (which is to be achieved after the self-motion) deserves wide attention and discussion. For instance, the joint-angle-drift phenomenon [11], [12] may occur when the manipulators are applied to industrial applications and cyclic tasks. In this case, the manipulators could readjust themselves from the final configuration to the initial configuration to resume working through self motion. So, the initial configuration before cyclic motion could be designed as the target configuration for self motion, while the current configuration after the end-effector performing a periodical motion could be deemed as the initial configuration of the self-motion.

In order to avoid joint limits for 6-DOF and 7-DOF redundant manipulators, Refs. [13] and [14] present as follows two different objective functions to be minimized:

Ref. [13]:

\[
H(\theta) = \frac{1}{6} \sum_{i=1}^{6} \frac{(\theta_i - \theta_{m(i)})^2}{(\theta_{m(i)} - \theta_i)^2}, \quad \tag{11}
\]

\[
\theta_{m(i)} = \frac{1}{2}(\theta^+i + \theta^-i), \quad \tag{12}
\]

Ref. [14]:

\[
H(\theta) = \sum_{i=1}^{7} \frac{(\theta^-i - \theta_i)(\theta_i - \theta^+i)}{(\theta^+i - \theta_i)(\theta_i - \theta^-i)}, \quad \tag{13}
\]

where \( \theta_i \) is the \( i \)th joint angle, \( \theta^+i \) and \( \theta^-i \) are the upper and lower limits on the joint angle \( \theta_i \), respectively. From the above two objective functions, we can see that the minimum values of \( H(\theta) \) could both be obtained at the median values of each joint physical limit [i.e., \( \theta_{m(i)} = (\theta^+i + \theta^-i)/2 \)]. By generalizing this idea to the desired configuration selection of our self-motion planning situation, we could assume the joint median values [i.e., \( \theta_m = (\theta^+ + \theta^-)/2 \)] as the desirable configuration for the SMP scheme (4)-(7).
In this section, the presented physically-constrained QP-based redundancy-resolution scheme (4)-(7) is applied to the self-motion planning of two multi-link planar robot arms, i.e., five-link and seven-link robot arms. The LVI-based primal-dual neural network [8], [11] is employed for the online solution of the resultant QP problem (8)-(10). Without loss of generality, we only consider the positioning of the manipulators’ end-effectors. For illustrative purposes, several different simulation tests are carried out to validate the applicability and efficiency of the presented SMP scheme. In these simulations, the parameters $\lambda = 4$, $\mu = 20$ and the SMP-task duration $T = 3s$.

A. Five-Link Planar Robot Arm

The five-link planar robot arm has five DOF with respect to the two-dimensional workspace, and thus has three degrees of redundancy. Its joint physical limits are set as: $\theta^+ = -\theta^- = [\pi/4, \pi/4, -\pi/2, \pi/2, -\pi/2]^T$ in radians and $\dot{\theta}^+ = -\dot{\theta}^- = [\pi, \pi, \pi, \pi, \pi]^T$ in radians per second. The desirable configuration is set as $\theta_m = (\theta^+ + \theta^-)/2 = [0, 0, 0, 0, 0]^T$ (i.e., the median joint vector) in the simulation. The initial configuration is firstly set as $\theta(0) = [\pi/4, \pi/4, -\pi/2, \pi/2, -\pi/2]^T$. The computer-simulation results are in Fig. 1 through 3.

Fig. 1 illustrates the planar motion trajectories and the end-effector positioning errors of the five-link planar robot arm when performing self-motion. We can see that the manipulator successfully realizes the self-motion with the end-effector remaining at the same position. The positioning errors of the end-effector appear to be tiny enough (less than $4.5 \times 10^{-6}$m) and to be acceptable in practice.

Fig. 2 shows the joint-angle and joint-velocity profiles of the five-link planar robot arm when performing self-motion. As seen from Fig. 2, all joint-angle variables and joint velocity variables are evidently approaching the median configuration $\theta_m$. We could also observe that the final configuration of the manipulator after self-motion is $\theta_1$ rather than $\theta_m$ (see “Test 1” in Table I as well). This is because $f(\theta(0)) \neq f(\theta_m)$, which does not satisfy the requirement of self-motion planning. Instead, we have $f(\theta(0)) = f(\theta_1)$, where the self-motion final configuration $\theta_1$ is the nearest configuration to the desirable configuration $\theta_m$, in the two-norm sense. This would also be shown in Fig. 3 and analyzed in Section IV. In addition, all joint profiles in the figure are
kept within their limited ranges, because the joint physical limits have already been taken into consideration.

Fig. 3 presents the curves of maneuverability $M(\theta)$ and nearness $||\theta(t) - \theta_m||^2_2$. We can see that the value of $M(\theta)$ increases to a larger one after self-motion, which means the manipulator could achieve a better maneuverability by readjusting its configuration. Meanwhile, the curve of the nearness $||\theta(t) - \theta_m||^2_2$ decreases to its minimum value during the process of self-motion. That is, the final configuration $\theta_f$ is the nearest one to the desirable configuration $\theta_m$. This is also an inherent requirement of the presented quadratic performance index for the self-motion planning.

Based on the successful simulation above, the presented SMP scheme is simulated with different initial configurations of the five-link robot. For illustration and better readability, the initial configurations and simulated final configurations are shown in Table I. Obviously, corresponding to different initial configuration $\theta(0)$, different final configurations $\theta_f$ could be obtained after the self-motion. This is the reason why we emphasize “no target-configuration” in the title. Similarly, the value of manipulator maneuverabilities increases and the nearness $||\theta(t) - \theta_m||^2_2$ decreases during self-motion, which also demonstrates the efficacy of the presented SMP scheme for redundant manipulators.

**B. Seven-Link Planar Robot Arm**

In this simulation-test, the presented SMP scheme (4)-(7) is applied to a seven-link planar robot arm, which has seven DOF while operating in the two-dimensional plane, thus with five degrees of redundancy. The desirable configuration is assumed as the median joint vector, $\theta_m = (\theta^+ + \theta^-)/2 = [0,0,0,0,0,0,0]^T$. Figs. 4 through 6 show the computer-simulation results.

From Fig. 4, we can see that the robot arm carries out the self-motion successfully. The maximal end-effector positioning errors during the self-motion process is less than $7 \times 10^{-6}$m. As seen from Fig. 5, all joint-angle variables and joint-velocity variables are evidently also approaching the desirable configuration $\theta_m$, and are kept within their limited ranges throughout the SMP task. Fig. 6 illustrates that the value of $M(\theta)$ is increasing, while the curve of the nearness $||\theta(t) - \theta_m||^2_2$ is decreasing. This shows that the manipulator achieves a better configuration and maneuverability via the presented SMP scheme (4)-(7). In addition, different initial configurations and their corresponding final configurations after self-motion are listed in Table II. This shows again the meaning and efficacy of the self-motion planning with no target-configuration assigned for planar robot arms.

In summary, the above simulations based on different planar robot arms demonstrate the efficacy of the presented QP-based SMP scheme for self-motion planning with no target-configuration assigned for multi-link planar robot arms.

**IV. SMP Performance-Index Analysis**

As the previous section illustrates the effectiveness of our proposed SMP scheme applied to multi-link planar robot arms, in this section we present two kinds of analysis methods so as to investigate the reason why the SMP performance
In this paper, a QP-based redundancy resolution scheme has been proposed and presented for the self-motion planning of redundant manipulators. It follows that expanding the position-change function $E(t) = \theta(t) - \theta_m$, we could define alternatively a vector-value performance index $\dot{E} = -\lambda E(t)$, which is exactly the same as the quadratic SMP performance index $\dot{\theta}^T \dot{\theta}/2 + c^T \dot{\theta}$ employed in the self-motion planning of redundant robot manipulators.

V. Conclusions

In this paper, a QP-based redundancy resolution scheme has been proposed and presented for the self-motion planning...
with no target-configuration assigned for robot manipulators. The desired configuration of the self-motion could be achieved with better manipulability. The joint physical limits are also taken into consideration in the SMP scheme formulation. Computer-simulation results based on two different multi-link planar robot arms have further demonstrated the efficacy of the presented self-motion planning scheme on the real-time kinematic control of joint-constrained redundant manipulators. In addition, two different design methods are employed for the theoretical analysis of the quadratic SMP performance index, which also substantiates well the efficacy of the presented SMP scheme on redundant robots.

REFERENCES


